

# ABSTRACTS

## INVITED TALKS

### Stabilization of Navier-Stokes equations by boundary and point feedback controllers

**Viorel Barbu**

A few recent results pertaining internal and boundary feedback stabilization of steady-state solutions to Navier-Stokes equations are surveyed. The stabilization via infinite dimensional Riccati equations, explicit construction of stabilizable feedback controllers, internal point stabilization and internal stabilization by Gaussian noises are the main topics.

### Are black holes real?

**Sergiu Klainerman**

Black holes are simple, stationary solutions of the Einstein field equations supposed to model extremely large concentrations of matter from which nothing can escape, including light. Since they cannot be seen how can one be sure that they are Real?

It turns out, I will argue, that Mathematics can provide some pertinent answers

### On conjugacy classes in the Lie group E8

**George Lusztig**

In this talk we will describe a method to organize the conjugacy classes in the Lie group E8 according to the conjugacy classes in a finite group (the Weyl group). The talk will include a definition of the Lie group E8 which is simpler than the standard one. We will also consider some questions concerning the interaction between several conjugacy classes in the Lie group E8.

### Geometry of homology jump loci and topology

**Stefan Papadima**

The representation varieties of discrete groups into Lie groups appear in various branches of mathematics. When the discrete group is the fundamental group of a space (manifold), jump loci of twisted homology endow those varieties with a natural stratification by subvarieties. I will explain how the geometry of these subvarieties informs on the topology (geometry) of the space (manifold).

# Vanishing theorems and holomorphic one-forms

**Mihnea Popa**

Vanishing theorems for cohomology groups are one of the essential tools in higher dimensional algebraic and complex geometry. They usually require various notions of positivity. For those projective manifolds that carry non-trivial global holomorphic one-forms, the last two decades have seen the development of generic vanishing theory, where one obtains suitable modifications of the standard vanishing theorems under much weaker positivity assumptions. I will present the main results of this theory, together with recent developments involving higher homological algebra and Hodge D-modules.

These developments allow us to go full circle and answer very basic questions about holomorphic one-forms themselves.

## Horn inequalities: finite and infinite dimensions

**Dan Timotin**

The classical Horn inequalities characterize the eigenvalues of three matrices  $A, B, C$  subjected to the condition  $A + B = C$ . Stated first by Horn as a conjecture, the result has been proved by Klyachko and Knutson-Tao-Woodward. It has shown unexpected connections with other areas of mathematics: symplectic geometry, combinatorics, group representations, invariant subspaces, etc.

After presenting the history and the context, the talk will focus on recent developments of the subject. The matrices are replaced by different classes of operators on infinite dimensional Hilbert spaces. To obtain these extensions, supplementary insight in the finite dimensional case is needed.

# TALKS IN SPECIAL SESSIONS

*The abstracts are ordered alphabetically by speaker (marked with \*).*

## Roots of characteristic polynomials and intersection points of line arrangements

**Takuro Abe**

We show that the roots of the characteristic polynomial of a line arrangement give a restriction on the number of intersection points. In other words, if we substitute the number of points on a line, then the value of the characteristic polynomial is always non-negative. This holds true not only for lines in an arrangement but also a line which can be added to the arrangement. Moreover, we can show that the arrangement is free if that value is zero. We will show several applications related to this result.

## A probabilistic interpretation of Gruss and Ostrowski type inequalities

**Ana Maria Acu\*, Daniel Florin Sofonea**

The applications of probability theory in insurance, econometrics and actuarial mathematics motivate in the last years many authors to develop some estimations of the moments, the covariance and the variance of random variables. Our goal is to find some estimations of the moments of the random variable using the classical inequalities of Gruss and Ostrowski type. A estimation of cumulative distribution function is given. Some inequalities of Gruss-type available in literature are extended for the weighted case.

## Exact factorizations of Hopf algebras. Applications

**Ana Agore\*, Gigel Militaru**

Let  $A \subseteq E$  be a given extension of Hopf (respectively Lie) algebras. We answer the classifying complements problem (CCP) which consists of describing and classifying all complements of  $A$  in  $E$ . If  $H$  is a given complement then all the other complements are obtained from  $H$  by a certain type of deformation. We establish a bijective correspondence between the isomorphism classes of all complements of  $A$  in  $E$  and a cohomological type object  $\mathcal{H}\mathcal{A}^2(H, A | (\triangleright, \triangleleft))$ , where  $(\triangleright, \triangleleft)$  is the matched pair associated to  $H$ . The factorization index  $[E : A]^f$  is introduced as a numerical measure of the (CCP). For two  $n$ -th roots of unity we construct a  $4n^2$ -dimensional Hopf algebra whose factorization index over the group algebra is arbitrary large. Applications to group theory are also provided.

## The latticial, categorical, and relative counterparts of the Osofsky - Smith theorem

**Toma Albu**

We present a latticial version of the renown *Osofsky-Smith Theorem* saying that a cyclic (finitely generated) right  $R$ -module such that all of its cyclic (finitely generated) subfactors are CS modules is a finite direct sum of uniform submodules. Though the Osofsky-Smith Theorem is a module-theoretical result, our contention is that it is a result of a strong latticial nature. Applications of the *Latticial Osofsky-Smith Theorem* to Grothendieck categories and module categories equipped with a torsion theory are given.

# Robotic ensembles in polyhedral computing

Sorin Alexe\*, Diana-Olimpia Alexandrescu

First, we introduce the concept of polyhedral computing as a process operating over a collection of states represented as complex ensembles of polyhedra, called modules. The process changes the state of the system by applying one of the available operations, and randomly selecting one of the possible valid states that are accessible from the current one. The validation of the states is performed by a special semantics, for example the one checking for spatial representability. The process of polyhedral computing takes as an input a collection of polyhedra of a certain type and computes the output represented by a geometrical expression. The generated geometrical expressions are constructions displaying various degree of freedom. Consequently some of them are displaying robotic behavior. Secondly, we analyze some particular cases of polyhedral computing that lead to robotic ensembles. In particular we investigate the properties of one robotic ensemble that can change its form from linear to circular on one axis and from square to ring on a second axis.

Key words: XColony, Mathematical Origami, Computation Architecture, Fractals, Combinatorial Geometry, Robotics, Spatial Cognition, Kinetic Sculpture

## An injectivity theorem

Florin Ambro

I will discuss generalizations of the injectivity theorem of Esnault-Viehweg, and give some applications to the structure of linear systems of logarithmic type.

## An interpolation problem for completely positive maps

Călin Ambrozie\*, Aurelian Gheondea

We give a procedure to find completely positive maps between matrix algebras, that takes prescribed values on given matrices from a finite set.

## The circular Morse-Smale characteristic of a manifold

Dorin Andrica\*, Cornel Pinte

The *Morse-Smale characteristic* of a compact smooth manifold is the number

$$\gamma(M) = \min\{\text{card}(C(f)) : f \in \mathfrak{F}(M)\},$$

where  $\mathfrak{F}(M)$  denotes the set of all real-valued Morse functions on  $M$ .

This number is related to the minimum number of cells in the *CW*-decompositions of  $M$  up to homotopy (for details we refer to the monograph [1]). The formulation of the circle-valued Morse theory as a new branch of differential topology with its own problems was outlined by S. P. Novikov in 1980. The circular version of the Morse-Smale characteristic was recently introduced by D. Andrica and D. Mangra [2], it is defined by

$$\gamma_{S^1}(M) := \min\{\text{card}(C(f)) : f \in \mathcal{F}(M, S^1)\},$$

where  $\mathcal{F}(M, S^1)$  is the set of all circle-valued Morse functions  $f : M \rightarrow S^1$ , and it is a special case of  $\varphi$ -category [4]. Some properties of the circular Morse-Smale characteristic were already proved by D.Andrica and D.Mangra [2], [3]. For instance, for every closed manifold (i.e. compact and without boundary) we have the inequality  $\gamma_{S^1}(M) \leq \gamma(M)$ , as the universal cover  $\exp : \mathbb{R} \rightarrow S^1$  composed with every Morse real valued function produces a circle valued Morse function. This property implies that  $\gamma_{S^1}(M)$  is finite whenever  $M$  is compact. We present some general properties of this number, we compute it for the closed connected surfaces, and derive a circular analogous formula to the formula, earlier proved by Kuiper, for the Morse-Smale characteristic.

## References

- [1] D. Andrica, *Critical Point Theory and Some Applications*, Cluj University Press, 2005.
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- [4] D. Andrica, C. Pinteá, *Recent results on the size of critical sets*, in "Essays in Mathematics and Applications" (In honor of Stephen Smale's 80th Birthday), P.Pardalos,Th.M.Rassias (Eds.), 17-35, Springer, 2012.

## Pointed Hopf algebras with finite simple group of Lie type

**Nicolás Andruskiewitsch\*, Giovanna Carnovale, Gastón Andrés García**

We will present recent results on the determination of finite-dimensional pointed Hopf algebras  $H$  whose group of group-likes  $G(H)$  is isomorphic to a finite simple group of Lie type. It is expected that there is a unique such Hopf algebra up to isomorphisms, namely the group algebra. Evidence in this direction will be presented.

## Combinatorics and essential coordinate components of characteristic varieties of line arrangements

**Enrique Artal Bartolo\*, Jose I. Cogolludo-Agustin**

Characteristic varieties are a powerful invariant of the fundamental group of the complement of a line arrangement  $\mathcal{A}$  in  $\mathbb{P}^2$  (over the complex numbers). These varieties can be computed from a presentation of the group but in order to understand the relationship with combinatorics a more geometrical approach is needed. The work of Libgober provides a way to find most irreducible components of these varieties (depending on the position of the multiple points of  $\mathcal{A}$ ), though their combinatorial nature is still an open problem. In this work, we focus on the so-called essential coordinate components which are not covered by Libgober's approach. The existence of such components, in the framework of line arrangement, was found by Cohen and Suciu. We provide a method to compute these components where all the steps but one are of combinatorial nature; the remaining one is topological and uses the description of the embedding  $\partial N(\mathcal{A}) \subset \mathbb{P}^2 \setminus \mathcal{A}$  where  $N(\mathcal{A})$  is a regular neighborhood of the arrangement; this description is due to Florens, Guerville and Marco and generalizes E. Hironaka's work for complexified line arrangements.

## Self-adjoint Toeplitz operators associated with representing measures on multiply connected planar regions and their eigenvalues

**Cyrus Aryana**

The existence of eigenvalues of self-adjoint Toeplitz operators acting on Hardy spaces associated with non-negative representing measures on 1-holed planar regions is established in the case where there exists one bounded component in the complement of the essential range of the symbol  $\phi$  of the operator. The analysis is done by using the zeros of translations of theta functions restricted to  $\mathbb{R}$  in  $\mathbb{C}$ .

## A short proof of a de Finetti-type theorem

**Dragu Atanasiu**

In [1] and [2] P. Ressel proved general theorems from which the theorem of Hewitt and Savage and other de Finetti-type results can be deduced. Our aim is to give a proof of a de Finetti-type theorem similar to the results proved by Ressel. Our proof use Choquet theory of integral representations.

## References

- [1] P. Ressel, *De Finetti-type theorems: An analytical approach*, Ann.Probab. **13** (1985), 898-922.
- [2] P. Ressel, *Exchangeability and semigroups*, Rend. Mat. Appl., VII. Ser. **28** (2008), 63-81.

## A resonance stratification for the Grassmannian

**Eric Babson\*, Alexandru Suciu**

To every subspace of the second exterior power of a finite dimensional vector space is associated a resonance subscheme of the projectivization of the given vector space. These together form a subscheme of the product. These resonance schemes have been useful for distinguishing groups which can not arise as fundamental groups for special classes of manifolds, such as compact Kähler or complex hyperplane complements. I will discuss the structures of these schemes and the associated decomposition of the Grassmannians in low dimension.

## Nonuniform exponential dichotomy on the half-line through $(C_b, L^p)$ admissibility

**Mihai-Gabriel Babuția\*, Adina Luminița Sasu, Bogdan Sasu**

In this talk we present a new sufficient condition for the existence of a nonuniform exponential dichotomy of evolution families on the half-line. The main idea is to associate an input-output equation to an evolution family having a (most general) nonuniform exponential growth and to obtain the nonuniform exponential behavior for the given family from the admissibility of the pair  $(C_b(\mathbb{R}_+, X), L^p(\mathbb{R}_+, X))$  with respect to associated equation. Furthermore we show that the converse implication is not generally valid and in addition we give an application to the case of uniform exponential dichotomy of evolution families. We consider this approach as a first step in our goal to obtain a nonuniform exponential dichotomy from the admissibility of the pair  $(L^p(\mathbb{R}_+, X), L^q(\mathbb{R}_+, X))$  with respect to the associated input-output equation. Our results are in press in the journal Bulletin des Sciences Mathématiques (see <http://dx.doi.org/10.1016/j.bulsci.2012.11.002>) and this work was supported by a grant of the Romanian National Authority for Scientific Research, CNCS UEFISCDI, project number PN-II-RU-TE-2011-3-0103.

## Two-level methods for variational and quasi-variational inequalities of the second kind

**Lori Badea**

We introduce and analyze some two-level multiplicative and additive Schwarz methods for variational and quasi-variational inequalities of the second kind. The methods are introduced as subspace correction algorithms for problems in a reflexive Banach space. We prove that these methods are globally convergent and give, under some assumptions, error estimates. In the finite element spaces, the introduced algorithms are in fact two-level Schwarz methods. In this case we prove that the assumptions we made for the general convergence result hold, and write the convergence rate depending on the overlapping and mesh parameters. We get that our methods have an optimal convergence rate, it is almost independent of the mesh and overlapping parameters, and also, the methods have an optimal computing complexity per iteration.

## Test functions, Schur-Agler classes and transfer-function realizations for the matrix-valued setting

**Joseph Ball**

Following work of Dritschel, Marcantognini, and McCullough, given a collection of test functions, we define the associated Schur-Agler class as the intersection of the contractive on which each test function is a contractive multiplier. We indicate extensions of this framework to the case where the test functions, kernel functions, and Schur-Agler-class functions are allowed to be matrix- or operator-valued. We discuss two salient examples: (1) the matrix-valued Schur class over a finitely-connected planar domain and (2) the matrix-valued version of the constrained Hardy algebra (bounded analytic functions on the unit disk with derivative at the origin constrained to have zero value). The results lead to precise Agler decompositions and transfer-function realizations for the matrix-valued Schur class in these settings, extending earlier results of Dritschel, McCullough, and Pickering. This talk reports on joint work with Moises Guerra Huaman.

# Expanders from groups

**Cristina Ballantine**

Expander graphs are well-connected yet sparse graphs. The expansion property of a regular or bi-regular graph is governed by the second largest eigenvalue of its adjacency matrix. Optimal expanders are called Ramanujan graphs. We will introduce the notion of primes for graphs and define the Ihara-Zeta function and the Riemann Hypothesis in the context of graphs. Graphs satisfying the Riemann Hypothesis are Ramanujan. We will discuss how methods from the representation theory of  $p$ -adic groups can be used to construct infinite families of (regular and bi-regular) Ramanujan graphs.

## Self-organized criticality: models and mathematical results

**Viorel Barbu**

We discuss mathematical models of self-organized criticality and give theoretical results and examples.

## Hilbert transforms along vector fields

**Michael Bateman\*, Christoph Thiele**

We survey recent developments related to Hilbert transforms in the direction of a vector field, especially those depending on one-variable. This family of problems is connected to classical problems involving singular integrals and differentiation of integrals, but we apply the modern tools of time-frequency analysis. Other key ingredients are geometric covering arguments and a new method for proving vector-valued inequalities.

## Domain decomposition approaches for Stokes problems

**Constantin Băcuță\*, Jiguang Sun**

We present a domain decomposition technique for the Stokes type systems based on the partition of unity method. The discrete spaces are conforming finite element spaces that are build by gluing subspaces associated with overlapping subdomains. By using partition of unity functions, we construct global approximation spaces from overlapping non-matching grids. The global discrete spaces preserve the local approximation properties and satisfy a discrete inf – sup (LBB) condition.

