Qualitative study of nonlinear PDE's

Research project submitted to GDRI ECO-Math
by **Maria-Magdalena Boureanu** and **Benedetta Noris**

1 Research team

The researchers implied in this project (alphabetically ordered), are:

- Maria-Magdalena Boureanu (University of Craiova, Romania – responsible for the Romanian part of the team)
- Olivier Goubet (Picardie Jules Verne University, Amiens, France)
- Benedetta Noris (Picardie Jules Verne University, Amiens, France– responsible for the UPJV part of the team)
- Vicenţiu Rădulescu (University of Craiova and Institute of Mathematics "Simion Stoilow" of the Romanian Academy, Romania)

Collaborations between the members of the team have been established long ago. In 2009, M.-M. Boureanu obtained her PhD degree under the careful supervision of V. Rădulescu. This tight collaboration has been carried out over the years, as it is illustrated by the partnership at three common research grants and by the joint papers [4, 5, 6]. Moreover, in 2011, during her four months research visit at University of Milano Bicocca, M.-M. Boureanu worked closely with B. Noris and later they published a paper, see [3] (joint work with S. Terracini). Another cooperation exists between O. Goubet and V. Rădulescu who know each other for many years and contributed at the paper [10]. Furthermore, being employed at the same university, B. Noris and O. Goubet, have been participating at numerous scientific activities together (the weekly seminar on analysis, department meetings, etc) and this provided a very good environment to start their collaboration. We add that there were several occasions on which the four partners in this project have attended the same summer schools or conferences, which led to strengthen relations.

All these allow us to say that there is cohesion in our team and that the participants work well together. In addition, all members have strong background in nonlinear PDEs with different specializations, which ensures the necessary experience and the diversity to generate new ideas. A much more complete mathematical portrait is emerging from their *curricula vitae*, which will accompany this application. Also, a short list of seven significant papers for each of them is provided at the end.

2 Description of the project

The aim of this project is to further investigate different classes of nonlinear boundary value problems by combining various topological and variational methods. We intend to approach multiple research directions.

1. Variable exponent problems with Leray-Lions type operators

With a large scale of possible applications (such are those involving non-Newtonian fluids, elastic materials, image restoration, mathematical biology etc), the variable exponent problems receive a lot of interest, see for example the comprehensive book of Rădulescu and Repovš [13] and the references therein. This is the case also for the problems involving Leray-Lions type operators whose name comes from the French mathematicians [11] who introduced them. A common feature of those generalized operators is that they can be particularized to various elliptic operators, the most popular being the Laplace type operators and the mean curvature type operators.

Over the years, many variants of such operators appeared, each of them being adapted to a specific type of problem, and we recall here the Leray-Lyons type operators for elliptic problems with variable exponent, isotropic, see [8, 12], or anisotropic, see [7, 5], where both Rădulescu and Boureanu have had contributions.

Recently, Boureanu introduced Leray-Lions type operators for the treatment of the elliptic systems with variable exponent [2] and treated a Dirichlet system with zero boundary condition. Our goal is to treat systems with nonlinear Neumann or Robin boundary condition, to consider different kind of assumptions on the nonlinearity F involved in the system (as opposed to the set of hypotheses considered in [2]), and to extend the set of the hypotheses that define the Leray-Lions operators from [2] by introducing a weight function. As the reader may know, the presence of a weight function offers a higher degree of generality to the study, but, at the same time, generates more difficulties, as the functional framework changes to the weighted Lebesgue and Sobolev spaces with variable exponent. Furthermore, in addition to this, there are other classes of problems involving Leray-Lions type operators with variable exponent that we intend to investigate, such as the fourth order PDEs.

2. Bifurcation methods for problems involving p-Laplace equations

The analysis of bifurcation problems for Elliptic PDEs involving classical Laplace operators is well-known and relies on the method introduced by Crandall and Rabinowitz. We aim here to make some progress in the understanding of bifurcation diagrams for Elliptic PDEs involving non-linear p-Laplace operators. In this case it is worth to point out that the famous method quoted above does not apply. To begin with, we will focus on equations on the unit ball as

$$-\Delta_p u + u^{p-1} = \lambda f(u),$$

for some special nonlinearities and with homogeneous Neumann boundary conditions. We plan to extend some results that appear for the classical Laplace operators in the article [9] (see also [1]).

