

Relations between dynamical systems and dimension theory

GDR1 Research project for the period 2017-2020

1 Research Team

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2 Scientific Project

The interrelated fields of Dynamical Systems, Smooth Ergodic Theory and Dimension Theory have seen a tremendous growth in the last 30 years. They started from the seminal work of Bowen ([2]), Ruelle ([16], [15]), and then expanded into many applications and topics (such as [14], [1], [6], [8], [10], [13], etc). They have many connections with other fields in Mathematics and in other sciences.

The main applications that we will envision in our project are to dimension theory for various classes of fractals and invariant measures for dynamical systems.

Among these we mention hyperbolic endomorphisms $f : M \rightarrow M$, which have invariant fractal sets Λ either of a saddle type (i.e where both contracting and expanding directions are present), as well as repellers (where only expanding directions are present). In both cases, the natural extensions $(\hat{\Lambda}, \hat{f})$ will play a significant role.

An important problem that we plan to investigate is to estimate Hausdorff dimensions (and other types of dimensions) for slices of the invariant set Λ and also of invariant probability measures μ which are supported on Λ . A large and important class of invariant measures is that of equilibrium measures μ_ϕ for Hölder continuous potentials ϕ on Λ (for eg [2], [15], [14], etc).

We will study some applications of thermodynamic formalism for dimensions of equilibrium measures and sections through fractals with overlaps (for eg [4], [12]).

For self-affine, or more generally non-conformal systems on invariant sets, the situation is different than for conformal ones, due mainly to the fact that one cannot cover directly with iterate images of small balls, but has to divide these into finer coverings ([1], [3], [5], etc.) If the basic sets are non-conformal, then one has to take in consideration also the various Lyapunov exponents of an invariant measure μ over Λ , along various different directions, and to employ Falconer's singular functions (see [3], [1]).

Various problems that will be studied are related to Lyapunov exponents, singular functions, and measure theoretic entropies, and how they can be applied for dimension estimates. We will study these notions also from the point of view of iterated function systems, sometimes employing certain symbolic lift spaces ([7], [11], etc.) Of interest will be also the Variational Principles (for eg [2], [15], [14]), besides the classical one for topological pressure on compact spaces and equilibrium states, that may be found on locally maximal sets.

References

- [1] L. Barreira and K. Gelfert, Dimension estimates in smooth dynamics, *Ergodic Theory Dynam. Systems* 31 (2011), no. 3, 64–671.
- [2] R. Bowen, *Equilibrium States and the Ergodic Theory of Anosov Diffeomorphisms*, Lecture Notes in Math, 470, Springer, Berlin-New York, 1975.
- [3] K. Falconer, Bounded distortion and dimension for non-conformal repellers, *Math Proc Cambridge*, 115, 315–334, 1994.
- [4] J. E Fornæss, E. Mihailescu, Equilibrium measures on saddle sets of holomorphic maps on \mathbb{P}^2 , *Math. Annalen* (2013) 356, 1471–1491.
- [5] D. Gatzouras, S. Lalley, Hausdorff and box dimensions of certain self-affine fractals, *Indiana Univ. Math. J.* 41 (1992), 533–568.
- [6] B. Hasselblatt and J. Schmeling, Dimension product structure of hyperbolic sets, in *Modern Dynamical Systems and Applications*, Cambridge Univ. Press, Cambridge, 2004, 331–345.
- [7] V. Mayer and M. Urbański, Countable alphabet random subshifts of finite type with weakly positive transfer operator, *J. Statistical Physics*, 160, no 5, 1405–1431, 2015.
- [8] V. Mayer, M. Urbański, Thermodynamical formalism and multifractal analysis for meromorphic functions of finite order, *Memoir AMS*, Vol. 203, Nr. 954 (2010).

- [9] V. Mayer, B. Skorulski, M. Urbanski, Regularity and irregularity of fiber dimension of non-autonomous dynamical systems, *Ann Acad Sci Fenn Math*, 38, No. 2, 489–514 (2013).
- [10] E. Mihailescu, B. Stratmann, Upper estimates for the stable dimension for fractal sets with variable number of foldings, *Int Math Res Notices*, Vol. 2014, No. 23, (2014), pp. 6474–6496.
- [11] E. Mihailescu, M. Urbański, Random countable iterated function systems with overlaps and applications, *Advances in Math*, vol 298, 726–758, 2016.
- [12] E. Mihailescu, M. Urbański, Measure-theoretic degrees and topological pressure for non-expanding transformations, *J. Functional Analysis*, 267 (2014), no. 8, 2823–2845.
- [13] E. Mihailescu and M. Urbański, Relations between stable dimension and the preimage counting function on basic sets with overlaps, *Bull. London Math. Soc.*, 42 (2010), 15-27.
- [14] Y. Pesin, *Dimension Theory in Dynamical Systems: Contemporary Views and Applications*, Chicago Lectures in Math., Chicago Univ Press, 1997.
- [15] D. Ruelle, *Thermodynamic Formalism*, Reading, Addison-Wesley, 1978.
- [16] D. Ruelle, Repellers for real-analytic maps, *Ergodic Th. and Dynam. Syst.*, 2 (1982), 99–107.

3 Some recent publications of the team

- 1) V. Mayer, M. Urbański, A. Zdunik, Real Analyticity for random dynamics of transcendental functions, preprint - submitted.
- 2) V. Mayer, M. Urbański, Random Dynamics of Transcendental Functions, *J. d'Analyse Math*, to appear.
- 3) V. Mayer, M. Urbański, Countable alphabet random subshifts of finite type with weakly positive transfer operator, *J Stat Phys*, 160, 5, 1405–1431, 2015.
- 4) V. Mayer, B. Skorulski, M. Urbanski, Regularity and irregularity of fiber dimension of non-autonomous dynamical systems, *Ann Acad Sci Fenn Math*, 38, No. 2, 489–514 (2013).
- 5) E. Mihailescu, M. Urbański, Random countable iterated function systems with overlaps and applications, *Advances in Math*, vol 298, 726–758, 2016.
- 6) E. Mihailescu, B. Stratmann, Upper estimates for stable dimensions on fractal sets with variable numbers of foldings, *Int Math Res Notices*, 23, 6474–96, 2014.

- 7) E. Mihailescu, M. Urbanski, Overlap functions for measures in conformal iterated function systems. *J Stat Phys*, 162 (2016), no. 1, 43–62.
- 8) J.E Fornæss, E. Mihailescu, Equilibrium measures on saddle sets of holomorphic maps on \mathbb{P}^2 , *Math Annalen*, 356, no. 4, 147–1491, 2013.
- 9) E. Mihailescu, M. Urbański, Measure-theoretic degrees and topological pressure for non-expanding transformations, *J Funct Analysis*, 267, 2823–2845, 2014.

4 Visits and Estimated Budget

We intend to make several exchange research visits during the 4-year period:

1 January 2017 – 31 December 2020

We intend to use the GDRI financial support to cover the costs, namely **transport, hotel and per diem**, for:

- 1 visit/year (each of approximately 10 days) of Volker Mayer to Bucharest.
- 1 visit/year (each of approximately 10 days) of Eugen Mihailescu to Lille.

Estimated costs per year:

Approximately 900 euro for V.Mayer's visit + 900 euro for E.Mihailescu's visit
= 1800 euros/year.