## Seshadri positive submanifolds of polarized manifolds

**Lucian Bădescu\*, Mauro C. Beltrametti**

Let  $Y$  be a submanifold of dimension  $y$  of a polarized complex manifold  $(X, A)$  of dimension  $k \geq 2$ , with  $1 \leq y \leq k - 1$ . We define and study two positivity conditions on  $Y$  in  $(X, A)$ , called Seshadri  $A$ -bigness and (a stronger one) Seshadri  $A$ -ampleness. In this way we get the natural generalization of the theory initiated by Paoletti in 1995 (which corresponds to the case  $(k, y) = (3, 1)$ ) and subsequently generalized and completed in a joint paper with M. Beltrametti and Paolo Francia in 1997 (regarding curves in a polarized manifold of arbitrary dimension). The theory presented here, which is new even if  $y = k - 1$ , is motivated by a reasonably large area of examples.

## Sharp estimates for the second variations of periodic functions

**Sorina Bârză**

The classes of functions of bounded  $(p, \alpha)$  second variation form a scale connecting the class of functions of  $p$ -bounded second variation with the Sobolev space of function with  $p$ -integrable second derivative. We prove sharp estimates of the norms in these intermediate spaces via fractional moduli of order  $2-1/p$  as well as some interpolation results. Our case is not a direct consequence of the case of functions of bounded variation studied previously by M. Lind, in "On fractional smoothness of functions related to  $p$ -variation", MIA, vol. 16, Nr. 1 (2013), 21-39. For the proofs we use techniques developed in the above mentioned paper. We will present also some historical remarks which led to the problem.

# Hopf algebras of dimension $8p$

Margaret Beattie\*, G.A. García

Even for dimension  $4p$ ,  $p$  an odd prime, although all the isomorphism classes of semisimple and pointed Hopf algebras are known, the general classification is not complete. However, Cheng and Ng proved that if a Hopf algebra  $H$  of dimension  $4p$  is nonsemisimple, then  $H$  is pointed if and only if  $|G(H)| > 2$ . For dimensions 12, 20, 28, 44 every Hopf algebra is semisimple, pointed or the dual is pointed (Natale, Cheng&Ng).

A Hopf algebra  $H$  is said to be of type  $(r, s)$  if  $|G(H)| = r$  and  $|G(H^*)| = s$ . In spite of the fact that the classification of semisimple Hopf algebras and pointed Hopf algebras of dimension  $8p$  is incomplete, we are able to utilize information about dimension  $4p$  to prove:

**Theorem** *Let  $H$  be a Hopf algebra of dimension  $8p$  over  $\mathbb{C}$  which is not semisimple, not pointed and  $H^*$  is not pointed.*

*(i)  $H$  is of type  $(r, s)$  where either  $r, s$  are both powers of 2, or  $r = 2p$  and  $s$  is 2 or 4.*

*(ii) If  $H$  has the Chevalley property then either*

- *$H$  is of type  $(4, 2p)$  with coradical  $\mathbb{k}^{\mathbb{D}_{2p}}$  and the dimension of  $(H^*)_0$  is  $6p$  or*
- *$H$  is self-dual of type  $(4, 4)$  and has coradical the unique self-dual semisimple Hopf algebra of dimension  $4p$  with grouplikes the Klein 4-group.*

Finally we show that these result can be improved for some small dimensions where counting arguments can be used.

## Noncommutative complex differential geometry

Edwin Beggs\*, S. Paul Smith

This talk is about the basic properties of noncommutative analogues of almost complex structures, integrable almost complex structures, holomorphic curvature, cohomology, and holomorphic sheaves. The starting point is a differential structure on a noncommutative algebra defined in terms of a differential graded algebra. This is placed in the context of starting work on a noncommutative version of Serre's GAGA - Geometrie algebrique et geometrie analytique - the correspondence between certain complex manifolds and certain algebraic varieties.

## Groups of virtual and welded links

Paolo Bellingeri\*, V. Bardakov, J. Gonzales-Meneses

Starting from the (classical) notion of group of a knot and (less classical...) invariants related to Wada representations we will introduce new invariants for virtual and welded links. This will be occasion to motivate the study of virtual braids and welded links in the realm of endomorphisms of finitely generated free groups.

## Diffusion with redistribution

Iddo Ben-Ari

We consider a diffusion process on a bounded domain with random redistribution. The redistribution is obtained through either one of the following mechanisms. The first is redistribution when the diffusion hits the boundary, and this is repeated indefinitely. The second is "instantaneous" redistribution, occurring at events of some time-changed Poisson process, and the diffusion is killed upon hitting the boundary. By "redistribution" we mean starting the diffusion afresh from some prescribed probability distribution on the domain, which may depend on its location immediately prior. For the first mechanism, we will focus on ergodicity, spectral gap and coupling. For the second, we will discuss the interplay between the "fast", continuous diffusion and the "slow", non-local redistribution process in a "non-elliptic" setting, as it appears through asymptotic behavior of the exit distribution from the domain.

# On the dual of the Procesi-Formanek lattice

**Esther Beneish**

Let  $G$  be the symmetric group on  $n$  letters. Procesi and Formanek have shown that  $C_n$ , the center of the generic division algebra of degree  $n$  defined over a field  $F$ , is stably isomorphic to  $F(B_n)^G$  where  $B_n$  is a specific  $ZG$ -lattice. We refer to  $B_n$  as the Procesi-Formanek lattice. The question of the stable rationality of  $C_n$  is a long standing problem for which few results are known. Let  $F$  be an algebraically closed field of characteristic zero, let  $p$  be an odd prime, and let  $B_p^* = \text{Hom}_Z(B_p, Z)$  be the dual of the Procesi-Formanek lattice. We show that  $F(B_p^*)^G$  is stably rational over  $F$ . An interesting question is whether there exists a connection between  $C_p$  and  $F(B_p^*)^G$ .

## Control of singular Fourier multipliers by maximal operators

**Jonathan Bennett**

We present some recent results on the control of singular (or "rough") Fourier multipliers by geometrically-defined maximal operators via general weighted  $L^2(\mathbb{R})$  norm inequalities. The multipliers involved satisfy certain weak Marcinkiewicz-type conditions, and permit highly oscillatory factors of the form  $e^{i|\xi|^\alpha}$  for both  $\alpha$  positive and negative. The maximal functions that arise are of some independent interest, involving fractional averages associated with tangential approach regions (related to those of Nagel and Stein), and improper fractional averages associated with "escape" regions. Some extensions to higher are discussed.

## The compactness property of bilinear commutators

**Arpad Benyi\*, Kabe Moen, Rodolfo H. Torres**

Commutators of a large class of bilinear operators, which includes bilinear Calderón-Zygmund operators, and multiplication by functions in a certain subspace of the space of functions of bounded mean oscillations are shown to be jointly compact. Under a similar commutation, bilinear fractional integral operators yield separately compact operators in the variable of commutation.

## Invariant subspaces and intersection theory

**Hari Bercovici**

There is a direct connection between intersection theory on Grassmannians and invariant subspaces of certain operators related with compressions of shifts. This connection explains why the Littlewood-Richardson rule appears in both contexts.

## Variational methods for the relativistic pendulum

**Cristian Bereanu**

In this talk we present some results concerning the relativistic forced pendulum. The classical case is also discussed. The proofs are mainly based on methods of the calculus of variations in the nonsmooth sense.

## On bilinear Bochner-Riesz multipliers

**Frederic Bernicot\*, L. Grafakos, L. Song and L.X. Yan**

Motivated by the problem of spherical summability of products of Fourier series, we study the boundedness of the bilinear Bochner-Riesz multipliers corresponding to the bilinear symbol  $(1 - |\xi|^2 - |\eta|^2)_+^\delta$ . We will describe some boundedness on Lebesgue spaces for this bilinear operator. Our results are based on a variety of techniques, that include Fourier series expansions, orthogonality, and bilinear restriction and extension theorems to deal with extremal points ( $L^2 \times L^2 \rightarrow L^1$ ,  $L^2 \times L^\infty \rightarrow L^2$ ,  $L^1 \times L^\infty \rightarrow L^1$ ).

**Lucian Beznea\*, Oana Lupaşcu**

We discuss the construction of the branching Markov processes on the space of finite configurations of the state space of a given Markov process, controlled by a branching kernel and a killing one. We indicate the natural connection between discrete branching processes and some nonlinear partial differential operators. In particular, we may start with a sup-Brownian motion, obtaining a branching process with state space the finite configurations of positive finite measures on an Euclidean set. A main tool in proving the path regularity is the existence of convenient superharmonic functions having compact level sets.

## On the optimal properties for splines

**Alexandru Mihai Bica**

The well-known Holladay's property of minimal curvature of natural cubic splines can be viewed as a measure of small oscillations for the graph of the cubic spline. This classical property is expressed in terms of the  $L^2$ -norm of the second derivative. In this talk, we focus the attention to another recent measure of small oscillation for splines, quadratic oscillation in average (developed for Hermite's type cubic splines in Appl. Math. Lett. 25 (2012) 2047-2051).

## The Robin eigenvalue problem for the $p(x)$ -Laplacian as $p \rightarrow \infty$

**Marian Bocea**

We study the asymptotic behavior, as  $p \rightarrow \infty$ , of the first eigenvalues and the corresponding eigenfunctions for the  $p(x)$ -Laplacian with Robin boundary conditions in an open, bounded domain  $\Omega \subset \mathbb{R}^N$  with sufficiently smooth boundary. We prove that the positive first eigenfunctions converge uniformly in  $\Omega$  to a viscosity solution of a problem involving the  $\infty$ -Laplacian with appropriate boundary conditions. Joint work with F. Abdullayev (NDSU).

## Strong shape derivative for the wave equation with Neumann boundary data

**Lorena Bociu\*, Jean-Paul Zolesio**

We provide a full shape derivative analysis for the wave equation with mixed boundary conditions on a moving domain  $\Omega_s$  in the case of non smooth Neumann boundary data. The key ideas are (i) bypassing the classical sensitivity analysis of the state by using parameter differentiability of a functional expressed in the form of Min-Max of a convex-concave Lagrangian with saddle point, (ii) using a new regularity result on the solution of the wave problem (where the Dirichlet condition on the fixed part of the boundary is essential) to analyze the strong derivative, and (iii) taking advantage of the "extractor strategy" introduced in [Delfour-Zolesio, '96] and Fourier transform techniques.

## Eisenstein-Schönemann-Dumas type irreducibility conditions that use arbitrarily many prime numbers

**Nicolae Ciprian Bonciocat**

The famous irreducibility criteria of Eisenstein-Schönemann and Dumas rely on information on the divisibility of the coefficients of a polynomial by a single prime number. We provide several irreducibility criteria of Eisenstein-Schönemann-Dumas-type for polynomials with integer coefficients, criteria that are given by some divisibility conditions for their coefficients with respect to two or more prime numbers.

# Deforming diamond

Ciprian Borcea\*, Ileana Streinu

For materials science, diamond crystals are almost unrivaled for hardness and a range of other properties. Yet, when simply abstracting the carbon bonding structure as a geometric bar-and-joint periodic framework, it is far from rigid. We study the geometric deformations of this type of framework in arbitrary dimension  $d$ , with particular regard to the volume variation of a unit cell.

## Large deviations of Wigner matrices without Gaussian tail

Charles Bordenave\*, Pietro Caputo

We consider a Wigner matrix: a random Hermitian matrix  $X$  of size  $n$  whose entries above the diagonal are independent and identically distributed with unit variance. Since the seminal work of Wigner in the 50's, it is known that the empirical distribution of the eigenvalues of  $X/\sqrt{n}$  converges to the semi-circular law. In 1997, Ben Arous and Guionnet have established a large deviation principle (LDP) around the semi-circular law when the entries are Gaussian. The associated rate function is the Voiculescu's non-commutative entropy. Their proof was based on the explicit formula for the joint law of the eigenvalues, and beyond this result, establishing LDP's for Wigner matrices remains largely open. When the entries are of Weibull type but not subgaussian (for example exponential) we will see that it is however possible to prove such LDP using ideas coming from random graphs.

## Factoriality of the Zassenhaus variety or how lucky can one be

Amiram Braun

The center of the enveloping algebra of a reductive Lie algebra, in prime characteristic, is a factorial domain. This 2010 result is due to R.Tange. We shall explain the non-commutative ring-theoretic origin of this result and outline a new proof for it. This also enabled us to prove the same for its quantum analog (at the root of unity case). The unlikely chain of events which led to this proof, involving modular invariants, Grothendieck-Serre correspondence etc, will be also described.

## A global Baer-Kaplansky type theorem

Simion Breaz

The Baer-Kaplansky Theorem states that two primary abelian groups with isomorphic endomorphism rings are necessarily isomorphic. This statement was extended to various classes of modules over commutative rings. However straightforward examples show that in order to obtain such extensions we need to impose restrictions on these classes. For instance the endomorphism rings of the Prüfer group  $\mathbb{Z}(p^\infty)$  and of the group of  $p$ -adic integers  $\widehat{\mathbb{Z}}_p$  are both isomorphic to the ring  $J_p$  of  $p$ -adic integers. In the setting of modules over complete valuation domains  $W$ . May was able to prove a result for reduced proper mixed modules (they are neither torsion nor torsion-free) which have a nice subgroup  $B$  such that  $M/B$  is totally projective: if  $M$  is such a module and  $N$  is an *arbitrarily* module such that they have isomorphic endomorphism rings then  $M \cong N$ . The main aim of this talk is to prove a global Baer-Kaplansky type theorem for modules over principal ideal domains: if  $R$  is a (commutative) principal ideal domain then the correspondence (from the class of  $R$  modules to the class of rings)

$$\Phi : G \mapsto \text{End}_R(R \oplus G)$$

reflects ring isomorphisms. Moreover, this property characterizes principal ideal domains in the class of Dedekind domains: if  $R$  is a Dedekind domain such that the correspondence  $\Phi$  reflects isomorphisms then  $R$  is a PID. The restriction to Dedekind domains is motivated by the fact that these domains have the *cancellation property*, i.e. the endofunctor  $R \oplus - : \text{Mod-}R \rightarrow \text{Mod-}R$  on the category of all  $R$ -modules reflects isomorphisms.

# Vector bundles on non-Kaehler elliptic principal bundles

Vasile Brînzănescu\*, Andrei D. Halanay, Guenther Trautmann

We shall study relatively semi-stable vector bundles and their moduli on non-Kaehler principal elliptic bundles over compact complex manifolds. The main technical tools used are the twisted Fourier-Mukai transform and a spectral cover construction.

## Monoidal ring and coring structures obtained from wreaths and cowreaths

Daniel Bulacu\*, Stefaan Caenepeel

Let  $A$  be an algebra in a monoidal category  $\mathcal{C}$ , and let  $X$  be an object in  $\mathcal{C}$ . We study  $A$ -(co)ring structures on the left  $A$ -module  $A \otimes X$ . These correspond to (co)algebra structures in  $EM(\mathcal{C})(A)$ , the Eilenberg-Moore category associated to  $\mathcal{C}$  and  $A$ . The ring structures are in bijective correspondence to wreaths in  $\mathcal{C}$ , and their category of representations is the category of representations over the induced wreath product. The coring structures are in bijective correspondence to cowreaths in  $\mathcal{C}$ , and their category of corepresentations is the category of generalized entwined modules. We present several examples coming from (co)actions of Hopf algebras and their generalizations. Various notions of smash products that have appeared in the literature appear as special cases of our construction.

## Fusion rules of equivariantizations of fusion categories

Sebastian Burciu\*, S. Natale

We determine the fusion rules of the equivariantization of a fusion category  $\mathcal{C}$  under the action of a finite group  $G$  in terms of the fusion rules of  $\mathcal{C}$  and group-theoretical data associated to the group action. As an application we obtain a formula for the fusion rules in an equivariantization of a pointed fusion category in terms of group-theoretical data. This entails a description of the fusion rules in any braided group-theoretical fusion category.

