Atelier de travail en Stochastique et EDP

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RÉSUMÉS

Optimal control for SDEs with feedback inputs and related Kolmogorov equations

Contrôle optimal en feedback pour des EDS et équations de Kolmogorov associées

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Our talk concerns the optimal control of some stochastic differential equations with feedback inputs. The problems are reduced to deterministic optimal control problems for some related Kolmogorov equations where the controls are of open-loop type. The existence of an optimal control is proved for the deterministic control problem in a particular case and necessary optimality conditions are derived. Some comments and further extensions are discussed.

The presentation is based on the results in [Aniţa, Ş.-L., *International Journal of Control* (2020), A stochastic optimal control problem with feedback inputs],
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Construction of Boltzmann type flows: the sewing lemma argument

Construction des flots de Boltzmann en employant le "sewing lemma"

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We are concerned with Boltzmann type equations, this means (in probabilistic terms) equations driven by a Poisson point measures with the intensity depending on the law of the solution itself. Both the analytical Boltzmann equation and the probabilistic interpretation initiated by Tanaka have intensively been discussed in the literature for specific models

related to the behaviour of gas molecules. In this paper we first consider general abstract coefficients and then we discuss the link with specific models as well. In contrast with the usual approach in which integral equations are used in order to state the problem, we employ here a new formulation of the problem in terms of áows of endomorphisms on the space of probability measure endowed with the Wasserstein distance. This point of view already appeared in the framework of rough differential equations. Our results concern existence and uniqueness of the solution, in the formulation of flows, but we also prove that the "flow solution" is a solution of the classical integral weak equation and we obtain an uniqueness result (both for the homogenous and in homogeneous classical Bolzmann equation) which is a slight generalisation of the already known results. Moreover we obtain stability results and regularity with respect to the time for such solutions. Finally we prove the convergence of empirical measures based on particle systems to the solution of our problem, and we obtain the rate of convergence. We discuss as examples the homogeneous and the in-homogeneous Boltzmann (Enskog) equation with hard potentials.

On traite des équations de Boltzmann; en termes probabilistes, il s'agit d'équations à sauts, portés par une mesure de Poisson ponctuelle dont l'intensité dépend de la solution elle même. Aussi bien l'équation analytique que sa version probabiliste (introduite par Tanaka) ont été abondamment traité dans la littérature, en relation avec des systèmes de particules qui modélisent le comportement d'un gaz. Dans notre travail nous commençons par considerer des coefficients généraux et en deuxième temps nous traitons des coefficients spécifiques qui apparaissent dans l'équation de Boltzmann classique. A la difference de la formulation classique du problème en termes d'équations intégrales nous donnons ici une formulation en termes de flots de diphéomorfismes sur l'espace des mesures de probabilités munis de la distance de Wasserstein. Ce point de vue est inspiré des travaux dans le domain des "rough path". On démontre l'existence et l'unicité de la solution dans le sens des flots et après on démontre que la solution flot résout aussi l'équation analytique. On démontre aussi des résultats de stabilité et de régularité en temps des solutions, ansi qu'un théorème d'approximation basé sur des systèmes de particules. Finalement on discute l'équation de Boltzmann homogène et in-homogène.

h-TRANSFORM OF DOOB AND NONLOCAL BRANCHING PROCESSES

h-transformation de Doob et processus de branchement nonlocal

Ana-Maria Boeangiu (Bucarest, e-mail: ana23mariamaxim@yahoo.com)

We study the h-transform of Doob for nonlocal branching processes, we show that the branching property is preserved provided that h is a coherent state, and we emphasize the

probabilistic representation of the solution to the associated nonlinear evolution equation. The tools are from the analytic and probabilistic potential theory.

The talk is based on a joint work with Lucian Beznea and Oana Lupaşcu-Stamate.

Nous étudions la h-transformation de Doob pour des processus de branchement nonlocal. Nous montrons que la propriété de branchement est préservée si la fonction h est un état cohérent et nous soulignions la représentation probabiliste de la solution de l'équation d'évolution associée. Nous utilisons des méthodes de la théorie analytique et probabiliste du potentiel.

