

Report on the projet GDRI ECO-Math “Bifurcation set of polynomial applications”

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The project is based on the recent joint work done by the authors (*Bifurcation values of families of real curves*. Proc. Royal Soc. Edinburgh Sect.A 147, 6 (2017), 1233-1242.) and aims to find further developments and applications.

In our project we study the bifurcation locus of polynomial maps $F : \mathbb{C}^n \rightarrow \mathbb{C}^p$, $n > p \geq 2$, aiming to find necessary and sufficient conditions for the local triviality of F in terms of the topology of the fibres. During our study, we realized that there is a basic problem that was not yet solved, as follows.

The image of a holomorphic map germ $F : (\mathbb{C}^n, 0) \rightarrow (\mathbb{C}^p, 0)$, $n \geq p \geq 2$ is not well-defined as a set germ. This problem should be well-known but has not been investigated since mathematicians worked with *complete intersections with isolated singularity*, a condition which insures that the map is open at 0.

However, this is not always the case. The image of F is well-defined as a set germ if the germ at 0 of $F(B_\varepsilon)$ is independent of the (small enough) radius $\varepsilon > 0$ of the ball B_ε . The simplest example is the blow-up map $(\mathbb{C}^2, 0) \rightarrow (\mathbb{C}^2, 0)$ given by $(x, y) \mapsto (x, xy)$. This problem arises, for example, when defining local fibrations of map germs.

We have therefore concentrated our efforts to classifying the complex map germs with respect to the image problem.

We were able to find classifying conditions for holomorphic map germs $(f, g) : (\mathbb{C}^n, 0) \rightarrow (\mathbb{C}^2, 0)$ and for real analytic map germs $f\bar{g} : (\mathbb{C}^n, 0) \rightarrow (\mathbb{C}, 0)$ in order that their images are well-defined as set germs. We have published the preprint *Images of analytic map germs*.

Mihai Tibăr visited IMAR between May 25 and June 6 2018 for scientific collaboration with Cezar Joița. During this visit we have established the final version of our article *Bifurcation set of multi-parameter families of complex curves* which was later published in a very good journal.

At the same time, between the 28th and 31st of May, we participated in an international conference in memory of Ștefan Papdima, which took place in Bucharest, see <http://math.univ-ovidius.ro/workshop/2018/TGSP/>.

In September (10 - 21), Cezar Joița visited Université de Lille and we worked on the preprint mentioned above.

Outcomes.

We established the final version of:

- Cezar Joița, Mihai Tibăr, *Bifurcation set of multi-parameter families of complex curves*. J. Topology 11 (2018), no. 3, 739-751.

We published the preprint:

- Cezar Joița, Mihai Tibăr, *Images of analytic map germs*. arXiv:1810.05158, October 2018