## Pseudomonoids and 3-cocycles

Stefaan Caenepeel\*, Bojana Femic

Let  $R$  be a commutative  $k$ -algebra, and  $A$  a commutative  $R$ -bialgebroid. To  $A$ , we can associate complexes of abelian groups. The cohomology of these complexes generalizes known cohomologies such as group cohomology, Amitsur cohomology, Sweedler cohomology, Harrison cohomology. We can also associate a complex of restricted Picard groupoids, giving rise to a sequence of bicategories  $\underline{Z}^n(A, \underline{\text{Pic}})$ . The equivalence classes in this bicategory define an abelian group  $H^n(A, \underline{\text{Pic}})$ ; this generalizes an old construction by Villamayor and Zelinsky. In this talk, we focus on  $\underline{Z}^3(A, \underline{\text{Pic}})$ . First we introduce the bicategory of commutative  $R \otimes R$ -rings  $\underline{R}\underline{\text{CR}}_R$ . Then we show that  $\underline{Z}^3(A, \underline{\text{Pic}})$  is isomorphic to a suitable subcategory  $\underline{\text{PM}}(A)$  of the bicategory of pseudomonoids in  $\underline{R}\underline{\text{CR}}_R$ . We explain how this construction is related to pseudobialgebras.

## Braid cohomology, principal congruence subgroups and geometric representations

Filippo Callegaro\*, Fred Cohen, Mario Salvetti

We describe the integral cohomology of the third braid group  $B_3$  and  $SL_2(\mathbb{Z})$  with local coefficients in a classical geometric representation given by symmetric powers of the natural symplectic representation. We also show how to compute the integral cohomology of classical principal congruence subgroups in  $SL_2(\mathbb{Z})$  as well as their analogues in the third braid group with local coefficients in symmetric powers of the natural symplectic representation. These cohomology groups have a description in terms of the so called "divided polynomial algebra". The results show a strong relation between torsion part of the computed cohomology and fibrations related to loop spaces of spheres. The work extends a classical computation of Shimura to integral coefficients.

Adrian Carabineanu

We linearize the system of equations of magneto-aero dynamics and we use the method of fundamental solutions in order to obtain integral representations of the solution. We find that the solution has a hyperbolic part which satisfies the waves equation and an elliptic part which satisfies Laplace's equation. For obtaining the elliptic part of the solution we have to solve an integral singular equation. Then we calculate the jump of the pressure across the airfoil, the lift, the velocity field and the magnetic induction field.

## Uniform distribution for a class of $k$ -paradoxical oriented graphs

Mihai Caragiu

By using estimates for incomplete character sums with polynomial arguments, we provide uniform distribution results for the dominating sets in a class of  $k$ -paradoxical regular oriented graphs, including the Paley tournaments. Moreover, we will explore a method of quasi-random tournament generation from finite sets of natural numbers, by using the greatest prime factor function.

## Endpoint multilinear Keakeya via Borsuk–Ulam

Anthony Carbery\*, S.I. Valdimarsson

We discuss a new proof of Guth's recent endpoint multilinear Keakeya theorem, which, instead of using a variety of tools from algebraic topology, uses only the Borsuk–Ulam theorem.

## Essential spectra and semigroups of perturbations of generalized SG-hypoelliptic-pseudo-differential operators on $L^p(\mathbf{R}^n)$

Viorel Catană

By using the classes of generalized SG-pseudo-differential operators introduced by Camperi in [1] and Erhling's inequality for  $L^p$ -Sobolev spaces  $H^{s,p}(\mathbf{R}^n)$ ,  $-\infty < s < \infty$ ,  $1 \leq p < \infty$  we prove an analogue of Agmon-Douglis-Nirenberg estimates for SG-hypoelliptic pseudo-differential operators perturbed by singular potentials on  $L^p(\mathbf{R}^n)$ ,  $1 < p < \infty$ , introduced and studied by Schechter in [4].

We also feature some facts concerning essential spectra of SG-hypoelliptic pseudo-differential operators  $T_\sigma$  on  $L^p(\mathbf{R}^n)$ ,  $1 < p < \infty$  perturbed or not by operators of the form  $\sum_{j=1}^r V_j T_{\tau_j}$ , where  $V_j$  are singular potentials on  $L^p(\mathbf{R}^n)$  and  $T_{\tau_j}$  are SG-pseudo-differential operators with suitable symbols  $\tau_j$ ,  $j = 1, 2, \dots, r$ . A self-adjointness result is also proved for such perturbations of SG-hypoelliptic pseudo-differential operators on  $L^2(\mathbf{R}^n)$  whose symbols are independent of spatial variables  $x$  in  $\mathbf{R}^n$ .

Finally, a perturbation result concerning strongly continuous semigroups of contractions generated by SG-hypoelliptic pseudo-differential operators on  $L^p(\mathbf{R}^n)$ ,  $1 < p < \infty$  is given.

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# Uniform decay rate estimates for Schrödinger and Plate equations with nonlinear locally distributed damping

**Marcelo Cavalcanti**

On a compact  $n$ -dimensional Riemannian manifold  $(\mathcal{M}, \mathbf{g})$ , we establish uniform decay rate estimates for the linear Schrödinger and plate equations subject to an internal nonlinear damping locally distributed on the manifold. Our approach can be also employed for other equations provided that inverse inequality for the linear model occurs. In the particular case of the wave equation, where the well known geometric control condition (GCC) is equivalent to the observability inequality, our method generalizes the results due to Cavalcanti et. al. [1] [2] regarding the optimal choice of dissipative regions.

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# Global well-posedness and exponential decay rates for a KdV-Burgers equation with indefinite damping

**Valeria Domingos Cavalcanti**

We consider the the KdV-Burgers equation  $u_t + u_{xxx} - u_{xx} + \lambda u + uu_x = 0$  and its linearized version  $u_t + u_{xxx} - u_{xx} + \lambda u = 0$  on the whole real line. We investigate their well posedness their exponential stability when  $\lambda$  is an indefinite damping.

# Qualitative aspects of Hardy-type inequalities

**Cristian-Mihai Cazacu**

In this talk we discuss some qualitative properties for Schrödinger operators of the form  $A_\lambda := -\Delta - \lambda V$ ,  $\lambda > 0$ , where  $V$  is a positive potential with quadratic singularities in  $\mathbb{R}^N$ ,  $N \geq 1$ . We are concerned with potentials  $V$  having either an isolated singularity or a finite number of singular poles. The range of parameters  $\lambda$  is characterized through Hardy-type inequalities. We show such inequalities and discuss their optimality in smooth domains  $\Omega \subset \mathbb{R}^N$ , in terms of the number and location of the singular poles. In addition, due to the presence of such singularities, standard elliptic regularity of the Dirichlet problem associated to  $A_\lambda$  fails. Finally, we point out these properties in some numerical experiments.

# On the solutions for a class of nonlinear differential inclusion

**Aurelian Cernea**

We study nonlinear differential inclusions of the form

$$x' \in Ax + F(t, x), \quad x(0) = x_0,$$

where  $A$  is a  $m$ -dissipative operator on a Banach space  $X$ ,  $x_0 \in X$  and  $F(\cdot, \cdot) : [0, T] \times X \rightarrow \mathcal{P}(X)$  is a set-valued map with nonconvex values that satisfies Fillipov type assumptions. Using certain selection theorems, some existence results are obtained when  $X$  is separable and nonseparable. We also establish several variational inclusions for solutions of the problem considered in separable Banach spaces.

# Linear strip preservers

**Matthew Chasse\*, Petter Brändén**

Let  $S \subset \mathbb{C}$  be a closed strip, let  $P_n(S)$  be the set of complex polynomials of degree  $n$  whose zeros lie in  $S$ , and let  $P(S) = \bigcup_{n \in \mathbb{N}} P_n(S)$ . We present a characterization for linear operators which *preserve*  $P_n(S)$  (those operators  $T$  which satisfy  $T[P_n(S)] \subseteq P(S) \cup \{0\}$ ). A transcendental characterization follows for linear operators which preserve  $P(S)$ . Necessary and sufficient conditions are obtained for the related problem with real polynomials, and some classical theorems of de Bruijn and Pólya are extended. Specifically, we reveal new differential operators which map real polynomials into real polynomials whose zeros lie in a prescribed narrower strip; this is one of the properties that characterize a “strong univocal factor” as defined by N. G. de Bruijn. The aforementioned results extend naturally to classes of entire functions whose zeros lie in a strip. Using elementary methods, we extend a related theorem of N. G. de Bruijn and J. Ilieff which states a sufficient condition for a function to have a Fourier transform with only real zeros.

## New and old factorizations in certain preduals

**Bernard Chevreau\*, Isabelle Chalendar**

In the context of classical sesquilinear maps associated to a completely nonunitary contraction on a Hilbert space we will revisit factorizations in the preduals of certain weak\* closed subspaces of the Banach algebra of bounded analytic functions in the open unit disk.

## Structural results for $\text{II}_1$ factors associated with actions of negatively curved groups

**Ionuț Chifan\*, Thomas Sinclair, Bogdan Udrea**

In this talk I will survey some recent classification results for  $\text{II}_1$  factors arising from measure preserving actions of negatively curved groups on probability spaces. Various structural properties like solidity, strong solidity, and uniqueness of Cartan subalgebra will also be studied in this context.

## Coxeter transformation and inverses of Cartan matrices for coalgebras

**William Chin\*, Daniel Simson**

Let  $C$  be a coalgebra over a field  $K$  and consider the Grothendieck groups of the categories of left and right socle-finite injective  $C$ -comodules. We study the Coxeter transformation for pointed coalgebras  $C$ , and relate the action of it on indecomposable  $C$ -comodules  $N$  with almost split sequences starting or ending at  $N$ . Under suitable finiteness conditions we show that the Cartan matrix  $\mathcal{C}$  is a (possibly infinite) invertible matrix, and we use it to define the Coxeter transformation and its inverse. This can be applied to the case where  $C$  is a pointed  $K$ -coalgebra such that every vertex of the left Gabriel quiver of  $C$  has only finitely many neighbors. For any indecomposable non-projective finite-dimensional left  $C$ -comodule  $N$ , there exists a unique almost split sequence  $0 \rightarrow \tau N \rightarrow E \rightarrow N \rightarrow 0$  of left  $C$ -comodules ending at  $N$ , where  $\tau$  is the Auslander-Reiten translation. If  $C$  is also a hereditary coalgebra our results show that translated dimension vector  $\dim \tau N$  is given by the Coxeter transformation  $\dim N \mapsto ((\dim N) \cdot \mathcal{C}^{-\text{tr}}) \cdot \mathcal{C} = \dim \tau N$ .

## An infinity criterion for the Brauer $p$ -dimensions of finitely-generated field extensions

**Ivan Chipchakov**

Let  $E$  be a field of absolute Brauer  $p$ -dimension  $\text{abrd}_p(E)$ , for some prime number  $p$ , and let  $F/E$  be a finitely-generated extension of transcendency degree  $\text{trd}(F/E) \geq 1$ . The first main result presented in this talk shows that if  $\text{abrd}_p$  is infinite, then for each pair  $(m, n)$  of positive integers with  $m \leq n$ , there exists a central division  $F$ -algebra  $D_{m,n}$  of exponent  $p^m$  and Schur index  $p^n$ . Moreover, it proves that the Brauer dimension  $\text{Brd}(F)$  is infinite whenever the absolute Brauer dimension  $\text{abrd}(E)$  is infinite. When  $\text{abrd}_p(E)$  is finite, the main result usually enables one to improve Nakayama’s lower bounds on the Brauer  $p$ -dimension  $\text{Brd}_p(F)$  by showing that  $\text{Brd}_p(F) \geq \text{abrd}_p(E) + \text{trd}(F/E)$  in a number of frequently used special cases.

The second main result of the talk indicates that if  $q = 0$  or  $q$  is a prime number, then there exist fields  $E_{q,k}$ :  $k \in \mathbb{N}$ , of characteristic  $q$  satisfying the following conditions, for each index  $k$ : (i)  $\text{Brd}(E_{q,k}) = k$ ; (ii)  $\text{abrd}_p(E_{q,k}) = \infty$ , for all prime numbers  $p$  with, possibly, finitely many exceptions explicitly determined by  $q$ . Thus our main results prove in a strong form that the class of fields of finite Brauer dimension is not closed under the formation of finitely-generated extensions. This solves in the negative a problem posed by Ariel, Brussel, Garibaldi and Vishne in their survey "Open problems on central simple algebras" (Transform. Groups **16** (2011), 219-264).

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## A subclass of biholomorphic mappings generated by $g$ -Loewner chains

**Teodora Chirilă**

In this talk we use the method of Loewner chains to generate certain subclasses of normalized biholomorphic mappings on the Euclidean unit ball  $B^n$  in  $\mathbb{C}^n$ , which have interesting geometric characterizations. To this end, we obtain the characterization of  $g$ -starlike and  $g$ -spirallike mappings of type  $\alpha \in (-\pi/2, \pi/2)$ , as well as of  $g$ -almost starlike mappings of order  $\alpha \in [0, 1)$ , by using  $g$ -Loewner chains. Also, we will use these results to prove that, under certain assumptions, the mapping  $F(z) = P(z)(z)$ ,  $z \in B^n$ , is  $g$ -starlike,  $g$ -spirallike of type  $\alpha \in (-\pi/2, \pi/2)$  and  $g$ -almost starlike of order  $\alpha \in [0, 1)$  on  $B^n$ , where  $P : B^n \rightarrow \mathbb{C}$  is a holomorphic function such that  $P(0) = 1$ . More general, we consider conditions under which  $F$  has  $g$ -parametric representation on  $B^n$ . Various applications of these results are also provided.

## Meet semilattices from sheaves on von Neumann algebras

**Alexandru Chirvășitu**

Recently, Brown and Capraro have introduced an invariant associated to any two von Neumann algebras  $N, M$ , which is roughly speaking the "moduli space" of not-necessarily-unital homomorphisms from  $N$  to  $M$ . It is always a commutative monoid, and under certain technical conditions its Grothendieck group is a vector space. I will explain how the natural ordering coming from the monoid structure in fact makes the monoid into a Dedekind complete semilattice, and how this question arises naturally from a conjecture of Brown and Capraro. I will also explain briefly how, more generally, one can associate to any von Neumann algebra a site, and to any sheaf on this site a Dedekind complete meet semilattice. This helps generalize the above discussion to other invariants one might be interested in, such as moduli spaces of completely positive maps.

## Families of simple compact quantum groups

**Alexandru Chirvășitu**

Recently, Wang introduced a notion of simple compact quantum group, and provided examples. These include the quantum automorphism group of a traced  $C^*$ -algebra, free orthogonal quantum groups (or rather their projective versions), and  $q$ -deformed versions of simple compact Lie groups. One common feature of all of the above examples is that they are "almost classical", in the sense that their representation rings are isomorphic to those of ordinary compact Lie groups. The aim of this talk is to explain how one can obtain a rich supply of examples with highly non-commutative representation rings (and hence not almost classical in the above sense), and why such examples are in fact quite abundant and natural.

## On some special classes of automorphisms in certain 2-groups

**Mihai Chiș\*, Codruța Chiș**

For a characteristic subgroup  $N$  of a group  $G$  we consider the automorphisms of  $G$  which preserve the cosets of the characteristic subgroup  $N$ . In particular, we study central automorphisms (when  $N = Z(G)$ ) and derivational automorphisms (for  $N = G'$ ) and their natural action on particular families of 2-groups.

# Wedge operations and torus symmetries

Suyoung Choi\*, Hanchul Park

A fundamental result of toric geometry is that there is a bijection between toric varieties and fans. More generally, it is known that some class of manifolds having well-behaved torus actions, called topological toric manifolds  $M^{2n} = M(K, \lambda)$ , can be classified in terms of combinatorial data containing simplicial complexes  $K$  with  $m$  vertices. We remark that topological toric manifolds are a generalization of smooth toric varieties. The number  $m - n$  is known as the Picard number when  $M$  is a toric variety. In this talk, we investigate the relationship between the topological toric manifolds over a simplicial complex and the complex obtained by simplicial wedge operation from the one. As applications, we do the following.