3 Plan of activities and budget

Our main activity will consist in treating problems as the ones exposed in the previous section and trying to obtain the desired results, to answer some questions and, as importantly, to discover new questions to be asked. It is worth mentioning that all the members of this team have great experience in at least one of the proposed directions of research, as one can deduce from their list of publications, see

http://www.ucv.ro/departamente_academice/dma/cv_en/CV_boureanu_magdalena_EN.pdf

http://www.lamfa.u-picardie.fr/goubet/

https://www.lamfa.u-picardie.fr/membres/benedetta-noris/

http://math.ucv.ro/ radulescu/

Therefore we are entitled to consider that we will progress in the research directions indicated above and that we will obtain original results that will be published in prestigious mathematical journals.

We are applying for financial support to help to organize meetings between the members of the team. Indeed, although the internet and the video-conferences help a lot a collaboration, the face-to-face contact still remains more efficient.

Thus we preview a seven days visit of Maria-Magdalena Boureanu to Picardie Jules Verne University, and, if applicable, a seven days visit from Picardie Jules Verne University to University of Craiova. The expenses are detailed below:

(i) travel=2x350=700 euros,

(ii) per diem (accommodation and meals)=2x7x100=1400 euros,

hence we require a budget of 2100 euros for the two visits.

We consider that these research visits would encourage the exchange of information between the two groups, the one from France and the one from Romania, so that each of them could acquire novel techniques. Moreover, the visits represent an opportunity to communicate our research during scientific seminars and to tighten the relationships between the two universities.

4 Significant papers

We end this application with a short list of significant papers for each of us.

1. Maria-Magdalena Boureanu

1. M.M. Boureanu, V. Rădulescu and D. Repoš, On a $p(\cdot)$-biharmonic problem with no-flux boundary condition, Computers and Mathematics with Applications, 72 (2016), 2505–2515.

2. M.M. Boureanu and C. Udrea, No-flux boundary value problems with anisotropic variable exponents, Communications on Pure and Applied Analysis, 14 (2015), 881-896.

3. M.M. Boureanu, A. Matei and M. Sofonea, Nonlinear problems with $p(\cdot)$-growth conditions and applications to antiplane contact models, Advanced Nonlinear Studies, 14 (2014), 295-313.

4. M.M. Boureanu and D.N. Udrea, Existence and multiplicity results for elliptic problems with p(.) - growth conditions, Nonlinear Anal. Real World Applications, 14 (2013) 1829-1844.

5. M.M. Boureanu and V. Rădulescu, Anisotropic Neumann problems in Sobolev spaces with variable exponent, Nonlinear Anal. TMA, 75 (2012), 4471-4482.

6. M.M. Boureanu and F. Preda, Infinitely many solutions for elliptic problems with variable exponent and nonlinear boundary conditions, Nonl. Diff. Eq. and Appl. (NoDEA), 19 (2012), 235-251.

7. M.M. Boureanu and A. Matei, Weak solutions for antiplane models involving elastic materials with degeneracies, Zeitschrift fur Angewandte Mathematik und Physik (ZAMP), 61 (2010), 73–85.

2. Olivier Goubet

1. O. Goubet and S. Labrunie, The Dirichlet problem for $-\Delta\varphi = e^{-\varphi}$ in an infinite sector. Application to plasma equilibria, Nonlinear Analysis, Theory, Method et Applications, Nonlinear Anal. 119 (2015), 115–126.

2. J. Davila and O. Goubet, Partial regularity for a Liouville system, DCDS-A, vol 34, n 6, (2014), 2495–2503.

3. L. Dupaigne, M. Ghergu, O. Goubet and G. Warnault, The Gel'fand problem for the biharmonic operator, Archive for Rational Mechanics and Analysis, 208 (2013), no. 3, 725–752.

4. F. Demengel and O. Goubet, Existence of boundary blow up solutions for singular or degenerate fully nonlinear equations, Commun. Pure Appl. Anal. 12 (2013), no. 2, 621–645.

5. O. Costin, L. Dupaigne and O. Goubet, Uniqueness of large solutions, J. Math. Anal. Appl. 395 (2012), no. 2, 806–812.

6. L. Dupaigne, M. Ghergu, O. Goubet and G. Warnault Entire Large Solutions for Semilinear Elliptic Equations, J. Differential Equations 253 (2012), no. 7, 2224–2251.

7. S. Dumont, L. Dupaigne, O. Goubet and V. Radulescu, Back to the Keller-Osserman condition for boundary blow-up solutions, Adv. Nonlinear Stud. 7 (2007), no. 2, 271–298.

