Logic Seminar: Proof Interpretations III

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In this seminar we will present the *monotone modified realizability*, in which we extend the results from modified realizability studied in the previous seminar to the case where large classes of non-effective sentences are added to the theories we are working with, without having an impact on the extractability and complexity of uniform bounds. Similarly to the monotone realizability, we will show

1. *Soundness theorem of* monotone *modified realizability*, which allows us to obtain (provable in $\mathbf{E}-\mathbf{HA}^\omega+\Theta$) *monotone* realizers for a formula $A(\underline{a})$, provided we can prove $A(\underline{a})$ in $\mathbf{E}-\mathbf{HA}^\omega+\mathrm{AC}+\mathrm{IP}^\omega_{\mathrm{ef}}+\Theta$ (Here Θ is a set of sentences of the form $\exists\underline{v}\leq_{\underline{\sigma}}\underline{r}B_{\mathrm{ef}}(\underline{v})$).

2. Some consequences of the *schema of comprehension in all types for arbitrary negated formulas*,
$$\mathrm{CA}^\omega_\neg :\equiv \exists\Phi^{0(\underline{\sigma})}\forall\underline{x}^{\underline{\sigma}}(\Phi(\underline{x})=_0 0\leftrightarrow\neg A(\underline{x})),$$
where A is an arbitrary formula not containing Φ free and $\underline{\sigma}$ an arbitrary tuple of types.

3. *Main theorem on program extraction by* monotone *modified realizability*, which, in $\mathbf{E}-\mathbf{HA}^\omega+\mathrm{AC}+\mathrm{IP}^\omega_\neg+\Omega$, from a proof of $\forall x^1\forall y\leq_\rho sx\exists z^\tau A(x,y,z)$, allows us to extract a term t such that we can prove $\forall x^1\forall y\leq_\rho sx\exists z^\tau A(x,y,tx)$, for any formula $A(x,y,z)$ in $\mathcal{L}(\mathbf{E}-\mathbf{HA}^\omega)$ (Here Ω is a set of sentences of the type $\forall\underline{u}^{\underline{\delta}}(C(\underline{u})\to\exists\underline{v}\leq_{\underline{\sigma}}\underline{r}\,\underline{u}\neg B(\underline{u},\underline{v}))$, where C and B are arbitrary formulas of $\mathbf{E}-\mathbf{HA}^\omega$, and ρ, τ, $\underline{\sigma}$, $\underline{\delta}$ are arbitrary (tuples of) types.)