1. We classify smooth toric varieties of Picard number 3. This is a reproving of a result of Batyrev.
2. We give a new proof of projectivity of smooth toric varieties of Picard number 3 originally proved by Kleinschmidt and Sturmfels.
3. We prove that every compact toric orbifold over a join of boundaries of simplices is projective. When the toric orbifold is smooth, it is a generalized Bott manifold.
4. We classify topological toric manifolds  $M(K, \lambda)$  when  $K$  is a join of boundaries of simplices. These are exactly quasitoric manifolds over a product of simplices studied by Choi, Masuda, and Suh. When  $M(K, \lambda)$  is a toric variety, then it becomes a generalized Bott manifold.
5. We classify and count real topological toric manifolds when  $m - n \leq 3$ .

## Linear algebraic methods in discrete mathematics and computer science

Sebastian Cioabă

Linear algebra is useful in many areas of discrete mathematics and computer science such as the theory of expander graphs, the theory of strongly and distance-regular graphs or (hyper)graph decomposition among others. In this talk, I will describe some of the applications of linear algebra in these areas.

## Stochastic homogenization of interfaces moving by oscillatory normal velocity

Adina Ciomaga\*, P.E. Souganidis, H.V. Tran

In this talk I will present some recent results concerning the behavior of moving interfaces in random environments, driven by oscillatory normal velocity

$$\begin{cases} u_t^\varepsilon + a\left(\frac{x}{\varepsilon}, \omega\right) |Du^\varepsilon| = 0 & \text{in } (0, \infty) \times \mathbb{R}^n \times \Omega, \\ u^\varepsilon(0, x, \omega) = u_0(x) & \text{on } \mathbb{R}^n \times \Omega. \end{cases}$$

The problem has been studied in great detail in the case when the Hamiltonian is coercive, i.e.  $a(\cdot) \geq a_0 > 0$ . However, the non-coercive case remained an open problem for a long time. Recently (2009) Cardaliaguet, Lions, and Souganidis provided new results in the periodic setting, when  $a(\cdot)$  changes sign. We extend their results to the stationary ergodic environment and we show that under sharp assumptions, fronts homogenize, i.e. as  $\varepsilon \rightarrow 0$  the equation averages to a deterministic Hamilton Jacobi equation.

## Accurate curvature computation in digital images

Adina Ciomaga\*, Pascal Monasse, Jean-Michel Morel

In processes of visual perception, it may be argued on neurological grounds that the human brain could not possibly use all the information provided by states of stimulation, but rather it strips away redundant information and encodes only important features. As such, when processing a visual image, the brain encodes the information along contours (i.e., regions where color changes abruptly), and furthermore those points on a contour at which its direction changes most rapidly (i.e., at angles or peaks of curvature). Yet, a direct computation of curvature on a raw image is impossible. I will explain in this talk how curvatures can

be accurately estimated by a computation on level lines after their independent smoothing. The algorithm, entitled *Level Lines Shortening*, provides results coherent to our visual perception, and gives a new method of features computation and selection (e.g. cores and ridges for fingerprints), similar to the human brain activity.

## The cocenter of a graded affine Hecke algebra and the density theorem

**Dan Ciubotaru\*, Xuhua He**

The affine Hecke algebras arise naturally in the theory of smooth representations of reductive p-adic groups. Motivated by the relation with abstract harmonic analysis for p-adic groups (such as the trace Paley-Wiener theorem and the Density theorem), as well as the study of affine Deligne-Lusztig varieties, it is important to describe the cocenter of affine Hecke algebras, i.e., the quotient of the Hecke algebra by the vector subspace spanned by all commutators. In this talk, we present an algebraic solution for the related problem for the graded affine Hecke algebras introduced by Lusztig. More precisely, we determine a basis of the (twisted) cocenter of graded affine Hecke algebras with arbitrary parameters. In this setting, we prove that the kernel of the (twisted) trace map is the commutator subspace (Density theorem) and that the image is the space of good forms (trace Paley-Wiener theorem).

## A triangular gap of side 2 in a sea of dimers in a 60 degree angle

**Mihai Ciucu\*, Ilse Fischer**

We consider a triangular gap of side 2 in a 60 degree angle on the triangular lattice whose sides are zigzag lines. We study the interaction of the gap with the corner as the rest of the angle being completely filled with lozenges. We show that the resulting correlation is governed by the product of the distances between the gap and its five images in the sides of the angle. This provides a new aspect of the parallel between the correlation of gaps in dimer packings and electrostatics developed by the first author in previous work.

## Integral closure of ideals in first general grade reductions

**Cătălin Ciupercă**

For an ideal  $I = (a_1, \dots, a_n)$  in a commutative noetherian ring  $R$ , a first general grade reduction of  $(R, I)$  is the  $R$ -algebra  $R[X_1, \dots, X_n]/(\sum a_i X_i)$ . As proved by Hochster, several properties of the ring  $R$  are preserved when passing to a first general grade reduction: being reduced and being an integral domain in the case when grade  $I \geq 2$ , and being integrally closed and being a UFD for grade  $I \geq 3$ . In this talk we discuss the behavior of the integral closure of the ideal  $I$  when passing to a first general grade reduction of  $(R, I)$ . We also present several applications.

## Recent progress on isolated singularities for nonlinear elliptic equations

**Florica Cîrstea**

We present recent results which completely classify the behaviour of all positive solutions near an isolated singularity for nonlinear elliptic equations. As a special feature, our approach is developed based on regular variation theory and has wide applicability to elliptic equations incorporating inverse square potentials and weighted nonlinearities. As a by-product, we obtain optimal conditions for the removability of all singularities, thus resolving an open question of Vázquez and Véron (1985).

Sanda Cleja-Țigoiu

The aim of the talk is to provide a mathematical framework able to describe the irreversible behaviour of crystalline materials, based on the lattice defects like dislocations, disclinations and matter distribution defects (or point defects). The geometrical nature of the defects is physically motivated by the crystal lattice attached to material microstructure. The microstructural defects are modeled in terms of the differential geometry concept, following Kröner [6], de Wit [4], Acharya and Fressengeas [1]. In our model the key point is given by the so-called plastic connection, which defines together with the plastic distortion the geometrical structure associated with dislocated plastically deformed material structure, see Cleja-Țigoiu [2], [3]. The curl of the plastic distortion is considered to be a measure of dislocations, while the second order tensorial field which enters the expression of plastic connection, through its curl, defines the disclinations. In the proposed here framework we simultaneously take into account the mentioned type of the structural defects, dislocations, disclinations as well as the point defects. Consequently, the plastic connection has to be modified by a third order tensorial field which is a measure of non-metricity of the appropriate connection, see Schouten [7]. The models are dissipative, all the constitutive and evolution equations being compatible with the principle of the free energy imbalance, see Gurtin, Fried and Anand [5], properly reformulated to capture the internal power expanded during the evolution of the defects.

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## Discrete features of circles and encircling

Cristian Cobeli\*, Florin Guga, Alexandru Zaharescu

We present discrete characteristics drawn out along the metaphoric line from circle packings and an encircling game.

In integer Apollonian circle packings, the set of circles that are tangent to a certain fixed arc has many exceptional properties that are of interest in Number Theory. Today they are most apparent when the arc has zero curvature. In this case, the set of radii of the tangent circles is directly related to  $\mathcal{F}_Q$ , the set of Farey fractions of order  $Q$ . Let  $\mathcal{F}_Q(c, d) \subset \mathcal{F}_Q$  denote the subset of fractions that are congruent to  $c$  modulo  $d$ . The existence of a limit probability measuring the distribution of  $s$ -tuples of consecutive denominators of fractions in  $\mathcal{F}_Q(c, d)$  shows that there is a limit set  $\mathcal{D}(c, d)$ , which is generated by the cluster of points  $(q_0/Q, q_1/Q, \dots, q_s/Q) \in [0, 1]^{s+1}$ , where  $q_0, q_1, \dots, q_s$  are consecutive denominators of members of  $\mathcal{F}_Q$ .

The local density of  $\mathcal{D}(c, d)$  is distinguished and can be expressed as a sum of constant densities on a superposition of a finite number of polygonal layers. Furthermore, in all investigated cases, the layers are tiled mosaics displaying exquisite qualities. This discrete structure behaves like a reflection of the rationality and discreteness of numbers beyond the limiting process.

Our purpose is to investigate the same objects in packings constructed through an iterated process in which more than one circle is inserted between any two neighbor circles.

In the second part we discuss the system of axioms in a natural surrounding game. The shape of the board, which is a bounded lattice, is fundamental, so it may be also considered as an independent axiom. We analyze different patterns, shapes and arrangements and argue the fact that the  $19 \times 19$  square lattice is the “roundest” of them all.

# Alexander polynomials and number theory over fields of functions

Jose I. Cogolludo-Agustin\*, E. Artal-Bartolo, A. Libgober

The purpose of this talk is to present a new connection between the Alexander polynomials of  $X = \mathbb{CP}^2 \setminus C$  for a plane curve  $C$  and the existence of solutions of certain functional equations of *quasi-toric* type

$$F_1 h_1^p + F_2 h_2^q = F_3 h_3^r. \quad (1)$$

In certain cases, the set of quasi-toric decompositions of  $C$  can be endowed with a group structure which can be interpreted as the Mordell-Weil group of an elliptic threefold (in the elliptic case), or with the group of  $S$ -units of the Pell equation (in the toric case). The rank of such groups is associated with the multiplicity of certain roots of the Alexander polynomials. This approach can be used to show the existence of Zariski pairs.

## Are we counting or measuring anything?

Miriam Cohen\*, Sara Westreich

Let  $H$  be a semisimple Hopf algebras over an algebraically closed field  $k$  of characteristic 0. We define Hopf algebraic analogues of commutators and their generalizations and show how they are related to  $H'$ , the Hopf algebraic analogue of the commutator subgroup. We introduce a family of central elements of  $H'$ , which on one hand generate  $H'$  and on the other hand give rise to a family of functionals on  $H$ . When  $H = kG$ ,  $G$  a finite group, these functionals are counting functions on  $G$ . It is not clear yet to what extent they measure any specific invariant of the Hopf algebra. However, when  $H$  is quasitriangular they are at least characters on  $H$ .

## Evolution of membranes via a phase field approach to the Willmore flow with volume and area constraints

Pierluigi Colli

Biological cell membranes define the border between the interior of the cell and its surrounding medium and can be roughly described as a lipid bilayer in which several kinds of lipids are assembled and through which proteins diffuse. The size of the cell (a few microns) is typically much larger than the thickness of the membrane (a few nanometers) and a possible approach to model the geometric properties of the latter is to assume the membrane to be a two-dimensional embedded surface in the three-dimensional space with a shape at equilibrium being determined by the Canham-Helfrich elastic bending energy. In a simplified setting, this energy reduces to the Willmore functional which is a well-known object in differential geometry. Two natural geometric constraints come along with cell membranes: the inextensibility of the membrane fixes the total area while a volume constraint follows from its permeability properties. Recently, experimental results have shown evidence of dynamic instabilities in membranes and provided the impetus for the development of dynamical models. A first approach is to consider the gradient flow associated to the Canham-Helfrich functional which describes the time evolution of a family of (smooth) surfaces. The difficulty with this approach is that it requires to solve a highly nonlinear free boundary problem which is hard to study analytically and costly to compute numerically. A well-known alternative to free boundary problems is the phase-field approach where the sharp interface is replaced by a diffuse interface which is nothing but a thin neighbourhood of thickness  $\varepsilon$  of the zero level set of an  $\varepsilon$ -dependent smooth function, the order parameter. Hence, a phase-field approximation to the Willmore flow with either a volume constraint or area and volume constraints is introduced and studied. The well-posedness of the phase-field approximation to the Willmore flow with either a volume constraint is proved in the general case, and with area and volume constraints is established when the functional approximating the area has no critical point satisfying the two constraints. The existence proofs rely on the underlying gradient flow structure of the problem: the time discrete approximation is solved by a variational minimization principle. The main difficulty stems from the nonlinearity of the area constraint. This talk reports on two joint works with Philippe Laurençot.

## Generic and special constructions of pure O-sequences

Alexandru Constantinescu\*, Thomas Kahle, Matteo Varbaro

In 1977 Stanley conjectured that the h-vector of a matroid is a pure O-sequence. We present two different strategies of approach to this problem and the cases in which they provide solutions. We also partition the set of all matroids in classes, and identify in each class matroids with minimal, respectively maximal, h-vector. These extremal matroids, matroids of Cohen-Macaulay type smaller than 6 and matroids which are truncations of Cohen-Macaulay simplicial complexes are among the families for which we proved that Stanley's conjecture holds. Consequences of our results for the computational search of a counterexample to the conjecture are discussed as well.

# Searching functional exponents for generalized Fourier series

Constantin Corduneanu

The generalized Fourier series whose terms are of the form  $a(n)\exp[i\omega(n,t)]$ , with  $f(n, t)$  real valued and  $n$  integer,  $t$ -real, have emerged some 20 years ago in connection with applications of Fourier Analysis in Engineering Sciences (mostly in IEEE Transactions, various sections). Two mathematical books have been published in 1992, by V.F. Osipov (Univ. of Sankt Petersburg Press, in Russian) and by Chuanyi Zhang (the so-called Pseudo Almost Periodic Functions). The A. is presenting a rather general method in obtaining exponents  $f(n, t)$  which can serve in constructing Banach spaces of oscillatory functions, more general than the classical spaces of periodic or almost periodic functions. The procedure of searching exponents that are adequate in such constructions is based on Cauchy's theorem of residues and on the concept of mean value on the real axis (Poincare). Part of the results have been published in journals and more are in press (including applications to functional equations).

## Mirror symmetry and Fano classification

Alessio Corti\*, Coates, Galkin, Golyshev, Kasprzyk

I will give a status-report on a program to classify Fano manifolds in 3 and 4 dimension from a classification of their mirrors. I will explain a stripped-down version of mirror symmetry for Fano manifolds and illustrate how to recover the classification of Fano 3-fold from the study of their mirrors. I will sketch recent results and open problems.

## De Branges-Rovnyak spaces and Dirichlet spaces

Constantin Costara

For  $b$  in the unit ball of  $H^\infty$ , let  $\mathcal{H}_b$  be the de Branges–Rovnyak space associated to it. For a positive finite Borel measure  $\mu$  on the unit circle  $\mathbb{T}$ , let  $\mathcal{D}_\mu$  be the generalized Dirichlet space associated to  $\mu$ . In 1997, Sarason proved that if  $\mu$  is a point mass measure, we have  $\mathcal{H}_b = \mathcal{D}_\mu$  (with equality of norms) for an appropriate chosen  $b$ . Conversely, Chevrot, Guillot and Ransford proved that if  $\mathcal{H}_b = \mathcal{D}_\mu$  with equality of norms, then necessarily  $\mu$  is a point mass. Can we have  $\mathcal{H}_b = \mathcal{D}_\mu$ , equality as sets, in this case the norms being only equivalent? We shall present necessary conditions for the equality. Sufficient conditions will also be obtained, by constructing explicitly examples of functions  $b$  and non-Dirac measures  $\mu$  on  $\mathbb{T}$  such that  $\mathcal{H}_b = \mathcal{D}_\mu$ . Applications of the equality case to the study of the de Branges–Rovnyak and Dirichlet spaces will also be presented.

## Multiplicity of critical points for non-smooth parametrized functionals

Nicușor Costea

In this talk we discuss a non-smooth Ricceri-type multiplicity result concerning parametrized functions defined on a real reflexive Banach space. Some applications to differential inclusions are also presented.

## An efficient model to handle a dynamic TSP with chaotic behavior

Gloria Cerasela Crișan\*, Elena Nechita

Current real-life situations need to process high volumes of dynamic data, with complex connections, and to quickly provide efficient decisions, that satisfy large set of complicated and inter-related constraints. The didactic problems are continuously adapted to these situations, forcing the researchers to devise new variants and new methods for approaching them. One such example is the well-known Traveling Salesman Problem (TSP): with more than 80 years from its formalization, having simple specification but being hard to solve, TSP is now under heavy investigation and has many generalizations (an example is the group of Vehicle Routing problems) or variants (i.e. with stochastic, robust or dynamic features). Basically, TSP seeks for a closed tour with a minimum length connecting all the cities from a map.

