Application

GDRI ECO-Math 2018-2019

October 20, 2017

Homogenization of a Class of Diffusion Problems with Flux Jump

Participants

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1 Scientific Project

Homogenization of a Class of Diffusion Problems with Flux Jump

The present project is a continuation of the LEA Math-Mode project "Homogenization of a Thermal Problem with Flux Jump", which ran in 2015, involving R. Bunoiu and C. Timofte. The goal of our LEA Math-Mode project was to analyze, using homogenization techniques, a class of diffusion problems in a periodic composite material formed by two constituents, separated by an imperfect interface where certain discontinuities occur. In the last three decades, the study of such problems has attracted a lot of attention, both from a theoretical point of view and also from the point of view of applications. In particular, the effective thermal transfer in a periodic composite material formed by two constituents, separated by an imperfect interface where the temperature has a jump, was a topic of huge interest for a broad category of researchers. We mention here the pioneering works [2], [3], [6] only. The main novelty brought by our LEA Math-Mode project consisted in allowing the presence of a jump in the flux across the imperfect interface, too. Indeed, up to our knowledge, only few studies of this type of problem are known by now (see, for instance, [1], [4], [5], [7]).

Let Ω be an open bounded material body in $\mathbb{R}^n (n \geq 2)$, with a Lipschitz - continuous boundary $\partial\Omega$. We assume that Ω is formed by two constituents, Ω_1^ε and Ω_2^ε, representing two materials with different thermal characteristics, separated by an imperfect interface Γ^ε. We also assume that the phase Ω_1^ε is connected and reaches the external fixed boundary $\partial\Omega$ and that Ω_2^ε is not connected. Actually, Ω_2^ε is the union of domains of size ε, periodically distributed in Ω with periodicity ε (ε is a small real parameter related to the characteristic size of the two constituents). We consider the problem of finding the asymptotic behavior, as ε tends to zero, of the solution $(u_1^\varepsilon, u_2^\varepsilon)$ of the following problem:

$$
\begin{cases}
-\text{div }(A^\varepsilon \nabla u_1^\varepsilon) = f_1(u_1^\varepsilon) \quad \text{in } \Omega_1^\varepsilon, \\
-\text{div }(\varepsilon^\gamma A^\varepsilon \nabla u_2^\varepsilon) = f_2(u_2^\varepsilon) \quad \text{in } \Omega_2^\varepsilon, \\
A^\varepsilon \nabla u_1^\varepsilon \cdot n^\varepsilon = \varepsilon^\alpha h^\varepsilon(u_1^\varepsilon, u_2^\varepsilon) + \varepsilon^\beta g^\varepsilon \quad \text{on } \Gamma^\varepsilon, \\
\varepsilon^\gamma A^\varepsilon \nabla u_2^\varepsilon \cdot n^\varepsilon = \varepsilon^\alpha h^\varepsilon(u_1^\varepsilon, u_2^\varepsilon) \quad \text{on } \Gamma^\varepsilon, \\
u_1^\varepsilon = 0 \quad \text{on } \partial\Omega.
\end{cases} \tag{1}
$$

Here, the given matrix A^ε represents the thermal conductivity of the material and γ, α, β are real constants taking different values in a range of parameters which needs to be precized. Several cases are considered for the functions f_1, f_2, h^ε, and g^ε.

A jump in the flux of the solution in observed across the common imperfect interface Γ^ε, as previously announced. Indeed, making the difference between the third and the fourth equations in system (1), one has

$$
A^\varepsilon \nabla u_1^\varepsilon \cdot n^\varepsilon - \varepsilon^\gamma A^\varepsilon \nabla u_2^\varepsilon \cdot n^\varepsilon = \varepsilon^\beta g^\varepsilon.
$$

Various problems, corresponding in (1) to given functions f_1, f_2 depending on x only, to $h^\varepsilon(u_1^\varepsilon, u_2^\varepsilon) = h^\varepsilon(x)(u_1^\varepsilon - u_2^\varepsilon)$ and to particular values of the parameters γ, α, β, have been studied in the frame of the already mentioned LEA Math-Mode project and gave rise to the following publications:

- R. BUNOIU, C. TIMOFTE, *On the homogenization of a two-conductivity problem with flux jump*, Communications in Mathematical Sciences, Vol. 15, No. 3, 745-763, 2017;
- R. BUNOIU, C. TIMOFTE, *Homogenization of a thermal problem with flux jump*, Networks and Heterogeneous Media, Vol. 11, No. 4, 545-562, 2016;
- R. BUNOIU, C. TIMOFTE, *On the homogenization of a diffusion problem with flux jump*, communication presented at "International Conference on Mathematical Methods and Models in Biosciences (Biomath 2016)", Bulgaria (abstract published in Biomath Communications, Vol 3, No. 1, 2016).

It was observed in these papers that the limit problem, obtained after homogenization, captured the influence of the two jumps in the limit temperature field, in an additional source term, and in the correctors, as well. Moreover, we observed that there is a subtle interplay between the scaling of the jump functions and that, in particular, the jump flux term g^ε leads in the limit problem to completely different effects.