3. Benedetta Noris

1. A. Boscaggin, F. Colasuonno and B. Noris, Multiple positive solutions for a class of p-laplacian Neumann problems without growth conditions, ESAIM : COCV, 2017, to appear.
http://arxiv.org/abs/1703.05727

2. D. Bonheure, J.-B. Casteras and B. Noris, Multiple positive solutions of the stationary Keller-Segel system, Calc. Var. PDE, 56(3), Art. 74, 35 pp, 2017. http://arxiv.org/abs/1603.07374

3. D. Bonheure, M. Grossi, B. Noris, and S. Terracini, Multi-layer radial solutions for a supercritical Neumann problem, J. Differential Equations, 261(1), 455–504, 2016. http://arxiv.org/abs/1508.01619

4. A. Aftalion, B. Noris, and C. Sourdis, Thomas-Fermi approximation for coexisting two component Bose-Einstein condensates and nonexistence of vortices for small rotation, Comm. Math. Phys., 336(2), 509–579, 2015. http://arxiv.org/abs/1403.4695

5. B. Noris, H. Tavares, and G. Verzini, Existence and orbital stability of the ground states with prescribed mass for the L2-critical and supercritical NLS on bounded domains, Analysis and PDE, 7(8), 1807–1838, 2014. http://arxiv.org/abs/1307.3981

6. D. Bonheure, B. Noris, and T. Weth, Increasing radial solutions for Neumann problems without growth restrictions, Ann. Inst. H. Poincaré Anal. Non Linéaire, 29(4), 573–588, 2012.
http://arxiv.org/abs/1109.4009

7. B. Noris, H. Tavares, S. Terracini, and G. Verzini, Uniform Hölder bounds for nonlinear Schrödinger systems with strong competition, Comm. Pure Appl. Math., 63(3):267–302, 2010.
http://arxiv.org/abs/0810.5537

4. Vicenţiu Rădulescu

1. N. Chorfi and V. Rădulescu, Small perturbations of elliptic problems with variable growth, Applied Mathematics Letters 74 (2017), 167–173.

2. K. Kefi and V. Rădulescu, On a p(x)-biharmonic problem with singular weights, Zeitschrift fuer angewandte Mathematik und Physik (ZAMP) 68 (2017), 68–80.

3. N. Papageorgiou, V. Rădulescu and D. Repoš, Multiple solutions for resonant problems of the Robin p-Laplacian plus an indefinite potential, Calculus of Variations and PDEs (2017) 56–63.

4. N. Papageorgiou, V. Rădulescu and D. Repoš, Robin problems with a general potential and a superlinear reaction, Journal of Differential Equations 263 (2017), 3244–3290.

5. G. Molica Bisci, V. Rădulescu and R. Servadei, Competition phenomena for elliptic equations involving a general operator in divergence form, Analysis and Applications 15 (2017), 51–82.

6. B. Alleche and and V. Rădulescu, Set-valued equilibrium problems with applications to Browder variational inclusions and to fixed point theory, Nonlinear Analysis: Real World Applications 28 (2016), 251–268.

7. V. Rădulescu, Nonlinear elliptic equations with variable exponent: old and new, Nonlinear Analysis: Theory, Methods and Applications, 121 (2015), 336-369

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[3] M.M. Boureanu, B. Noris and S. Terracini, Sub and supersolutions, invariant cones and multiplicity results for p-Laplace equations, *Contemporary Mathematics – Recent Trends in Nonlinear Partial Differential Equations II: Stationary Problems*, 595 (2013), 91–119.

[4] M.-M. Boureanu, P. Pucci and V. Rădulescu, Multiplicity of solutions for a class of anisotropic elliptic equations with variable exponent, *Complex Variables and Elliptic Equations*, 56 (2011), 755–767.

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[6] M.-M. Boureanu, V. Rădulescu and D. Repovš, On a $p(\cdot)$-biharmonic problem with no-flux boundary condition, *Computers and Mathematics with Applications*, 72 (2016), 2505–2515.

[7] M.M Boureanu and C. Udrea, No–flux boundary value problems with anisotropic variable exponents, *Commun. Pure Appl. Anal.*, 14 (2015), 881–896.

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[9] D. Bonheure, JB. Casteras and B. Noris, Multiple positive solutions of the stationary Keller-Segel system, *Calc. Var.* (2017) 56–74.

[10] S. Dumont, L. Dupaigne, O. Goubet and V. Rădulescu, Back to the Keller-Osserman condition for boundary blow-up solutions, *Advanced Nonlinear Studies*, 7 (2007), 271–298.

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[13] V. Rădulescu and D. Repovš, *Partial Differential Equations with Variable Exponents: Variational Methods and Quantitative Analysis*, CRC Press, Taylor $\wp$ Francis Group, Boca Raton FL, 2015.