Ant Colony Optimization (ACO) is a biologically-inspired metaheuristic devised in 1999 for solving graph-represented optimization problems (like TSP). This method uses artificial agents that concurrently construct tours and deposit pheromone on traversed edges; the following agents are guided by the local intensity of the pheromone when constructing their paths, like real ants do. At the end, the procedure returns with the shortest tour found.

This talk describes a dynamic variant of TSP, with chaotic selection of the nodes that suffer local modification. During the execution of the solving procedure, some nodes are chosen using a chaotic dependency, and the lengths of all their adjacent edges are slightly modified and then brought back to their initial values. This is a model of real situations when heavy weather conditions could impact on travel

time between two neighbor cities. This dynamic TSP version is solved with an ACO algorithm that splits the execution time in epochs, each one consisting of fixed number of iterations, and having several fixed nodes that initially increase the distances to all their neighbors, and then decrease these distances down to the correct values. As the problem is dynamic, there is no solution in the classic sense, but the performance is measured using the recovery speed, meaning how many iterations were needed to obtain a solution at least as good as the one from before the perturbation was manifested.

For experiments we used five instances from the TSP Benchmarks publicly available, and we showed that, like their natural models, the artificial ants are robust and effective. This encouraging behavior allows us to further investigate more TSP instances, and to expand our model to other TSP variants.

## Finitely accessible categories versus module categories

Septimiu Crivei

An additive category is called *finitely accessible* if it has direct limits, the class of finitely presented objects is skeletally small, and every object is a direct limit of finitely presented objects. Any finitely accessible category  $\mathcal{C}$  may be embedded via the covariant Yoneda functor as a full subcategory of the category  $\text{Mod}(A)$  of unitary right modules over a certain ring with enough idempotents  $A$ , called the *functor ring* of  $\mathcal{C}$ . This allows one to relate properties of  $\mathcal{C}$  with properties of the module category  $\text{Mod}(A)$ . We shall review the main connections between them, and present some new ones, especially related to purity and approximations.

## Control of the model for HIV infection of $CD4^+T$ cells

Ana-Maria Croicu

We will present some results of the control for the HIV Infection of  $CD4^+T$  cells by applying the classical control theory. The model under consideration is characterized by a system of nonlinear differential equations on the concentration of susceptible  $CD4^+T$  cells,  $CD4^+T$  cells infected by the HIV viruses and free HIV virus particles in the blood

$$\begin{aligned} \frac{dT}{dt} &= q - \alpha T + rT \left(1 - \frac{T+I}{T_{\max}}\right) - kVT, \quad T(0) = r_1 \\ \frac{dI}{dt} &= kVT - \beta I, \quad I(0) = r_2, \quad 0 \leq t \leq R < \infty \\ \frac{dV}{dt} &= \mu\beta I - \gamma V, \quad V(0) = r_3. \end{aligned}$$

## Extensions of Hopf algebras with nonzero integral

Juan Cuadra

The Haar measure on a compact group induces a linear functional  $\int$  on the Hopf algebra of its representative functions. The invariance property of the Haar measure reads as a condition on  $\int$  that can be expressed in Hopf algebraic terms. Sweedler defined an algebraic notion of integral for Hopf algebras using this condition. Hopf algebras having a nonzero integral are also called co-Frobenius. Quantized coordinate algebras of simple algebraic groups figure among the most relevant examples of them. In this talk we will discuss extensions of co-Frobenius Hopf algebras. Given an extension of Hopf algebras  $1 \rightarrow A \rightarrow B \rightarrow C \rightarrow 1$  with  $B$  faithfully coflat as a  $C$ -comodule, we will show that  $B$  is co-Frobenius if and only if  $A$  and  $C$  are co-Frobenius. We will also show that  $B$  has an integral that restricted to  $A$  is nonzero if and only if  $A$  is co-Frobenius and  $C$  is cosemisimple. We will derive from this that  $B$  is cosemisimple if and only if  $A$  and  $C$  so are. New characterizations of co-Frobenius Hopf algebras will be established to achieve these results.

These results appear in the joint work with Nicolás Andruskiewitsch *On the structure of (co-Frobenius) Hopf algebras*, J. Noncommut. Geom. **7** (2013), 83-104; arXiv:1011.3457.

## Semigroups from algebraic number theory and operator algebras

Joachim Cuntz\*, Siegfried Echterhoff, Xin Li

The  $C^*$ -algebra associated with the  $ax + b$ -semigroup (or the multiplicative semigroup) of a ring of algebraic integers has an interesting structure. Its  $K$ -theory reflects properties of the corresponding number field.

# Non-positivity of the semigroup generated by the Dirichlet-to-Neumann operator

Daniel Daners

Let  $\Omega \subseteq \mathbb{R}^N$  be a bounded open set with smooth boundary, and let  $\lambda \in \mathbb{R}$ . The Dirichlet-to-Neumann operator  $D_\lambda$  is a closed operator on  $L^2(\partial\Omega)$  defined as follows. Given  $\varphi \in H^{1/2}(\Omega)$  solve the Dirichlet problem

$$\Delta u + \lambda u = 0 \quad \text{in } \Omega, \quad u = \varphi \quad \text{on } \partial\Omega.$$

A solution exists if  $\lambda$  is not an eigenvalue of  $-\Delta$  with Dirichlet boundary conditions. If  $u$  is smooth enough we define

$$D_\lambda \varphi := \frac{\partial u}{\partial \nu},$$

where  $\nu$  is the outer unit normal to  $\partial\Omega$ . It is known that  $D_\lambda$  extends uniquely to a closed operator in  $L^2(\partial\Omega)$  again by  $D_\lambda$ , and that  $-D_\lambda$  generates an analytic semigroup  $e^{-tD_\lambda}$  on  $L^2(\partial\Omega)$ . Let  $0 < \lambda_1 < \lambda_2 < \lambda_3 < \dots$  be the strictly ordered Dirichlet eigenvalues of  $-\Delta$  on  $\Omega$ . It was shown by Arendt and Mazzeo that  $e^{-tD_\lambda}$  is positive and irreducible if  $\lambda < \lambda_1$ . The question left open was whether or not the semigroup is positive for any  $\lambda > \lambda_1$ . The aim of this talk is to explore this question by explicitly computing the semigroup explicitly for the disc in  $\mathbb{R}^2$ . The example demonstrates some new phenomena:

1. The semigroup  $e^{-tD_\lambda}$  is positive and irreducible for all  $\lambda < \lambda_1$ , and for  $\lambda$  in a left neighbourhood of every simple eigenvalue.
2. The semigroup  $e^{-tD_\lambda}$  is eventually positive and irreducible for all  $\lambda \in (\lambda_3, \lambda_4)$ . More precisely, there exists  $T > 0$  such that  $e^{-tD_\lambda}$  is positive and irreducible for all  $t \geq T$  and all  $\lambda \in (\lambda_3, \lambda_4)$ .
3. The semigroup  $e^{-tD_\lambda}$  is not positive for  $\lambda$  in a neighbourhood of every double eigenvalue, and in a right neighbourhood of every simple eigenvalue.

In particular, the semigroup  $e^{-tD_\lambda}$  can change from not positive to positive between two eigenvalues. This happens for  $\lambda \in (\lambda_3, \lambda_4)$ . Moreover, it is possible that  $e^{-tD_\lambda}$  is positive (and irreducible) for large enough  $t$ , but not for small  $t$ . This seems to be a kind of semigroup not observed before. We finally discuss some conclusions and conjectures regarding more general domains.

## Geodesics in the space of Kähler metrics

Tamas Darvas

Given a compact Kähler manifold  $(X, \omega)$  let  $\mathcal{H}$  be the set of Kähler potentials of  $\omega$ . As observed by Mabuchi,  $\mathcal{H}$  has a natural infinite dimensional Riemannian manifold structure. Following Donaldson, existence and regularity of geodesics in this space is of great interest. The endpoint problem for the geodesic equation is a PDE of complex Monge-Ampère type. In this talk we discuss regularity issues, as well as methods of obtaining weak solutions to this problem.

## Frobenius algebras of corepresentations: gradings

Sorin Dăscălescu\*, Constantin Năstăsescu, Laura Năstăsescu

We investigate Frobenius algebras in the monoidal category of right comodules over a Hopf algebra  $H$ . If  $H$  is a group Hopf algebra, we study a more general Frobenius type property and uncover the structure of graded Frobenius algebras. Graded symmetric algebras are also considered. The connection to Frobenius functors is discussed.

## Hitting times of Bessel processes

Mădălina Deaconu\*, Samuel Herrmann

The aim of this talk is to present new methods for the simulation of the hitting times of Bessel processes. Explicit formulas, via Laplace transform, are already known for these hitting times but they are not easy to use for numerical purposes. First, we will consider Bessel processes with integer dimension. We will construct a new method, called the walk on moving spheres algorithm. In this new approach, we combine the method of images for the first hitting time, of a non-linear boundary for the Brownian motion, with the random walk on the spheres method, for the heat equation. The connexion between the Bessel process and the square root of the Euclidean norm of a Brownian motion is also an important tool for this study. Finally, an ongoing work on Bessel processes with non-integer dimension will be presented. The idea is to use the additivity property of the class of Bessel processes and to decompose, at each step, the algorithm in two parts. Our procedure can be adapted for the Cox-Ingersoll-Ross process and can be applied in problems arising from finance and neurosciences.

# Group actions on graphs and Doplicher-Roberts algebras

Valentin Deaconu

Let the group  $G$  act on a directed graph  $E$ . This determines a representation  $\rho$  of  $G$  on the  $C^*$ -correspondence  $\mathcal{H}_E$  and an action on the Cuntz-Pimsner algebra  $C^*(E)$ . Our goal is to study the crossed product  $C^*(E) \rtimes G$  and the fixed point algebra  $C^*(E)^G$  when  $G$  is compact and the action is arbitrary. We define the Doplicher-Roberts algebra  $\mathcal{O}_\rho$  associated to  $\rho$ , constructed from intertwiners  $(\rho^m, \rho^n)$ , where  $\rho^n = \rho^{\otimes n}$  on  $\mathcal{H}_E^{\otimes n}$ . In some cases,  $\mathcal{O}_\rho$  is strongly Morita equivalent to  $C^*(E) \rtimes G$  and their K-theory can be computed. We also discuss the crossed product of a  $C^*$ -correspondence by a group  $G$ , with examples. In particular, if  $G$  and  $E$  are finite, we prove that  $C^*(E) \rtimes G$  is SME to a graph algebra.

## Minimality of toric arrangements

Emanuele Delucchi\*, Giacomo d'Antonio

A toric arrangement is given by a family  $A$  of level sets of characters of a complex torus  $T$ . The focus of this talk will be on the topology of the complement  $M := T \setminus A$ , and in particular on the extent to which it is determined by the combinatorial data of the arrangement  $A$ . After an introduction to toric arrangements, I will present some recent joint work with Giacomo d'Antonio, proving that  $M$  is a minimal space (and thus homologically torsion-free). Our methods include a generalization of Discrete Morse Theory to nonregular cell complexes, providing a sequence of cellular collapses leading to a minimal complex.

## Progress on the discrete restriction problem

Ciprian Demeter

I will discuss a few new lines of approach for the discrete restriction problem to the sphere and the paraboloid. Part of the work is joint with Jean Bourgain.

## Modularity and the reciprocal plane

Graham Denham\*, Mehdi Garrounian, Ștefan Tohăneanu

Let  $\mathcal{A}$  be a collection of  $n$  linear hyperplanes in  $K^\ell$ , where  $K$  is an algebraically closed field. The Orlik-Terao algebra of  $\mathcal{A}$  is the subalgebra  $R(\mathcal{A})$  of the rational functions generated by reciprocals of linear forms vanishing on hyperplanes of  $\mathcal{A}$ . It determines an irreducible subvariety  $Y(\mathcal{A})$  of  $\mathbb{P}^{n-1}$ . We show that a subspace  $X$  in the arrangement is modular if and only if  $R(\mathcal{A})$  is a split extension of the Orlik-Terao algebra of the subarrangement  $\mathcal{A}_X$ . This provides another refinement of Stanley's Modular Factorization Theorem, and an analogue of the Modular Fibration Theorem. We deduce that if  $\mathcal{A}$  is supersolvable, then its Orlik-Terao algebra is Koszul. In certain cases, the algebra is also a complete intersection, and we characterize when this happens.

## Fourier series near $L^1$ and bilinear Hilbert transform near $L^1 \times L^2$

Francesco Di Plinio\*, Ciprian Demeter

The classical counterexample by Kolmogorov exhibits an  $L^1$  function with Fourier series divergent almost everywhere on the torus. Subsequent refinements by Antonov, Konyagin et al. have led to the conjectures that  $L \log L$  is the sharp order of integrability sufficient for almost everywhere convergence of the full Fourier series, and that  $L \log \log L$  suffices for almost everywhere convergence along any lacunary subsequence. In terms of the Carleson maximal operator, and respectively of its lacunary version, this corresponds to conjecturing boundedness from the Orlicz space  $L \log L$  into weak  $L^1$  and respectively from  $L \log \log L$  into weak  $L^1$ . One of the main difficulties arising in the treatment of the Carleson operator, and of the related bilinear Hilbert transform, acting on function spaces close to  $L^1$ , is that the usual Calderón-Zygmund decomposition fails to be effective, due to the modulation invariance properties of both operators. In this talk, we present several endpoint (near  $L^1$  and  $L^1 x L^2$ ) bounds for both the Carleson operator and the Walsh analogue of the bilinear Hilbert transform, relying on appropriate multi-frequency versions of the Calderón-Zygmund decomposition and of refinements of the classical proofs of boundedness of Lacey-Thiele. In particular, we are able to improve on the best known results, due respectively to Lie and Do-Lacey, on almost everywhere convergence of Fourier and Walsh-Fourier lacunary series.

Andrei Drăgănescu\*, Ana Maria Soane

We construct multigrid preconditioners to accelerate the solution process of optimal control problems constrained by the Stokes/Navier-Stokes equations. Our approach for the Stokes control problem is to eliminate the state and adjoint variables from the optimality system and to construct efficient multigrid preconditioners for the Schur-complement of the block associated with these variables. Similar preconditioners are constructed for the reduced Hessian in the Newton-PCG method for the optimal control of the stationary Navier-Stokes equations.

## Classical invariant theory for free metabelian Lie algebras

Vesselin Drensky\*, Sehmus Findik

Let  $KX_d$  be the vector space with basis  $X_d = \{x_1, \dots, x_d\}$  over a field  $K$  of characteristic 0. One of the main topics of classical invariant theory is the study of the algebra of invariants  $K[X_d]^{SL_2(K)}$ , where  $KX_d$  is a module of the special linear group  $SL_2(K)$  isomorphic to a direct sum  $V_{k_1} \oplus \dots \oplus V_{k_r}$  and  $V_k$  is the  $SL_2(K)$ -module of binary forms of degree  $k$ . Noncommutative invariant theory deals with the algebra of invariants  $F_d(\mathfrak{A})^G$  of the group  $G < GL_d(K)$  acting on the relatively free algebra  $F_d(\mathfrak{A})$  of a variety of  $K$ -algebras  $\mathfrak{A}$ . When the group  $G$  is finite and  $\mathfrak{A}$  is a variety of associative algebras (including the case of the variety of all associative algebras) the picture is quite clear, see e.g. [6]. Not too much is known when  $\mathfrak{A}$  is a variety of Lie algebras, see [1, 3]. The invariants  $F_d(\mathfrak{A})^{UT_2(K)}$  of the unitriangular group  $UT_2(K)$  were considered in [5, 4]. Recently the special case  $F_d(\mathfrak{A}^2)^{UT_2(K)}$ , where  $\mathfrak{A} = \mathfrak{A}^2$  is the variety of metabelian Lie algebras was studied in detail in [2].

