

**Journée Mathématique Régionales: Third Romanian-Turkish  
Mathematics Colloquium**

Organized by: Grigore Moisil Romanian–Turkish Joint Laboratory of Mathematical Research from Faculty of Mathematics and Informatics, Ovidius University, Constanta, General Consulate of the Republic of Turkey in Constanta, Romania, Galatasaray University, Istanbul, Turkey, Simion Stoilow Institute of Mathematics of the Romanian Academy

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**Journée Mathématique Régionales  
Third Romanian-Turkish Mathematics  
Colloquium  
September 18 - 22, 2019  
Constanta, Romania**

**ABSTRACTS**

*Algebraic Structure of Generalized Splines*

**Selma ALTINOK**

Hacettepe University, Turkey

In this presentation, we talk about generalized splines on an edge labeled graph  $G$  whose edges are labeled by the ideals of a base ring  $R$ . Given an edge labeled graph, a generalized spline is a labeling of each vertex by an element of  $R$  so that adjacent vertices differ by an element of the ideal of  $R$  associated to the edge. It is easy to see that the set of generalized splines has a ring and an  $R$ -module structure. We focus on some algebraic properties of the generalized spline modules such as the freeness by using combinatorial and linear algebraic techniques and finding bases for these modules.

*Rational Point on Curves over Finite Fields*

**Alp BASSA**

Boğaziçi University, Turkey

In this talk I will be interested in rational points on curves over finite fields. Although the Riemann Hypothesis for the classical Riemann zeta function is an open question for more than 150 years, its analogue for the zeta function of curves over finite fields has been proved by Hasse and Weil in the first half of the 20th century.

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It is directly related with the number of rational points on these curves, which will be the focus of this talk. Hence I will start by introducing the Riemann Hypothesis (Hasse–Weil Theorem) for curves over finite fields and some of its consequence. The theory can naturally be divided into two regimes depending on whether one considers curves of small or large genus. I will focus on the latter and study the asymptotics with large genus, in particular the maximal possible number of rational points on such curves. I will finish with some recent results of myself with Beelen–Garcia–Stichtenoth and with Ritzenthaler giving strong lower bounds over (almost) all finite fields.

### *Algebraic methods for solutions of Bloch-Iserles Hamiltonian systems*

**Vasile BRÎNZĂNESCU**

Simion Stoilow Institute of Mathematics of the Romanian Academy, Romania

We study this Hamiltonian system by establishing its algebraic complete integrability in the case of distinct eigenvalues of the spectral matrix.

This is a joint work with Cristina Maria Sandu.

### *Fuzzy Analytical Hierarchy Process In Theory of Insurance Premiums*

**Sezen CIORABAI**

Ovidius University, Romania

The calculation of the premium is one of the main objectives of study for actuaries. The insurance premium should reflect both expected claims and certain loadings, as all theoreticians in the field approve. The certain loadings depend a lot on the insurer's ability to estimate expected claims (including costs) and on the selection of a fair risk loading. Therefore a complex decision should be taken in order to thank both the insurance company and the insured.

The Analytic Hierarchy Process (AHP) is a powerful device for managing such complex choice and may aid the decision maker to set priorities and make the best choice from a portfolio. By lessening complex decisions to a series of pairwise comparisons through policies of an reliable insurance policy, and then synthesizing the results, the AHP helps to capture both subjective and objective aspects of a decision. Moreover, the AHP joins a valuable procedure for checking the consistency of the decision maker's evaluations, thus reducing the bias in the decision making process. Fuzzy logic is acquainted with dispose of the objectiveness of the decision.

Key-words: insurance premium, ahp, fuzzy logic, decision making.

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*A new view on  $D(-1)$ -sets*

**Mihai CIPU**

Simion Stoilow Institute of Mathematics of the Romanian Academy, Romania

For a fixed integer  $n$ , a  $D(n)$ -set consists of integers with the property that by adding  $n$  to the product of any two distinct elements one obtains a perfect square.  $D(-1)$ -sets have many peculiar properties that distinguish them from other  $D(n)$ -sets. The aim of the talk is to present an innovative approach to the study of the conjecture that there is no  $D(-1)$ -set with four elements. Most of the results reported here are obtained in collaboration with N. C. Bonciocat (IMAR Bucharest) and M. Mignotte (Univ. of Strasbourg).

*On the cover ideals of chordal graphs*

**Nursel EREY**

Gebze Technical University, Turkey

Given a graph  $G$ , the vertex cover ideal of  $G$  is generated by the monomials that correspond to the vertex covers of  $G$ . It is known that when  $G$  is a chordal graph, its independence complex has shellable property. In this talk, we consider such property and discuss its applications for vertex cover ideals of chordal graphs.

*Differential Forms On Stratified Spaces*

**Serap GÜRER**

Galatasaray University, Turkey

First, we extend the notion of stratified spaces to diffeology. Then we characterise the subspace of stratified differential forms, or zero-perverse forms in the sense of Goresky-MacPherson, which can be extended smoothly into differential forms on the whole space. For that we introduce an index which outlines the behaviour of the perverse forms on the neighbourhood of the singular strata.

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*Commutator and stable commutator lengths in groups*

**Mustafa KORKMAZ**

Middle East Technical University, Turkey

This talk is aimed to general audience. For an element  $x$  in the commutator subgroup of a group, the commutator length of  $x$  is defined to be the minimal number of commutators needed to express  $x$  as a product of commutators. The stable commutator length of  $x$  measures how the commutator length of the power  $x^n$  grows compared to  $n$ .

The mapping class group of a closed oriented surface is the group of isotopy classes of orientation–preserving diffeomorphisms of the surface. The algebraic properties, in particular the commutator lengths of elements, of the mapping class group is of interest in the theory of the topology of low–dimensional manifolds.

