

Description of the area of research:

Matroids are abstract combinatorial structures that capture the notion of independency in vector spaces or in graphs. Combinatorial properties of matroids are linked to algebraic properties of the associated toric base ring. White (1980) conjectured that its defining ideal is generated by quadrics corresponding to symmetric exchange relations.

So far this was proved for some classes: graphic matroids (Blasiak, 2008), rank at most three (Kashiwabara 2010), lattice path matroids (Schweig, 2011), etc. Lason and Michalek (Advances in Math. 2014) verified that it holds up to saturation, and for the class of strongly base orderable matroids. We will investigate topics related to that question of White, and more generally, how invariants of matroids are captured by the associated toric ideal.

For instance, not much is known about their Betti numbers or about the number of generators of these toric ideals. Only recently, García-Marco and Ramírez Alfonsín (SIAM J. Discrete Mathematics, 2015) characterized the matroids whose toric ideal is a complete intersection ideal.

Research team:

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Selected publications:

- K. Knauer, L. Martinez-Sandoval, **J.L. Ramírez Alfonsín**, *On lattice path matroid polytopes: integer points and Ehrhart polynomial*, Discrete and Computational Geometry, in press.
- K. Knauer, L. Martinez-Sandoval, **J.L. Ramírez Alfonsín**, *A Tutte polynomial inequality for lattice path matroids*, Adv. in Appl. Math., 94 (2018), 23–38.
- I. García-Marco, **J.L. Ramírez Alfonsín**, *Matroid toric ideal: complete intersection, minors and minimal systems of generators*, SIAM J. Discrete Mathematics, 29(4) (2015), 2267–2276.

- J. Chappelton, I. García-Marco, L. Montejano, **J.L. Ramírez Alfonsín**, *Möbius function of semigroup poset through Hilbert series*, J. Combinatorial Theory Ser. A, 136 (2015), 238–251.
- **D.I. Stamate**, *Betti numbers for numerical semigroup rings*, to appear in: Multigraded Algebra and Applications (V. Ene, E. Miller, Eds.), Springer Proceedings in Mathematics & Statistics.
- J. Herzog, **D.I. Stamate**, *Quadratic numerical semigroups and the Koszul property*, Kyoto Journal of Mathematics, Volume 57, Number 3 (2017), 585–612.
- V. Reiner, **D.I. Stamate**, *Koszul incidence algebras, affine semigroups, and Stanley-Reisner ideals*, Advances in Mathematics 224 (2010), 2312–2345.

Proposed activities for 2018:

- (1) visit of D. Stamate at the Université de Montpellier, up to 2 weeks, in April-May 2018. Estimated budget: 1700 euro
- (2) visit of J. Ramírez-Alfonsín at the S. Stoilow Institute of Mathematics of the Romanian Academy, up to 2 weeks.