We study the algebra  $F_d(\mathfrak{A}^2)^{SL_2(K)}$  of invariants of the free metabelian Lie algebra  $F_d(\mathfrak{A}^2)$ . We describe the cases when the algebra is finitely generated. This happens if and only if  $KX_d \cong V_1 \oplus V_0 \oplus \dots \oplus V_0$  or  $KX_d \cong V_2$  as an  $SL_2(K)$ -module (and in the trivial case  $KX_d \cong V_0 \oplus \dots \oplus V_0$ ). For small  $d$  we give a list of generators even when  $F_d(\mathfrak{A}^2)^{SL_2(K)}$  is not finitely generated.

The methods for establishing that the algebra  $F_d(\mathfrak{A}^2)^{SL_2(K)}$  is not finitely generated work also for other relatively free algebras.

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## Dilations and constrained algebras

Michael Dritschel

It is well known that contractive representations of the disk algebra are completely contractive. Let  $\mathcal{A}$  denote the subalgebra of the disk algebra consisting of those functions  $f$  for which  $f'(0) = 0$ . We prove that there are contractive representations of  $\mathcal{A}$  which are not completely contractive, and furthermore characterize those contractive representations which are completely contractive.

## Some examples of two-dimensional regular rings

Tiberiu Dumitrescu\*, Cristodor Ionescu

Let  $B$  be a ring and  $A = B[X, Y]/(aX^2 + bXY + cY^2 - 1)$  where  $a, b, c \in B$ . We study the smoothness of  $A$  over  $B$ , and the regularity of  $B$  when  $B$  is a ring of algebraic integers.

# A "super-diagonal form" decomposition theorem in $\text{II}_1$ factors

Ken Dykema

A classical result of Ringrose is that every compact operator on Hilbert space can "diagonal + upper triangular" form with respect to an increasing family of invariant subspaces. Here, the "strictly upper triangular" part is quasinilpotent. In joint work with Fedor Sukochev and Dmitriy Zanin, and building upon results of Uffe Haagerup and Hannah Schultz, we prove an analogous result in  $\text{II}_1$  factors. In particular, an arbitrary element  $T$  of a  $\text{II}_1$  factor can be written as  $T = N + Q$ , where  $N$  is a normal operator whose Brown measure agrees with that of  $T$ , and where  $Q$  is a strong-operator-topology quasinilpotent operator.

## Quantum symmetric states on universal free product $C^*$ -algebras

Ken Dykema\*, Claus Koestler, John Williams

We study quantum symmetric states on universal free product  $C^*$ -algebras of the form  $*_1^\infty A$ , for an arbitrary  $C^*$ -algebra  $A$ . This is a generalization of the notion of quantum exchangeable random variables. By extending and building on the proof of the noncommutative de Finetti theorem of Koestler and Speicher, we prove a de Finetti type theorem that characterizes quantum symmetric states in terms of amalgamated free products, with amalgamation over the tail algebra. This allows a convenient description of the set of all quantum symmetric states and a convenient characterization of the extreme quantum symmetric states.

## $K$ -theory for crossed products by group actions on totally disconnected spaces and of semi-group algebras

Siegfried Echterhoff\*, Joachim Cuntz, Xin Li

In this lecture we give a report on joint work with Joachim Cuntz and Xin Li on the computation of the  $K$ -theory for crossed products by certain actions of groups on totally disconnected spaces. We apply the results to the computation of the  $K$ -theory for certain semi-group  $C^*$ -algebras. In particular, we obtain explicit computations for the  $ax + b$ -semigroups  $R \rtimes R^*$ , where  $R$  is the ring of integers in a number field.

## On some convexity results for mean curvature type equations

Cristian Enache

Convexity is an issue of interest for a long time in pde's, being intimately related to the study of geometric properties of solutions. The starting point in this research is a result of M. Shiffman [Ann. of Math., 1956], which says that if a 2-dimensional minimal surface  $M$  is bounded by two convex plane curves  $C_1$  and  $C_2$ , lying in parallel planes, then every intersection of  $M$  by a plane parallel to the plane of  $C_1$  and  $C_2$  is again a convex curve. One year later, R. Gabriel [J. of LMS, 1957] proved that the level sets of the Green function in 3-dimensional convex domains are strictly convex. After other 14 years, L.G. Makar-Limanov [Math. Notes. Acad. Sci. USSR, 1971] considered the torsion problem in a bounded convex planar domain and using an elegant argument, involving a maximum principle for an appropriate auxiliary function, he proved that  $\sqrt{u}$  is strictly concave. Then, H.J. Brascamp-E.H. Lieb [J. Funct. Anal. 1976] established the log-concavity of the first eigenfunction of the Laplacian in convex domains. New proofs of the Brascamp-Lieb's result were later obtained by A. Acker-L.E. Payne-G.A. Philippin [Z. Angew Math. Phys., 1981] (using the methods of Makar-Limanov) or by N.J. Korevaar [Indiana Univ. Math. J., 1983] and L.A. Caffarelli-J. Spruck [Comm. on PDE's, 1982]. Also, in 1982, making use of an ingenious comparison technique due to A.D. Alexandrov [Amer. Math. Soc. Transl., 1962], J.T. Chen and W.H. Huang [Invent. Math., 1982] proved the convexity of capillary surfaces in outer space. One year later, in another paper, N.J. Korevaar [Indiana Univ. Math. J., 1983] used a concavity maximum principle to establish convexity results for some mean curvature type equations. B. Kawohl [Math. Meth. Appl. Sci., 1986] and A.U. Kennington [Indiana Univ. Math. J., 1985] improved Korevaar's concavity principle and obtained a higher-dimensional extension of Makar-Limanov's result (see also X.N. Ma-S. Shi-Y. Ye [Comm. on PDE's, 2013], for an extension to higher dimension using Makar-Limanov's idea). In a fundamental work of L.A. Caffarelli-A. Friedman [Duke Math. J., 1985], a deformation technique and a constant rank theorem were used to establish the strict convexity of level sets of solution to a semilinear elliptic problem in planar convex domain (see also I. Singer-B. Wong-S.T. Yau-S.S.T. Yau [Ann. Sc. Norm. Sup. Pisa, 1985] or D. Finn [Comm. Pure Appl. Anal., 2008]). Caffarelli-Friedman's result was later extended to higher dimensions by N.J. Korevaar-J. Lewis [Arch. Ration. Mech. Anal., 1987]. Also, motivated by some differential geometry problems, constant rank theorems were recently obtained by P. Guan-X.-N. Ma [Invent. Math., 2003], L.A. Caffarelli-P. Guan-X.-N. Ma [Comm. Pure Appl. Math., 2007] or B.J. Bian-P. Guan [Invent. Math., 2009]. More details on the convexity issue may be also found in a book of B. Kawohl [Springer Verlag, 1985]. Despite all these important achievements, similar questions regarding convexity properties for several classical mean curvature type problems have been open for long time. The aim of this talk is to present some new achievements in this direction.

# Microstructure models for composites with imperfect interface via the periodic unfolding method

Horia Ene\*, Claudia Timofte

Using the periodic unfolding method, a macroscopic model for an  $\varepsilon$ -periodic elastic composite formed by two interwoven and connected components with imperfect contact at the interface is analyzed. We assume that on the interface there is a jump in the displacement vector. The order of magnitude of this jump with respect to the small parameter  $\varepsilon$  defines the macroscopic elastodynamic equations and the analysis shows three different important cases. More precisely, we obtain one or two equations at the macroscale, with different stiffness tensors: (i) if the intensity of the jump is of order  $\varepsilon^{-1}$ , we obtain only one equation at the macroscale, with the stiffness tensor depending on the jump coefficient; (ii) if the intensity of the jump is of order  $\varepsilon$ , we obtain a system of two coupled equations with classical stiffness tensors; (iii) if the intensity of the jump is of order 1, we obtain at the macroscale only one equation, with no influence in the macroscopic tensor. The convergence of the homogenization process is proven in all the cases.

## Binomial ideals and graphs

Viviana Ene

In this talk we discuss about binomial ideals arising from graphs. Given a simple graph  $G$  on the vertex set  $[n]$ , one may associate with it a binomial ideal  $J_G$  in the polynomial ring  $K[X]$  over a field  $K$ , where  $X = \begin{pmatrix} x_1 & x_2 & \dots & x_n \\ y_1 & y_2 & \dots & y_n \end{pmatrix}$ .  $J_G$  is generated by maximal minors of  $X$ ,  $f_{ij} = x_i y_j - x_j y_i$  with  $\{i, j\}$  edge of  $G$ , and is called the *binomial edge ideal* of  $G$ . Later on, the notion of binomial edge ideal was generalized to a pair of graphs. The interest in studying (generalized) binomial edge ideals partially comes from the fact that they turned out to have applications in statistics. In our talk, we are going to discuss various algebraic and homological properties of binomial edge ideals. We mainly focus on some recent results obtained in joint papers with J. Herzog, T. Hibi, A. Qureshi, A. Zarojanu.

## Smoothing for dispersive PDE's and applications

M. Burak Erdogan\*, Nikos Tzirakis

In this talk we will discuss a smoothing phenomena for various dispersive PDE on the torus. Examples include the nonlinear Schrödinger equation, the KdV equation and the Zakharov system. As applications we will discuss nonlinear Talbot effect, existence and structure of global attractors for dispersive PDE with dissipation, and bounds for higher order Sobolev norms.

## Semilocal categories, local functors and applications

Alberto Facchini

A ring  $R$  is *semilocal* if  $R/J(R)$  is semisimple artinian, that is, a finite direct product of rings of matrices over division rings. A preadditive category  $\mathcal{A}$  is a *null* category if all its objects are zero objects. A preadditive category is *semilocal* if it is non-null and the endomorphism ring of every non-zero object is a semilocal ring. The following are examples of full semilocal subcategories of the category  $\text{Mod-}R$  of all right modules over an associative ring  $R$ :

1. the full subcategory of all artinian right  $R$ -modules (Camps and Dicks);
2. the full subcategory of all finitely generated  $R$ -modules, for  $R$  a semilocal commutative ring (Warfield);
3. the full subcategory of all finitely presented modules right  $R$ -modules, for  $R$  a semilocal ring (Facchini and Herbera);
4. the full subcategory of all serial modules of finite Goldie dimension (Herbera and Shamsuddin);
5. the full subcategory of all modules of finite Goldie dimension and finite dual Goldie dimension (Herbera and Shamsuddin).

An additive functor  $F: \mathcal{A} \rightarrow \mathcal{B}$  between preadditive categories  $\mathcal{A}$  and  $\mathcal{B}$  is said to be a *local functor* if, for every morphism  $f: A \rightarrow A'$  in  $\mathcal{A}$ ,  $F(f)$  isomorphism in  $\mathcal{B}$  implies  $f$  isomorphism in  $\mathcal{A}$ . This notion must not be confused with the notion of *isomorphism reflecting* functor: for every  $A, A'$  objects of  $\mathcal{A}$ ,  $F(A) \cong F(A')$  implies  $A \cong A'$ . We will present the interplay between the concepts of semilocal category, local functor, Jacobson radical of the category and maximal ideals. Our main concern will be the study of the canonical functor  $\mathcal{A} \rightarrow \mathcal{A}/\mathcal{I}_1 \times \dots \times \mathcal{A}/\mathcal{I}_n$ , where  $\mathcal{A}$  is a preadditive category and  $\mathcal{I}_1, \dots, \mathcal{I}_n$  are ideals of  $\mathcal{A}$ . We will consider and characterize the case where this canonical functor is local. An application will be given.

# Torelli problem for arrangements of divisors

Daniele Faenzi

Given a divisor  $D$  in the projective  $n$ -space, we consider the sheaf  $T(-\log D)$  of tangent fields with logarithmic poles along  $D$ . The Torelli problem for  $D$  is the question whether  $D$  is recovered by this sheaf. One way to formulate this is to ask when do the irreducible components of  $D$  appear as the only unstable divisors of  $T(-\log D)$ .

## A combinatorial model for the Milnor fiber of the discriminant

Michael Falk\*, Thomas Brady, Colum Watt

For a finite real reflection group  $G$ , we describe a regular cell complex built from the non-crossing partition lattice  $NCP_G$  of  $G$  that has the homotopy type of the Milnor fiber of the corresponding discriminant  $\Delta_G$ . This cell complex has the structure of an iterated mapping cone, yielding a combinatorially-defined cochain complex computing the cohomology of  $\Delta_G$ .

## Singularities of moduli spaces of level curves

Gavril Farkas\*, Alessandro Chiodo

I will describe a graph-theoretic formalism to describe the singularities of the compactified moduli space of genus  $g$  curves together with a point of order  $n$  in its Jacobian variety. In particular, we determine all non-canonical singularities of the moduli space and show that for  $n \geq 5$  they impose no adjunction conditions

## A classical Perron method for existence of smooth solutions to boundary value and obstacle problems for degenerate differential operators via holomorphic maps

Paul Feehan

We prove existence of solutions to boundary value problems and obstacle problems for degenerate-elliptic, linear, second-order partial differential operators with partial Dirichlet boundary conditions using a new version of the Perron method. The elliptic operators considered have a degeneracy along a portion of the domain boundary which is similar to the degeneracy of a model linear operator identified by Daskalopoulos and Hamilton (1998) in their study of the porous medium equation or the degeneracy of the Heston operator (1993) in mathematical finance. Existence of a solution to the Dirichlet problem on a half-ball, where the operator becomes degenerate on the at boundary and a Dirichlet condition is only imposed on the spherical boundary, provides the key additional ingredient required for our Perron method. Surprisingly, proving existence of a solution to this Dirichlet problem with "mixed" boundary conditions on a half-ball is a harder problem than one might expect. Due to the difficulty in developing a global Schauder estimate and due to compatibility conditions arising where the "degenerate" and "non-degenerate boundaries" touch, one cannot directly apply the continuity or approximate solution methods. However, in dimension two, there is a holomorphic map from the half-disk to the infinite strip in the complex plane and one can extend this definition to higher dimensions to give a diffeomorphism from the half-ball to the infinite "slab". The solution to the Dirichlet problem on the half-ball can thus be converted to a Dirichlet problem on the slab, albeit for an operator which now has exponentially growing coefficients. The required Schauder regularity theory and existence of a solution to the Dirichlet problem on the slab can nevertheless be obtained using previous work of the author and Camelia Pop in arXiv:1210.6727. Our Perron method relies on weak and strong maximum principles for degenerate-elliptic operators, suitable concepts of continuous subsolutions and supersolutions for boundary value and obstacle problems for degenerate-elliptic operators, and maximum and comparison principle estimates developed by the author in arXiv:1204.6613.

## Twisted partial actions of groups on semiprime rings and Goldie rings

Miguel Ferrero

Partial actions of groups have been defined and studied first in  $C^*$  algebras by R. Exel. Then the notion of partial action in a pure algebraic context has been defined by M. Dokuchaev and R. Exel. Later on the notion of twisted partial actions and partial crossed products have been defined by M. Dokuchaev, R. Exel and J. Simon. In this talk we recall the basic definitions. Then we consider a twisted partial action  $\alpha$  of a group  $R$  on a semiprime ring  $R$ . We show that  $\alpha$  can be extended to a twisted partial action on the Martindale ring of quotients and also to the maximal ring of quotients of  $R$ . Using these extensions we study the transfer of the property of being a right (left) Goldie ring between  $R$  and the partial crossed product  $R \rtimes_{\alpha} G$  under some assumption on  $G$ . The results of this talk are contained in a joint work with L. Bemm, W. Cortes and S. de la Flora.

# Differentiability for solutions of linear integral equations with weakly singular kernels

Mikil Foss\*, Petronela Radu

Let  $\Omega \subset \mathbb{R}^n$  be an open bounded domain. With  $\delta > 0$ , suppose that  $u \in L^2(\mathbb{R}^n; \mathbb{R}^N)$  is smooth outside of  $\Omega$  and satisfies

$$\int_{\Omega} \int_{B_{\delta}(x)} [u(x) - u(y)][\phi(x) - \phi(y)]\mu(|x - y|) dy dx = 0$$

for all  $\phi \in L^2(\mathbb{R}^n; \mathbb{R}^N)$  that are 0 outside of  $\Omega$ . The primary assumption on  $\mu$  is that there is a constant  $C > 0$  and an  $\alpha$ .

