

Financement GDRI/IRN Eco Math

For the year 2023, our interest is to study control problems associated to phase field equations. A phase-field model is a mathematical model for solving interfacial problems. We shall consider materials which can have two states of aggregation, e.g. solid and liquid. The domain has two parts in which the material has the different two states, but there is an interface where the two states can co-exist. The method substitutes boundary conditions at the interface by a partial differential equation for the evolution of an auxiliary field (the phase field) that takes the role of an order parameter. This phase field takes two distinct values (for instance $+1$ and 1) in each of the phases, with a smooth change between both values in the zone around the interface, which is then diffuse with a finite width. A discrete location of the interface may be defined as the collection of all points where the phase field takes a certain value (e.g., 0). A phase-field model is usually constructed in such a way that in the limit of an infinitesimal interface width (the so-called sharp interface limit) the correct interfacial dynamics are recovered. This approach permits to solve the problem by integrating a set of partial differential equations for the whole system, thus avoiding the explicit treatment of the boundary conditions at the interface. An example of a phase field model are the celebrated Cahn-Hilliard equations. In order also to take account of the movement of the material (e.g. blocks of ice floating and melting down the stream of a river) one should couple these equations with the Navier-Stokes equations, which model the fluid dynamics. Moreover, in order to take into account of possible random perturbations of the model and measurements, and to have a more accurate and realistic model we shall perturb the system by multiplicative noise. Therefore, we shall study stochastic Cahn-Hilliard-Navier-Stokes equations. We shall associate a stabilization problem to the model. To be more exact, we shall look for a boundary controller such that once plugged into the equations ensures that the solution of the corresponding controlled equation approaches exponentially fast to a given target, e.g. the parabolic Poiseuille profile. In fact we shall continue the work of I. Munteanu [1], where he designed proportional boundary stabilizer controllers for the linearised deterministic Cahn-Hilliard-Navier-Stokes equations around the Poiseuille profile. In this project we want to show that the same controller ensures asymptotic exponential stability for the full-nonlinear system as-well, and also for the stochastic version. We intend to use a fixed point argument. We define the functional which gives the solution as a fixed point and show that it is a contraction. The difficulty will be represented by fine estimates of the multiple complex terms of the functional, in order to determine that it is a contraction in a suitable functional space.

References

- [1] I. Munteanu, Boundary stabilizing actuators for multi-phase fluids in a channel, *Journal of Differential Equations* 285, 2021, pp. 175-210.

Activities within the project:

1) At the beginning of September I. Munteanu will have an academic visit for two weeks at INSA Rouen at prof. I. Ciotir, where he will also give a talk about the exposed subject

2) In October I. Ciotir will make an academic visit at the University of Iasi at prof. I. Munteanu where also she will give a talk with the occasion of the Scientific Anniversary days of the University Cuza

3) We intend to publish a paper on the exposed subject

There are not previous results to declare within the project since last year prof. I. Ciotir was ill therefore no visits and research work was possible.