Motivated by these encouraging results, we would like to go further in our study, extending it to more general frameworks, as for example:

- functions f_1 and f_2 depending on the unknowns $u_1^\varepsilon, u_2^\varepsilon$;
- non linear jump functions h^ε;
- more general geometries of the interwoven phases;
- parabolic problems.

The research visits, whose financial support is requested in this application, would be an essential help for the finalization of our common work in progress and for the accomplishment of the new announced projects.

Finally, we point out that both participants to this project already worked on problems with interfacial barriers and published together or independently (alone or with co-authors) a series of papers on this topic.

References

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[5] M. Gahn, P. Knabner, M. Neuss-Radu, "Homogenization of reaction-diffusion processes in a two-component porous medium with a nonlinear flux condition at the interface, and application to metabolic processes in cells", SIAM J. Appl. Math. 76, pp. 1819-1843, 2016.

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[7] E.R. Ijioma, A. Muntean, T. Ogawa, "Pattern formation in reverse smouldering combustioon: a homogenization approach", Combustion Theory and Modelling 17(2), pp. 185-223, 2013.

2 Short Curricula Vitae

Renata Bunoiu

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Professional address: Institut Elie Cartan, UMR 7502, Université de Lorraine, 3 rue Augustin Fresnel, F-57073, Metz, FRANCE
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Position : Maître de Conférences Hors Classe, Université de Lorraine, Metz (since September, 1998)

Education

1997 PhD in Mathematics, Université de Metz (très honorable)
1994 D.E.A. de Mathématiques (mention bien), Université de Metz.

Research Interests

- Homogenization theory
- Asymptotic analysis in thin domains
- Theory of waveguides
- Spectral theory
- Scattering theory

Last Five Publications

[1] R. BUNOIU, C. TIMOFTE, *On the homogenization of a two-conductivity problem with flux jump*, Communications in Mathematical Sciences, Vol. 15, No. 3, 745-763, 2017.

[2] R. BUNOIU, R. PRECUP, C. VARGA, *Multiple positive standing wave solutions for Schrödinger equations with oscillating state-dependent potentials*, Communications on Pure and Applied Analysis, Vol. 16, No. 3, 954-972, 2017.

[3] R. BUNOIU, G. CARDONE, *Bingham flow in porous media with obstacles of different size*, Mathematical Methods in the Applied Sciences, Vol. 40, No. 12, 4514-4528, 2017.

[4] R. BUNOIU, P. DONATO, *Unfolding homogenization in doubly periodic media and applications*, Applicable Analysis, Vol. 96, No. 13, 2218-2235, 2017.

[5] R. BUNOIU, C. TIMOFTE, *Homogenization of a thermal problem with flux jump*, Networks and Heterogeneous Media, Vol. 11, No. 4, 545-562, 2016.

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Position : Professor, Bucharest University (since February, 2008).

Education

- 2017 Habilitation in Mathematics, University of Bucharest, Romania
- 1996 Ph. D. in Mathematics, Institute of Mathematics "Simion Stoilow" of the Romanian Academy, Romania
- 1988 Master of Science, University of Bucharest, Faculty of Mathematics, Romania.

Research Interests

- Homogenization theory
- Macrotransport processes
- Upscaling in chemical reactive processes in porous media
- Mathematical models in biology

Last Five Publications

[1] R. BUNOIU, C. TIMOFTE, *On the homogenization of a two-conductivity problem with flux jump*, Communications in Mathematical Sciences, Vol. 15, No. 3, 745-763, 2017.

[2] C. TIMOFTE, *Homogenization results for the calcium dynamics in living cells*, Mathematics and Computers in Simulation, Vol. 133, No. 3, 165-174, 2017.

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[5] R. BUNOIU, C. TIMOFTE, *Homogenization of a thermal problem with flux jump*, Networks and Heterogeneous Media, Vol. 11, No. 4, 545-562, 2016.

3 Activities to be Supported by the Project

We propose two research visits per year, for each year, with financial support needed for travel expenses, accommodation and local expenses.

2018

- One research visit (14 days) to Metz of Claudia Timofte.

Required financial support 1650 €, as follows:

- 390 € for travel expenses (flight, train and bus tickets)
- 1260 € (14 x 90 €) for accommodation and local expenses.

- One research visit (10 days) to Bucharest of Renata Bunoiu.

Required financial support 1290 €, as follows:

- 390 € for travel expenses (flight, train and bus tickets)
- 900 € (10 x 90 €) for accommodation and local expenses.

Required financial support for 2018: ≈ 2940 €.

2019

- One research visit (14 days) to Metz of Claudia Timofte.

Required financial support 1650 €, as follows:

- 390 € for travel expenses (flight, train and bus tickets)
- 1260 € (14 x 90 €) for accommodation and local expenses.

- One research visit (10 days) to Bucharest of Renata Bunoiu.

Required financial support 1290 €, as follows:

- 390 € for travel expenses (flight, train and bus tickets)
- 900 € (10 x 90 €) for accommodation and local expenses.

Required financial support for 2019: ≈ 2940 €.