## Positivity properties of numerical cycles on projective varieties

Mihai Fulger\*, Brian Lehmann

Where the geometry of curves and divisors on algebraic varieties is reasonably well-studied and understood, higher (co)dimensional cycles are seen as a land of pathology. We investigate notions of pseudoeffectivity and movability for all numerical cycle classes. We use these to define Zariski decompositions for cycles, and to approach a conjecture on the structure of pseudoeffective classes contracted by pushforwards via projective morphisms.

## Finite quotients of symplectic groups and mapping class groups

Louis Funar

The aim of this talk is to explore the relations between the residual finiteness of central extensions, the 2-cohomology and the set of finite quotients for both symplectic and mapping class groups.

## Drift and the risk free rate

Anda Gadidov\*, Marcus Carl Spruill

Consider the Black-Scholes model of a market with a single equity whose price follows a geometric Brownian motion on  $[0, \infty)$  satisfying the stochastic differential equation:

$$dS_t = \mu(t)S_t dt + \sigma(t)S_t dB_t.$$

Suppose that the volatility  $\sigma(t)$ , the drift  $\mu(t)$  and the risk-free interest rate  $r(t)$  satisfy the following assumptions:

- (A1) The functions  $\mu(t), r(t)$  are continuous on  $[0, \infty)$  and  $\sigma(t)$  is absolutely continuous with a derivative bounded on compact intervals;
- (A2)  $\sigma(t)$  and  $r(t)$  are uniformly bounded;
- (A3) the risk premium  $\rho(t) = \sigma^{-1}(t)(\mu(t) - r(t))$  is uniformly bounded.

We prove that under assumptions (A1) - (A3) the drift rate must be close to the risk free rate. Moreover, if the drift rate  $\mu$  and the risk free rate  $r$  are constant, then  $\mu = r$  and the price process is the same under both the empirical and the risk free measures. Statistical tests performed on empirical data collected at various times over an assortment of equities and relatively short durations are consistent with our result in that in none of them could the null hypothesis (of equality) be rejected.

## Real models of arrangements and polytopes

Giovanni Gaiffi

There are several real spherical models associated to a root arrangement, depending on the choice of a "building set". These models can be used to construct special polytopes ("permutonestohedra") equipped with an action of the Coxeter group.

# Nonlocal Cahn-Hilliard equations

**Ciprian Gal**

We wish to present the latest developments in the theory of phase transitions where the classical Cahn-Hilliard equation is central to the understanding of the behavior of binary mixtures. We will consider the nonlocal Cahn-Hilliard equation in a bounded domain and discuss various issues: well-posedness, regularity and long time behavior of solutions as time goes to infinity.

## Global solutions for the 2D NS-CH model for a two-phase flow of viscous, incompressible fluids with mixed partial viscosity and mobility

**Ciprian Gal\*, C. Cao**

Whether or not global solutions of the 2D Navier-Stokes-Cahn-Hilliard system without full viscosity and mobility can develop finite time singularities is a difficult issue. A major part of this talk deals with global regularity of strong solutions for the NS-CH system with mixed partial viscosity and mobility. In addition, we will also discuss the 2D NS-CH system without viscosity but with full mobility. We wish to discuss the global existence and uniqueness of classical solutions.

## Modelling a tumour avascular spheroid as a two-fluid Bingham-like system

**Alberto Gandolfi\*, A. Bertuzzi, A. Fasano, C. Sinigalli**

This talk illustrates a model for the evolution of a tumour spheroid that assumes a structure in which the central necrotic region contains an inner liquid core surrounded by dead cells that keep some mechanical integrity. This partition is a consequence of assuming that a finite delay is required for the degradation of dead cells into liquid. The phenomenological assumption of constant local volume fraction of cells is also made. The above structure is represented in a mechanical two-phase scheme that views the cell component as a Bingham-like fluid and the extracellular liquid as an inviscid fluid. By imposing the continuity of the normal stress throughout the whole spheroid, we can describe the spheroid time evolution and characterize the possible steady state. Depending on the values of mechanical parameters, the model predicts either an evolution toward the steady state or an unbounded growth.

## Multivariate random fields and Hilbert $B(X)$ -modules

**Păstorel Gaşpar\*, Dumitru Gaşpar**

In this talk we use Hilbert  $\mathcal{B}(\mathcal{X})$  - modules of Banach space operators, of vector valued holomorphic functions and of vector measures in the study of multivariate (Banach space valued) random fields. An extension from random fields to random fields distributions is also considered. The talk concludes with a Wold decomposition and some considerations on prediction theory in this setting.

## Global regularity for a 2+1-dimensional Skyrme model

**Dan-Andrei Geba\*, Kenji Nakanishi, Xiang Zhang**

The aim of this talk is to present a new small data global well-posedness result for a 2+1-dimensional field theory, which has applications in condensed matter physics and cosmology. This is joint work with Kenji Nakanishi and Xiang Zhang.

## Composition and Blaschke products

**Pamela Gorkin**

When is a polynomial the composition of two non-trivial polynomials? A recent article in *The Monthly* by J. Rickards shows how to determine whether or not a polynomial is a composition without knowledge of all of its roots. This raises the question of whether or not there is a good algorithm for doing the same thing for Blaschke products. We will begin by discussing the way the geometry of finite Blaschke products can be used to answer this question and then we will discuss infinite Blaschke products. The work on infinite Blaschke products is joint work with I. Chalendar and J. R. Partington.

# Higher analytic indices and symbolic index pairing

Alexander Gorokhovsky\*, Henri Moscovici

Higher index theory was started in the work of A. Connes and H. Moscovici on the Novikov conjecture. The goal of my talk is to reinterpret their theorem, extend the definition of higher indices to new situations, and to describe a theorem computing them in topological terms.

## Schreier dynamical systems, totally nonfree actions, and self-similar $C^*$ algebras

Rostilav Grigorchuk

We will begin with topologies in the spaces of subgroups of a countable group and Schreier graphs and explain what is a Schreier Dynamical system. Then we will discuss the dichotomy: (essentially) free action - totally non-free action. We will show how non-free actions can be used to construct invariant random subgroups (IRS). This will be done for branch groups and topological full groups of minimal homeomorphisms of a Cantor set. We will show that the lamplighter type groups have “zoo” of invariant random subgroups and that simplex of IRS on them is Poulsen simplex. Finally we will show how actions of self-similar groups on regular rooted trees lead to self-similar  $C^*$ -algebras and recurrent trace on them, and how this can be used for construction of asymptotic expanders and asymptotic Ramanujan graphs.

## Direct and indirect control of open quantum systems

Andreea Grigoriu\*, H. Rabitz, G. Turinici

Realistic physical situations include circumstances where a quantum system is not isolated, but interacting with an environment (e.g., a molecule in a solvent). These type of systems, also called open quantum system may be difficult to control because the environment can contain irreversible dynamics that fights against control mechanisms. We consider in this work the situation when the environment can be engineered i.e. its characteristics chosen at will. First the dynamics of a quantum system interacting with an engineered environment is described by a non-Markovian master type equation. Secondly the problem of controllability is addressed. The manipulations of the dynamics is simultaneously realized with both a laser field and a tailored non-equilibrium, and generally time-dependent, state of the surrounding environment. Lie algebra theory is used to characterize the structures of the reachable state sets and to prove controllability. The theoretical results are supported by examples.

## Regularizing properties of the twisted Kähler-Ricci flow

Vincent Guedj

Let  $X$  be a compact Kähler manifold. We show that the Kähler-Ricci flow (as well as its twisted versions) can be run from an arbitrary positive closed current with zero Lelong numbers and immediately smoothes it. This is joint work with Ahmed Zeriahi.

## Spectral triples for the Sierpinski gasket

Daniele Guido\*, F. Cipriani, T. Isola, J-L. Sauvageot

We construct a 2-parameter family of spectral triples for the Sierpinski Gasket  $K$ . For suitable values of the parameters we determine the dimensional spectrum and recover the Hausdorff measure of  $K$  in terms of the residue of the functional  $a \rightarrow \text{tr}(a |D|^{-s})$  at the abscissa of convergence  $d$ , which coincides with the Hausdorff dimension of the fractal. We determine the associated Connes' distance showing that it is bi-Lipschitz equivalent to a suitable root of the Euclidean metric of the plane, and show that the pairing of the associated Fredholm module with (odd)  $K$ -theory is non-trivial. We recover also the unique, standard Dirichlet form on  $K$ , as the residue of the functional  $a \rightarrow \text{tr}(|D|^{-s/2} |[D, a]|^2 |D|^{-s/2})$  at the abscissa of convergence  $\delta$ , which we call the *energy dimension*. The fact that the volume dimension differs from the energy dimension,  $d \neq \delta$ , reflects the fact that on  $K$  energy and volume are distributed singularly.

# Periodic solutions for systems of delay differential equations modeling leukemia under treatment

Andrei Halanay

One type of results on the existence of periodic solutions concerns periodic non autonomous models coupling the Mackey type model of leukopoiesis to a periodic treatment. It is proved that there exists a guiding function so, under a specific condition, a theorem of Krasnoselskii can be applied. When a constant dose treatment is considered, periodic solutions can also appear due to the existence of a Hopf bifurcation of equilibria. In this case the stability of the limit cycles is studied using the Lyapunov coefficient. Numerical simulations will be given, too.

## A large box limit for the cubic nonlinear Schrödinger equation

Zaher Hani\*, Erwan Faou, Pierre Germain

Inspired by the general paradigm of weak turbulence theory, we consider the 2D cubic nonlinear Schrödinger equation on a box of size  $L$  with periodic boundary conditions. In an appropriate "large box regime" ( $L$  very large), we derive a continuum equation on  $\mathbb{R}^2$  that governs the dynamics of the discrete frequency modes over nonlinear time scales. This equation turns out to satisfy several surprising symmetries and conservation laws, as well as several families of explicit solutions.

## Energy-critical Schrödinger equations on manifolds

Sebastian Herr\*, Daniel Tătăru, Nikolay Tzvetkov

In this talk I will present recent small data global well-posedness results for energy-critical nonlinear Schrödinger equations on specific compact manifolds, such as flat tori and spheres. Key ingredients are certain multilinear estimates of Strichartz type which are based on  $L^p$ -estimates for exponential sums and spectral clusters. The classical dispersive estimate fails in this setup.

## Non-overlapping discretization methods: a means of achieving 100% parallelization

Ismael Herrera Revilla

At present, parallel computing is a very effective means of increasing computation speed. The main difficulties of parallel computing are the coordination of the many processors that carry out the different tasks and the information-transmission between them. The paradigm that parallel programming seeks is to carry out such tasks with the processors working independently of each other. The most effective procedures for solving PDEs in parallel -especially of elliptic type- are domain-decomposition-methods (DDM). For DDM, the corresponding paradigm is: "to solve the global BVP by solving local problems exclusively", which is easier to achieve when the subdomains are separated from each other. This explains in part why non-overlapping methods have prevailed. However, an important limitation is that even in non-overlapping methods interface-nodes are shared by two or more subdomains of the domain decomposition. To overcome this limitation, I. Herrera has introduced a new class of discretization procedures [1-3], known as non-overlapping discretizations, which are formulated using systems of nodes with the property that each node belongs to one and only one partition subdomain. This kind of discretizations has been very successful in producing massively parallelized algorithms for the solution of partial differential equations [1]. Here, we present the DVS-discretization method, discuss some of its properties and open questions.

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# Geometric Fox Calculus

**Eriko Hironaka**

We define a variant of Fox Calculus that can be used, for example, to compute growth rates of families of Torelli free group automorphisms.

## A Lefschetz hyperplane theorem with an assigned base point

**June Huh**

I will explain how to assign a base point when applying the Lefschetz hyperplane theorem. This gives an affirmative answer to a conjecture of Dimca and Papadima on the Milnor numbers of projective hypersurfaces.

## Sharp weighted bounds: towards rough operators

**Tuomas Hytonen**

The  $A_2$  conjecture (a sharp quantitative bound for the weighted norm of singular integrals operators) was first confirmed for special classical transforms with infinitely smooth kernels, then finally under general Calderon-Zygmund standard conditions. More recently, I have been able to further relax the assumptions to minimal Dini-type continuity on the kernel. I will also discuss an interesting open problem concerning rough (bounded but non-continuous) kernels for which weighted bounds are known qualitatively, but the precise quantitative estimate remains unknown.

## Gorenstein injective envelopes

**Alina Iacob**

We prove that the class of Gorenstein injective modules is enveloping over commutative noetherian rings with dualizing complexes. This is joint work with Edgar Enochs.

## How to compute the multigraded Hilbert depth of a module

**Bogdan Ichim**

We introduce a method for computing Hilbert decompositions (and consequently the Hilbert depth) of a finitely generated multigraded module  $M$  over the polynomial ring  $K[X_1, \dots, X_n]$  by reducing the problem to the computation of the finite set of Hilbert partitions. We show that Hilbert partitions may also be used for computing the Stanley depth of the module  $M$ . Thus, we provide solutions to some problems proposed by Jurgen Herzog.

## Two dimensional water waves in holomorphic coordinates

**Mihaela Ifrim\*, John Hunter, Daniel Tătăru**

This talk is concerned with the infinite bottom water wave equation in two space dimensions. We consider this problem expressed in position-velocity potential holomorphic coordinates. Viewing this problem as a quasilinear dispersive equation, we establish two results: (i) local well-posedness in Sobolev spaces, and (ii) almost global solutions for small localized data. Neither of these results are new; they have been recently obtained by Alazard-Burq-Zuily, respectively by Wu using different coordinates and methods. Instead our goal is improve the understanding of this problem by providing a single setting for both results, as well as new, somewhat simpler proofs.

Liviu Ignat\*, Tatiana Ignat, Denisa Stancu-Dumitru

In this talk we will present some nonlocal evolution problems that involve operators of the type:

$$\mathcal{L}u(x) = \int_{\mathbf{R}^d} J(x-y)(u(y) - u(x)) dy$$

We analyze the asymptotic behaviour of the solutions of the following nonlocal convection-diffusion equation

$$u_t = J * u - u + G * u^2 - u^2.$$

The results are mainly obtained by scaling arguments and a new compactness argument that is adapted to nonlocal evolution problems.

**Theorem** *Let  $1 \leq p < \infty$  and  $\Omega \subset \mathbf{R}^d$  be an open set. Let  $\rho : \mathbf{R}^d \rightarrow \mathbf{R}$  be a nonnegative smooth radial function with compact support, non identically zero, and  $\rho_n(x) = n^d \rho(nx)$ . Let  $\{f_n\}_{n \geq 1}$  be a sequence of functions in  $L^p((0, T) \times \Omega)$  such that*

$$\int_0^T \int_{\Omega} |f_n|^p \leq M \tag{1}$$

and

$$n^p \int_0^T \int_{\Omega} \int_{\Omega} \rho_n(x-y) |f_n(t, x) - f_n(t, y)|^p dx dy dt \leq M. \tag{2}$$

1. If  $\{f_n\}_{n \geq 1}$  is weakly convergent in  $L^p((0, T) \times \Omega)$  to  $f$  then  $f \in L^p((0, T), W^{1,p}(\Omega))$  for  $p > 1$  and  $f \in L^1((0, T), BV(\Omega))$  for  $p = 1$ .
2. Let  $p > 1$ . Assuming that  $\Omega$  is a smooth bounded domain in  $\mathbf{R}^d$ ,  $\rho(x) \geq \rho(y)$  if  $|x| \leq |y|$  and that

$$\|\partial_t f_n\|_{L^p((0, T), W^{-1,p}(\Omega))} \leq M \tag{3}$$

then  $\{f_n\}_{n \geq 1}$  is relatively compact in  $L^p((0, T) \times \Omega)$ .