In this talk, we will first discuss the commutator length and the stable commutator length functions on arbitrary groups. Then we turn our attention to the free groups and to mapping class groups, in particular the lengths of Dehn twists.

*Capparelli's Identities and the Kanade-Russell Conjectures*

**Kağan KURŞUNGÖZ**

Sabancı University, Turkey

We will review the basics of integer partitions, then we will give a very rough, somewhat subjective, classification of partition identities. We will impress on the impact of Capparelli's identities on integer partition theory, and talk about Kanade-Russell conjectures as time allows.

*Graded semisimple algebras are graded symmetric*

**Laura-Elena NĂSTĂSESCU**

Institute of Mathematics Simion Stoilow of the Romanian Academy, Romania

We study graded symmetric algebras, which are the symmetric monoids in the monoidal category of vector spaces graded by a group. We show that a finite dimensional graded semisimple algebra is graded symmetric.

This is a joint work with Sorin Dăscălescu and Constantin Năstăsescu.

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### *Gosset Polytopes of $(-2)$ -curves in the Picard Groups of Degenerate del Pezzo Surfaces*

**Ovidiu PĂSĂRESCU**

Simion Stoilow Institute of Mathematics of the Romanian Academy, Romania

The Gosset polytope  $4_{21}$  from the 8-dimensional real geometry is a vertex-transitive (i.e. all its vertices are equivalent for the symmetric transformations of the figure) which is constructed by some (uniform) 7-dimensional polytope faces, which is not regular, having 240 vertices. Jae-Hiouk Lee and JongJo Shin constructed such polytopes in the Picard group of smooth del Pezzo surfaces and blow ups of Hirzebruch surfaces having lines (seen as points in the Picard group) as vertices. However, the notion is connected to the symmetries of the group  $E_8$  and its roots.

We construct a  $4_{21}$  polytope on degenerate del Pezzo of degree 1 surfaces having as edges  $(-2)$ -curves instead of lines (in the Picard group of the desingularization of the surface). We also consider the analogous polytopes  $3_{21}$  and  $2_{21}$  from 7 and 6 dimensions, using degenerate del Pezzo surfaces of degrees 2 and 3, respectively. We mention that, in physics, in the new approach for Quantum Gravitation (named The Emergent Universe), combining information, energy and consciousness, one consider that the 4–dimensional space-time space is "pixelated" by small tetrahedrons, representing projections from dimensions of a lattice of polytops of type  $4_{21}$  (Herman Velinde, Klee Irwin and others).

### *Average + Learning in financial markets*

**Ionel POPESCU**

Simion Stoilow Institute of Mathematics of the Romanian Academy, Romania

One of the problems in financial market is to understand how trading works and why the market comes to a consensus.

We introduce several models for trading in the financial markets which is inspired by how traders think and act. We will discuss several examples of convergence to consensus. This is joint work with Tushar Vaidya.

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*The Shuffle Variant of a Diophantine equation of Miyazaki and Togbé*

**Gökhan SOYDAN**

Department of Mathematics, Bursa Uludağ University, Turkey

Suppose that  $a$  and  $b$  are odd positive integers. In 2012, T. Miyazaki and A. Togbé, [3], gave all of the solutions of the Diophantine equations  $(2am - 1)^x + (2m)^y = (2am + 1)^z$  and  $b^x + 2^y = (b + 2)^z$  with  $a > 1$  and  $b \geq 5$  in positive integers. In this work, we propose a similar problem (which we call the shuffle variant of a Diophantine equation of Miyazaki and Togbé). Here we first prove that the Diophantine equation  $(2am + 1)^x + (2m)^y = (2am - 1)^z$  has only the solution  $(a, m, x, y, z) = (2, 1, 2, 1, 3)$  with any fixed positive integer  $a > 1$  in positive integers. Then using this result, we show that the Diophantine equation  $b^x + 2^y = (b - 2)^z$  has no solutions with any fixed odd positive integer  $b \geq 7$  in positive integers. On the proofs, we use elementary methods and Laurent's refinement of Baker's theorem.

This is joint work with Elif Kızıldere, Qing Han and Pingzhi Yuan. Elif Kızıldere and Gökhan Soydan were supported by TÜBİTAK (the Scientific and Technological Research Council of Turkey) under Project No: 117F287.

**Keywords and phrases:** Exponential Diophantine equation, Baker's method.

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*The outer automorphism of  $PGL(2, \mathbb{Z})$  and a transcendence conjecture*

**A. Muhammed ULUDAĞ**

Galatasaray University, Turkey

I will discuss the involution of the real line induced by the outer automorphism of  $PGL(2, \mathbb{Z})$  and the related comumerator function. This involution conjecturally sends the algebraic numbers of degree at least three to transcendental numbers. I will also discuss a strong version of this transcendence conjecture.

*Design of nematic bulk potentials*

**Arghir ZĂRNESCU**

Simion Stoilow Institute of Mathematics of the Romanian Academy and Basque Center for Applied Mathematics, Spain

We consider a Landau-de Gennes model for a suspension of small colloidal inclusions in a nematic host. We impose suitable anchoring conditions at the boundary of the inclusions, and we work in the dilute regime - i.e., the size of the inclusions is much smaller than the typical separation distance between them, so that the total volume occupied by the inclusions is small. By studying the homogenised limit, and proving rigorous convergence results for local minimisers, we compute the effective free energy for the doped material. In particular, we show that not only the phase transition temperature, but any coefficient of the quartic Landau-de Gennes bulk potential can be tuned, by suitably choosing the surface anchoring energy density.

This is a joint work with Giacomo Canevari.