Logic Seminar: Proof Interpretations II

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We will continue our study on Modified realizability, and we will present a few important results about it:

1. *Soundness theorem of modified realizability*, which allows us to obtain (provable in $\mathbf{E}-\mathbf{HA}^\omega$) realizers for a formula A, provided we can prove A in $\mathbf{E}-\mathbf{HA}^\omega+\mathrm{AC}+\mathrm{IP}^\omega_{\mathrm{ef}}$. (Here AC is the axiom of choice principle, and $\mathrm{IP}^\omega_{\mathrm{ef}}$ is the independence of premise principle for $\exists$-free formulas).

2. *Characterization theorem for modified realizability.*

3. *Main theorem on program extraction by modified realizability*, which, in $\mathbf{E}-\mathbf{HA}^\omega+\mathrm{AC}+\mathrm{IP}^\omega_{\mathrm{ef}}$, from a proof of $\forall x\exists y A(x,y)$, allows us to extract a term t such that we can prove $\forall x A(x,tx)$, for any formula A in $\mathcal{L}(\mathbf{E}-\mathbf{HA}^\omega)$.

We will finish with some applications of modified realizability.