Ces résultats sont issus d'un travail commun avec Lucian Beznea et Oana Lupaşcu-Stamate.

On the construction of Hunt processes with applications to Mehler semigroups

Sur la construction des processus de Hunt et applications aux semi-groupes de Mehler

Iulian Cîmpean (Bucarest, e-mail: iulian.cimpean@imar.ro)

The aim of this talk is to take yet another look at the construction of Hunt processes on general spaces through compact excessive functions, with applications to Mehler semigroups in infinite dimensions. Joint work with Lucian Beznea and Michael Röckner.

Le but de l'exposé est de presenter une autre perspective sur la construction des processus de Hunt ayant l'espace d'états general, en utilisant les fonctions excessives compacts, avec des applications aux semi-groupes de Mehler en dimension infinie. Travail en collaboration avec Lucian Beznea et Michael Röckner.

Conditional propagation of chaos for interacting systems of neurons

Propagation du chaos conditionnelle pour des systèmes de neurones en interactions

Eva Löcherbach (Paris, e-mail: locherbach70@gmail.com)

We consider a system of N interacting neurons. Each neuron emits action potentials (spikes) at a rate depending on the current value of its membrane potential. When spiking, all other neurons in the system receive an additional amount of potential which is random,

centred and of order $1/\sqrt{N}$. In between successive spikes, each neuron's potential follows a deterministic flot (so this is an instance of a so-called Piecewise Deterministic Markov process). We show that as N tends to infinity the system converges to a limit SDE driven by an additional Brownian motion which is created by the CLT. This Brownian motion gives rise to a common noise factor such that in the limit, particles are only conditionally independent - a property which is called "conditional propagation of chaos". We obtain an explicit rate of convergence by using semi-group techniques. In the second part of the talk I will discuss a "variable memory length" version of the above model where each spiking neuron's potential value is reset to a resting value after each spike. Work in collaboration with Xavier Erny and Dasha Loukianova

Travail en collaboration avec Xavier Erny et Dasha Loukianova.

Nous étudions un système de N neurones en interactions. Chaque neurone émet des décharges électriques ("spikes") avec un taux dépendant de son potentiel de membrane. Je discuterai d'abord un premier modèle dans lequel au moment de chaque spike, tous les autres neurones dans le système reçoivent une quantité supplémentaire de potentiel ("poids synaptique"). Cette quantité est aléatoire, centrée et de l'ordre de $1/\sqrt{N}$*. De plus, entre deux spikes successifs, le potentiel de membrane de chaque neurone suit un flot déterministe. Nous démontrons que le système converge, lorsque N tend vers l'infini, vers une EDS limite dirigée par un mouvement Brownien supplémentaire qui est créé par le théorème central limite. Ce mouvement Brownien est sous-jacent à l'évolution de chaque neurone dans le système limite et engendre ainsi un facteur de bruit commun à tous les neurones. Par conséquent, pour le système limite, les différents neurones sont conditionnellement indépendants, sachant le mouvement Brownien - ce qui peutêtre exprimé comme propriété de "propagation du chaos conditionnelle". Nous obtenons un taux de convergence explicite en utilisant des techniques de semi-groupe.*

h-TRANSFORM FOR BOCHNER SUBORDINATE L^p-SEMIGROUPS

h-transformation pour les semigroupes subordonnés au sens de Bochner sur l'espace L^p

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We show that the subordination induced by a convolution semigroup (subordination in the sense of Bochner) of a C_0-semigroup of sub-Markovian operators on an L^p space is actually associated to the subordination of a right (Markov) process. As a consequence, we solve the martingale problem associate with the L^p-infinitesimal generator of the subordinate semigroup. It turns out that an enlargement of the base space is necessary. A main step

in the proof is the preservation under such a subordination of the property of a Markov process to be a Borel right process. We also investigate the h-transform of a subordinate C_0-semigroup of sub-Markovian operators on an L^p space.

The results were obtained jointly with Lucian Beznea and Ana-Maria Boeangiu.

