GDRI: *ECO-Math*

Project 2022

Nonlinear PDEs, Lévy bases, and branching mechanisms

Equipe :

Lucian BEZNEA (IMAR, Bucarest)

Oana LUPAȘCU-STAMATE (ISMMA, Bucarest)

Madalina DEACONU (Univ. de Lorraine & Inria, Nancy)

Radu STOICA (Univ. de Lorraine & Inria, Nancy)

Activites:

- 2 visites scientifiques de deux semaines a l'Universite de Lorraine, Nancy.
- 2 visites scientifiques d'une semaine a l'IMAR

Research project

I) Branching processes and nonlinear PDEs. We intend to model the solutions of certain equations and nonlinear boundary value problems with physical relevance. We investigate the two-dimensional (2d) *vorticity equation* that describes the time evolution of the vorticity of a fluid, namely the local rotation of the fluid in a bounded Euclidean domain. In the paper [S. Benachour, B. Roynette, P. Vallois, *Rev. Mat. IberoAm.* **17** (2001)] the solutions of the vorticity equation are represented by a stochastic model, through a branching process with state space the set of all finite configurations of the closure of the domain. The branching mechanism on the boundary leads to a stochastic interpretation of the creation or disappearance of the vorticity on the boundary layer. However, as the authors emphasize, the proof for the convergence result of the proposed numerical algorithm is not yet available. Consequently, our first aim is to study the phenomenon that corresponds to the creation-annihilation of the vortex localized at the boundary of the domain and the convergence for the corresponding numerical approaches. We intend to use an appropriate branching processes as developed in [L. Beznea, O. Lupașcu-Stamate, C. Vrabie, *Nonlinear Analysis* **200** (2020)] and very recent stochastic numerical methods as in [B. Leimkuhler, A. Sharma, M.V. Tretyakov, *Simplest random walk…*, Preprint 2020].

II) Lévy bases with a branching mechanism. The Cox point processes represent one of the most important and versatile classes of point process models for clustered point patterns, see [G. Hellmund, M. Prokesova, E. B. Vedel Jensen, *Adv. Appl. Prob.* (SGSA) **40** (2008)]. During the last decades several new classes of Cox point process models appeared in the literature, e.g., shot noise Cox processes, log Gaussian Cox processes, and log shot noise Cox processes. These models share some common properties and differ in others, depending on how the driving intensity measure of the Cox process is constructed. To handle such driving intensity measures, Lévy bases are used. The

Lévy bases include Poisson random measure, mixed Poisson random measures, and Gaussian random measures. Our objective is to construct Cox point processes having driving intensity measures induced by a non-local branching mechanism, in particular, to study the associated Choquet capacities. The purpose of this study is to obtain for the processes on hand, summary statistics able to be described by analytical closed formula. Hence, these statistics might be used to characterize point patterns arising from real data sets. Among the application domains where these datat sets are encountered, we mention : environmental sciences, astrophysics, epidemiology and geosciences.

The 2022 tasks

I) Firstly, we shall describe the dynamics of the particles system which occurs as solution of the vorticity equation in a bounded planar domain, with Neumann boundary conditions and based on a specific non-local branching process. We shall complete the model with a numerical approximation. The branching mechanism on the boundary will offer a stochastic interpretation of the creation or disappearence of vorticity on the boundary and should lead to the convergence of an associated numerical scheme.

II) Secondly, we intend to consider and study a new class of Lévy-driven Cox processes, by adding a nonlocal branching mechanism to the driving intensity measure. Such processes can be regarded as marked or Boolean point processes. More precisely, the points of a Lévy Poisson base can be replaced by new particles (descendants), distributed according to a given "branching" Markovian kernel. For that, we study the n^{th}-order product densities, the generating functional, and the capacity functional.

Articles:

1. L. Beznea, **M. Deaconu**, **O. Lupaşcu-Stamate**, Scaling property for fragmentation processes related to avalanches**.** In: *Applications of Mathematics and Informatics in Natural Sciences and Engineering*, Springer Proceedings in Mathematics & Statistics **334** (2020), https://doi.org/10.1007/978-3-030-56356-1_3

2. L. Beznea, **M. Deaconu**, and **O. Lupascu-Stamate,** Numerical approach for stochastic differential equations of fragmentation; application to avalanches,Mathematics and Computers in Simulation, **160 (**2019), 111-125.

3. L. Beznea, **M. Deaconu**, and **O. Lupascu**, Stochastic equation of fragmentation and branching processes related to avalanches, *Journal of Statistical Physics,* **162** (2016), 824–841.

4. . L. Beznea, **M. Deaconu**, and **O. Lupascu**, Branching processes for the fragmentation equation, *Stochastic Processes and their Applications*, **125** (2015), 1861-1885.