Graded modules over polynomial rings with applications to free divisors (Modules gradués sur anneaux de polynômes avec applications aux diviseurs libres)

Scientific Program / Programme Scientifique

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Courbes rationnelles et courbes libres

Exposé 1: courbes rationnelles
- syzygies associées à une paramétrisation;
- lien avec les singularités de la courbe, résultant d'Abhyankar/Tessier;
- équations de l'algèbre de Rees associée (au passage);
- courbes rationnelles cuspidales;

Exposé 2: courbes rationnelles libres
- courbes libres;
- invariants algébriques associés;
- critère de liberté et presque-liberté à partir d'une paramétrisation.

Référence: L. Busé, A. Dimca, G. Sticlaru: *Freeness and invariants of rational plane curves,* arXiv:1804.06194.

Alexandru Dimca
Université Côte-d'Azur

Plane curves and Jacobian syzygies

We consider reduced curves $C : f = 0$ in the complex projective plane. First we discuss upper bounds on the number and degrees of a set of generators for the module of Jacobian syzygies. Then we discuss the extreme cases: either very small number of generators (which corresponds to free curves, nearly free curves, plus-one generated curves), or maximal number of generators (which corresponds to curves having large global Tjurina numbers).

References:

1. A. Dimca, G. Sticlaru: *Plane curves with three syzygies, plus-one generated curves, and nearly cuspidal curves*, arXiv: 1810.11766.

2. A. Dimca, G. Sticlaru, *Jacobian syzygies and plane curves with maximal global Tjurina numbers*, arXiv: 1901.05915.

Mihai Fulger
University of Connecticut

Local and global methods in the study of Hilbert functions

We survey techniques employed in the study of Hilbert functions associated to linear series on projective varieties. The main characters are Seshadri constants and Newton–Okounkov bodies. They are both local-to-global invariants. Seshadri constants measure positivity around a point, and in a sense determine freeness, and more generally higher jet separation at the point. Newton–Okounkov bodies are refined invariants of graded families of linear series that encode both volume (a global invariant) and the Seshadri constants.

Michel Granger
Université d'Angers

Diviseurs libres

Exposé 1: Notion de diviseur libre. Dualité entre 1-formes et champs logarithmiques. Une liste d'exemples clé. Diverse caractérisations équivalentes :

- par une formule de type résidu;
- par le critère déterminantiel de Saito;
- par une propriété de Cohen-Macaulay du lieu singulier.

Cas homogène et lien avec la définition de liberté pour les hypersurfaces projectives. Discussion de deux problèmes typiques sur les diviseurs libres:

i) Singularités ayant la propriété de comparaison logarithmique.

ii) Caractérisation de certaines singularités par leurs modules de résidus logarithmiques.

Exposé 2: Diviseurs linéairement libres. Exemples et caractérisation comme discriminants de certains espaces préhomogènes. Cas réductif en lien avec la propriété de comparaison logarithmique. Le cas particulier des espaces de représentation d'un carquois. Discussion du cas des carquois de type de représentation finie ou tame. Diviseurs libres et systèmes de racine de certaines algèbres de Lie.

Anca Măcinic
Simion Stoilow Institute of Mathematics of the Romanian Academy

Freeness and variations

I will survey some new *freeness-adjacent* properties, such as near freeness and next to freeness, with particular focus on line arrangements. In this context we will take a look at how the weak combinatorics influences or is influenced by the freeness and freeness-type properties.

Jean Vallès
Université de Pau et des Pays de l'Adour

Triangular arrangements

First of all, I will present new techniques developed with Daniele Faenzi (Bourgogne University) and based on projective duality to study Hyperplane Arrangements. Thanks to these techniques we recover many results, some of them are well known and some of them are new.

In a second part of this talk, I will present a new work (written in collaboration with Simone Marchesi from Campinas University) concerning line arrangements formed by lines passing through three non aligned points, that we call Triangular Arrangements. Fixing the number of lines through each vertex, we describe a combinatorics for all possible exponents of free triangular arrangements. Terao's conjecture says that: if two arrangements have the same combinatorics and one is free then the other is also free. This conjecture is still open after 30 years. A weak version concerns the weak combinatorics, that is the data of the numbers t_i of points with multiplicity $i \geq 2$. We disprove this weak version by exhibiting two triangular arrangements, having the same weak combinatorics (that means the same numbers t_i of points with multiplicity $i \geq 2$) but such that one is free, when the other is not.

Marius Vlădoiu
Bucharest University

Minimal graded free resolutions

In these lectures we review the classical results on the (short) minimal graded free resolution of a finitely generated graded module, its computational aspects, and the information it provides on the module.

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Claudia Andrei-Ciobanu
Bucharest University

Powers of t-spread principal Borel ideals

We prove that t-spread principal Borel ideals are sequentially Cohen-Macaulay and study their powers. We show that these ideals possess the strong persistence property and compute their limit depth. This is a joint work with V. Ene and B. Lajmiri.

Filip Chindea
Simion Stoilow Institute of Mathematics of the Romanian Academy

Stability conditions on K3 surfaces

Working in the setting of projective complex geometry, we start with a short review of derived categories and Fourier-Mukai transforms. After introducing μ-stability, torsion theories for coherent sheaves and Grothendieck groups, we give a short exposition of stability conditions on abelian categories, in particular the derived bounded categories of curves, surfaces and K3 surfaces.

Mircea Cimpoeaş
Simion Stoilow Institute of Mathematics of the Romanian Academy

Polarization and spreading of monomial ideals

We characterize the monomial ideals $I \subset K[x_1, \ldots, x_n]$ with the property that the polarization I^p and $I^{\sigma^n} :=$ the ideal obtained from I by the n-th iterated squarefree operator σ are isomorphic via a permutation of variables. We give several methods to construct such ideals. We also compare the depth and sdepth of I and I^{σ^n}, using their associated LCM-lattices.

Rodica Dinu
Bucharest University

Gorenstein t-spread Veronese algebras

We characterize the t-spread Veronese algebras which are Gorenstein.

Irina Ilioaea
Georgia State University

On the Frobenius Complexity Sequence of Stanley-Reisner Rings

The Frobenius complexity of a local ring R measures asymptotically the abundance of Frobenius actions of order e on the injective hull of the residue field of R. It is known that, for Stanley-Reisner rings, the Frobenius complexity is either $-\infty$ or 0. However, a complete description of the Frobenius complexity sequence $c_e(R)$ for all values of e is not yet known when R is Stanley-Reisner. We will provide the answer for a large class of Stanley-Reisner rings, generalizing work of Alvarez Montaner, Boix and Zarzuela.