On démontre que la subordination induite par un semi-groupe de convolution (la subordination au sens de Bochner) d'un C_0 -semi-groupe d'opérateurs sous-markoviens sur l'espace L^p est associée à la subordination de processus droit de Markov. En conséquence, on résout le problème des martingales associé au L^p -générateur infinitésimal d'un semi-groupe subordonné. Il s'avère qu'un élargissement de l'espace de base est nécessaire. La principale étape de la preuve est la préservation sous une subordination de la propriété d'un processus de Markov d'être un processus droit borélien. En suite, on étudie la h-transformation d'un C_0 -semi-groupe d'opérateurs sous-markoviens sur l'espace L^p.

Ces résultats ont été obtenus en collaboration avec Lucian Beznea et Ana-Maria Boeangiu.

A regime switching on Covid-19 analysis and prediction in Romania

Changement de régime dans l'analyse de Covid-19 et prédiction en Roumanie

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In this presentation we propose a regime separation for the analysis of Covid19 on Romania combined with mathematical models of SIR and SIRD. The main regimes we study are the free spread of the virus, the quarantine and partial relaxation and the last one is the relaxation regime. The main model we use is SIR which is a classical model, but because we can not fully trust the numbers of infected or recovered people we base our analysis on the number of deceased people which is more reliable. To actually deal with this we introduce a simple modification of the SIR model to account for the deceased separately. This in turn will be our base for fitting the parameters. We actually use the classical SIR model to detect the regime switching and in fact prove a proposition which shows that we can recover the parameters in a unique way from the daily observation of the number of infected and susceptible. This is the basis for guessing the main parameters in the model. The actual estimation of the parameters in our SIRD model is done in two steps. The first one consists in training a neural network based on SIR models to detect the regime changes. Once this is done, we fit the main parameters of the SIRD model using a grid search near the values suggested by the neural network. At the end, we make some predictions on what the evolution will be in a timeframe of a month with the fitted parameters

Covariance matrices and the recovery of the spectrum

Matrices de covariance et récupération du spectre

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We will show some results about recovering the spectrum of a covariance matrix from the empirical covariance matrix from a finite number n of samples. Typically this is done using some scheme from free probability for very large n, though the main interest is in finite n.

Random matrices in Wiener chaos

Matrices aléatoires sur le chaos de Wiener

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We study the fluctuations, as $d, n \to \infty$, of the Wishart matrix $\mathcal{W}_{n,d} = \frac{1}{d}\mathcal{X}_{n,d}\mathcal{X}_{n,d}^T$ associated to a $n \times d$ random matrix $\mathcal{X}_{n,d}$ with non-Gaussian entries. We analyse the limiting behaviour in distribution of $\mathcal{W}_{n,d}$ in two situations: when the entries of $\mathcal{X}_{n,d}$ are independent elements of a Wiener chaos of arbitrary order and when the entries are partially correlated and belong to the second Wiener chaos.

Stochastic solutions to evolution equations of non-local branching processes

Solutions stochastiques des équations d'évolution des processus de branchement non-local

Cătălin Ioan Vrabie (Bucarest, e-mail: catalin.ioan.vrabie@gmail.com)

Branching processes arise in the modelling of the time evolution of a system of particles. Informally, we can describe the procedure as follows: a parent particle moves according to a Markov process until a random terminal time when it is destroyed and replaced by second generation particles, which again move according to the same Markov process as their parent until their own terminal time and so on. The offspring need not to start at the same position where the parent died, hence the name *non-local*. The killing of the process is controlled by a measure μ, general enough to accept applications of interest.

We approach the problem from two perspectives. The first one is a probabilistic point of view, that is, we are interested in proving the existence of a branching process with the

above described dynamics. As it turns out, the transition semigroup will be obtained from an evolution equation that will involve a stochastic integral term. The second perpective is that of PDEs, where we view the solution of the evolution equation as a "mild" solution of a corresponding Neumann problem and give a probabilistic interpretation of it based on the above constructed branching process. As an application of the above results, we consider the case where the base process is the reflected Brownian motion on a smooth bounded Euclidean domain and μ is the surface measure on the boundary.

The presentation is based on a joint work with Lucian Beznea and Oana Lupaşcu-Stamate.