## The cross-over from symmetric to asymmetric transition layers in a nonlocal and nonconvex variational model

Radu Ignat\*, Lukas Doering, Felix Otto

We study the Landau-Lifshitz model for the energy of multi-scale transition layers in thin ferromagnetic films. Our main result is the rigorous derivation of a reduced model for the energy of the optimal transition layer, based on Gamma-convergence. The minimal energy splits into a contribution from an asymmetric, divergence-free core, and a contribution from two symmetric, logarithmically decaying tails. The contribution from the symmetric tails is computed explicitly, while the asymmetric core is analyzed via the harmonic map problem for  $S^2$ -vector fields satisfying a divergence constraint. As a consequence, we describe the bifurcation phenomenon from symmetric to asymmetric transition layers.

## The cross-over from symmetric to asymmetric transition layers in a nonlocal and nonconvex variational model

Stelian Ion

Our aim is to deduce a partial differential equations model of water flow on a hillslope with two dimensional variation on its slope. Beside the kinetics of the flow, the model takes into account the water mass transfer and surface modification due to soil erosion. The base of the model is given by the integral form of mass and momentum balance equations for a viscous fluid. By introducing a surface based curvilinear coordinate system we are able to obtain a mediate form of the balance equations and show the contribution of the surface geometry to the balance equation. Then, an asymptotic analysis of the mediate equations are performed and several simplified models are obtained. Some numerical applications are also presented.

**Ioan Ionescu**

We have developed a robust numerical algorithm for the visco-plastic Saint-Venant model with topography. For the time discretization an implicit (backward) Euler scheme was used. To solve the resulting nonlinear equations, a four steps iterative algorithm was proposed. To handle the non-differentiability of the plastic terms an iterative decomposition-coordination formulation coupled with the augmented Lagrangian method was adopted. The proposed algorithm is consistent, i.e. if the convergence is achieved then the iterative solution satisfies the nonlinear system at each time iteration. The equations for the velocity field are discretized using the finite element method, while a discontinuous Galerkin method, with an upwind choice of the flux, is adopted for solving the hyperbolic equations that describe the evolution of the thickness. The algorithm permits to solve alternatively, at each iteration, the equations for the velocity field and for the thickness. The iterative decomposition coordination formulation coupled with the augmented Lagrangian method works very well and no instabilities are present. The proposed algorithm has a very good convergence rate, with the exception of large Reynolds numbers ( $Re \gg 1000$ ), not involved in the applications concerned by the shallow viscoplastic model. The discontinuous Galerkin technique assure the mass conservation of the shallow system. The model has the exact C-property for a plane bottom and an asymptotic C-property for a general topography. Some boundary value problems were selected to analyze the robustness of the numerical algorithm and the predictive capabilities of the mechanical model. The comparison with an exact rigid flow solution illustrates the accuracy of the numerical scheme in handling the non-differentiability of the plastic terms. The influence of the mesh and of the time step are investigated for the flow of a Bingham fluid in a talweg. The role of the material cohesion in stopping a viscoplastic avalanche on a talweg with barrier was analyzed. Finally, the capacities of the model to describe the flow of a Bingham fluid on a valley from the broken wall of a reservoir situated upstream were investigated.

## Operator algebra models and positivity results for amalgamated conditionally monotone products of some operator-valued bimodule maps

**Valentin Ionescu**

We construct amalgamated (full and reduced) conditionally monotone products (in T. Hasebe's ordinary sense, from the scalar-valued case) of pairs of conditional expectations in  $C^*$ -algebraic or von Neumann algebraic context, by analogy with corresponding objects in the operator-valued free, and scalar-valued conditionally free probability theory, due to D. Voiculescu, Y. Ueda, or E. Blanchard and K. Dykema, and, respectively, M. Bozejko and R. Speicher. These extend constructions in the (conditionally) monotone probability theory due to N. Muraki or M. Popa. Then we show the amalgamated conditionally monotone product of some bimodule maps defined on involutive algebras preserves the complete positivity. This extends a result by M. Popa, parallel to F. Boca's theorem from the  $C^*$ -algebraic conditionally free setting. The direct proof is inspired by a scalar case method due to M. Bozejko, M. Leinert, and R. Speicher in the conditionally free probability theory, and uses a matrix non-commutative variance-covariance type inequality.

## Variational derivation of the Green-Naghdi model for shallow-water wave propagation

**Delia Ionescu-Kruse**

We consider the two-dimensional irrotational water-wave problem with a free surface and a flat bottom. In the shallow-water regime and without smallness assumption on the wave amplitude, we derive by a variational approach in the Lagrangian formalism the Green-Naghdi equations. The second equation is a transport equation, the free surface is advected by the fluid flow. We show that the first equation of the system yields the critical points of an action functional in the space of paths with fixed endpoints, within the Lagrangian formalism. The Lagrangian used in the variational derivation is not a metric.

## Deformations of Levi flat structures

**Andrei Iordan\*, Paolo de Bartolomeis**

We study intrinsic deformations of Levi flat structures on a smooth manifold by means of an associated DGLA. A Levi flat structure on a smooth manifold  $L$  is a couple  $(\xi, J)$  where  $\xi \subset T(L)$  is an integrable distribution of codimension 1 and  $J : \xi \rightarrow \xi$  is a bundle automorphism which defines a complex integrable structure on each leaf. We define a complex whose cohomology group of order 1 contains the infinitesimal deformations of a Levi flat structure. In the case of real analytic Levi flat structures, this cohomology group is  $H^1(\mathcal{Z}^*(L), \delta) \times H^1(\Lambda^0, *(\xi) \otimes \xi, \partial_J)$  where  $(\mathcal{Z}^*(L), \delta, \{\cdot, \cdot\})$  is the DGLA associated to  $\xi$ .

# Superposition operators between higher-order Sobolev spaces: well-definedness, continuity and a multivariate Faà di Bruno formula

Florin Isaia

The problem of finding necessary and sufficient conditions on a function  $g : \mathbb{R} \rightarrow \mathbb{R}$  in order that the superposition operator  $N_g : u \mapsto g \circ u$ , generated by  $g$ , is well-defined, continuous, and bounded from a Sobolev space  $W^{m,p}$  into a Sobolev space  $W^{l,q}$  is solved only in the case when the domain  $W^{m,p}$  is "very close" to the codomain  $W^{l,q}$  (as Bourdaud and Sickel noticed in their survey [2]). Marcus and Mizel [5, 6, 7] solved this problem in the case when  $m = l = 1$  and  $p \geq q \geq 1$ , while Bourdaud [1] solved this problem in the case when  $m = l \geq 2$  and  $p = q \geq 1$ . The conditions on  $g$  found by Marcus and Mizel which ensure the well-definedness, the continuity, and the boundedness of  $N_g : W^{1,p} \rightarrow W^{1,q}$ , ensure the validity of the (first-order) chain rule as well, i.e.

$$\partial_i (g \circ u) = (g' \circ u) \partial_i u, \quad \text{a.e., for all } i, \text{ for all } u \in W^{1,p},$$

where  $\partial_i$  denotes the weak derivative with respect to  $x_i$ . Surprisingly, the conditions on  $g$  found by Bourdaud which ensure the well-definedness, the continuity, and the boundedness of  $N_g : W^{m,p} \rightarrow W^{m,p}$  do not generally ensure the validity of the higher-order chain rule (which is in fact a multivariate Faà di Bruno formula). In [3, 4], we obtained sufficient conditions on  $g$  such that  $N_g$  is well defined, continuous, and bounded from an arbitrary space  $W^{m,p}$  into an arbitrary space  $W^{l,q}$ , with  $m \geq l \geq 1$  and  $p, q \geq 1$ , and in addition, the higher-order chain rule is valid for weak partial derivatives. This talk aims to present these results and to reveal in a heuristic manner how they have been produced. This manner could be a start point in the attempt to demonstrate that the sufficient conditions formulated on  $g$  in [3, 4] are also necessary. We believe that a superposition operator  $N_g : W^{m,p} \rightarrow W^{l,q}$  which is bounded, continuous, and satisfies the higher-order chain rule could be a powerful tool for various problems in nonlinear partial differential equations. For example, it could be useful to study the existence or nonexistence of sufficiently regular strong solutions to such problems.

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## The primitive cohomology of theta divisors

Elham Izadi

In joint work with Csilla Tamas and Jie Wang, we prove that the primitive cohomology of the theta divisor of an abelian fivefold satisfies the general Hodge conjecture, i.e., it is contained in the image of the cohomology of a threefold via a Gysin map.

## Classification of flexible Kokotsakis polyhedra

Ivan Izmistiev

A Kokotsakis polyhedron is an open polyhedral surface in  $R^3$ , consisting of 9 rigid quadrilaterals glued together as a  $3 \times 3$ -grid. Generically, a Kokotsakis polyhedron is rigid. Several classes of flexible polyhedra were known, but a complete classification was missing. In this talk we provide such a classification. The analysis is based on the study of a diagram of branched covers between configuration spaces of pieces of a Kokotsakis polyhedron. For example, the link of a vertex is a spherical quadrilateral, and its configuration space is generically an elliptic curve. In this case the dependence between the dihedral angles at two adjacent edges is given by a shift on an elliptic curve. One of the classes of flexible polyhedra appears when one requires the sum of four shifts along the perimeter of the central quadrilateral to be a period.

# Dirichlet problems with mean curvature operator in Minkowski space

**Petru Jebelean**

We survey some recent results on the existence and multiplicity of solutions for Dirichlet problems involving the mean curvature operator in Minkowski space:

$$u \mapsto \operatorname{div} \left( \frac{\nabla u}{\sqrt{1 - |\nabla u|^2}} \right).$$

Both of the radial and non-radial cases are discussed. The approach relies on upper and lower solutions method, Leray-Schauder degree type arguments, as well as on critical point theory for  $C^1$ -perturbations of convex lower semicontinuous functionals.

## Riesz transforms in Operator algebras and Harmonic Analysis

**Marius Junge**

In this talk we discuss how Riesz transform appearing from cocycles on discrete group appear naturally in different context, and can be used to prove harmonic analysis estimate and also have some typical commutative properties in Popa's rigidity/deformation theory.

## Semisimple Hopf algebras of dimension 32

**Yevgenia Kashina**

In this talk we will classify semisimple Hopf algebras of dimension 32 with an abelian group of grouplike elements of order 16. We first describe such Hopf algebras as abelian extensions and classify them up to equivalence. Then we describe the ones which are non-isomorphic and compute their invariants such as fusion rules and Frobenius-Schur indicators of irreducible representations. We will see that some of these non-isomorphic Hopf algebras are indistinguishable by these invariants.

## Vector-valued $L^\infty$ variational problems and their PDE systems

**Nicholas Katzourakis**

We will describe some basic concepts of vector-valued Calculus of Variations in the space  $L^\infty$  for the functional

$$E_\infty(u, \Omega) := \|H(Du)\|_{L^\infty(\Omega)},$$

Here  $u$  is a mapping  $u : \Omega \subseteq \mathbb{R}^n \rightarrow \mathbb{R}^N$  and  $H = H(P)$  is a Hamiltonian in  $C^2(\mathbb{R}^{N \times n})$ . Associated to  $E_\infty$  is the PDE system

$$A_\infty u := \left( H_P \otimes H_P + H[H_P]^\perp H_{PP} \right) (Du) : D^2 u = 0$$

which plays the role of "Euler-Lagrange" system for  $E_\infty$  and we call it Aronsson system. The vector case  $N \geq 2$  has recently been initiated by the speaker, while the related scalar case  $N = 1$  has a 50-year history. A particularly important case of the Aronsson system is the  $\infty$ -Laplacian and arises for  $H(P) = |P|^2$ .  $A_\infty u = 0$  is a quasilinear nondivergence form 2nd order system with discontinuous coefficients. By indicating the existence of "singular solutions" for  $A_\infty u = 0$ , we will motivate the introduction of a PDE theory which extends viscosity solutions of Crandall-Ishii-Lions and applies to general fully nonlinear 2nd order systems. We will conclude with some recent PDE results obtain within this framework for the  $\infty$ -Laplacian and 1st order PDE systems.

## Trivial representation in conformal field theory and operator algebras

**Yasuyuki Kawahigashi**

We sometimes encounter trivial representation in theory of operator algebras. In some cases, it is really trivial and nothing is interesting, but in other cases, some interesting structure is hidden. A local conformal net, an object in the operator algebraic approach to chiral conformal field theory, reveals the latter case. We construct certain local conformal nets with trivial representation as extensions of a tensor power of the Virasoro net with  $c = 1/2$ , which is one of the most fundamental examples, with a pair of binary codes  $(C, D)$  satisfying the conditions given by Lam and Yamauchi in the setting of vertex operator algebras. Our result is an operator algebraic counterpart of theirs, but our proof is entirely different. We apply the  $\alpha$ -induction in order to identify the representation theory of "code local conformal net" and this gives rise to the existence of the desired local conformal net. The famous Moonshine vertex operator algebra is one such example.

# Global bifurcations in wave equations

Eduard Kirr

I will summarize recent results, based on variational methods, regarding the existence and bifurcation properties of coherent states in wave equations. Then I will focus on questions that the variational methods have not answered and show how techniques developed for variational methods, such as concentration compactness and spectral properties of linear elliptic operators, can be combined with global bifurcation techniques and ODE estimates along branches, to obtain the global picture of all coherent states and their bifurcations.

## Curve complex theory for right-angled Artin groups

Thomas Koberda\*, Sang-hyun Kim

I will discuss an analogue of the curve complex for right-angled Artin groups and describe some of its properties. I will then show how the action of the right-angled Artin group on the curve complex analogue guides parallel results between the theory of mapping class groups and the theory of right-angled Artin groups. The central result is the acylindricity of the right-angled Artin group action on the curve complex analogue. From there, we develop a Nielsen-Thurston classification, a characterization of purely loxodromic subgroups, and an analogue of the Masur-Minsky machinery.

## Quantum symmetry in homological representations of braid groups and applications

Toshitake Kohno

Homological representations of braid groups are defined as the action of homeomorphisms of a punctured disk on the homology of an abelian covering of its configuration space. These representations were extensively studied by Krammer and Bigelow. In this talk we show that specializations of the homological representations of braid groups are equivalent to the monodromy of the KZ equation with values in the space of null vectors in the tensor product of Verma modules when the parameters are generic. We will give some applications of this construction. First, we describe the image and the kernel of the action of braid groups and mapping class groups on the space of conformal blocks. We give how combinatorial structures of braid groups such as dual Garside structures are recovered from homological representations of braid groups. Finally, we describe recent work in progress concerning the categorification of the above construction.

## Extremal properties associated with Loewner chains and the Loewner differential equation on the unit ball in $\mathbb{C}^n$

Gabriela Kohr\*, Ian Graham, Hidetaka Hamada, Mirela Kohr

In this talk we survey recent results related to extreme points, support points and reachable families of holomorphic mappings generated by the generalized Loewner differential equation on the unit ball  $B^n$  in  $\mathbb{C}^n$ . Certain applications and conjectures will be also considered. For a linear operator  $A \in L(\mathbb{C}^n)$ , let  $k_+(A)$  be the upper exponential index of  $A$  and let  $m(A) = \min\{\Re\langle A(z), z \rangle : \|z\| = 1\}$ . Under the assumption  $k_+(A) < 2m(A)$ , we consider the family  $S_A^0(B^n)$  of mappings which have  $A$ -parametric representation, i.e.  $f \in S_A^0(B^n)$  iff there exists an  $A$ -normalized univalent subordination chain  $f(z, t)$  such that  $f = f(\cdot, 0)$  and  $\{e^{-tA}f(\cdot, t)\}_{t \geq 0}$  is a normal family on  $B^n$ . We are concerned with extremal properties associated with the compact family  $S_A^0(B^n)$ . These results generalize to higher dimensions related results due to Pell, Kirwan and Schober. We also give applications to distortion and coefficient bounds for the family  $S^0(B^n)$ . In the second part of the talk, we use ideas from control theory to consider extremal problems related to bounded mappings in  $S_A^0(B^n)$ . For this aim, we investigate the (normalized) time-log  $M$ -reachable family  $\tilde{\mathcal{R}}_{\log M}(\text{id}_{B^n}, \mathcal{N}_A)$  generated by the Carathéodory mappings, where  $M \geq 1$  and  $k_+(A) < 2m(A)$ . Every mapping  $f$  in this reachable family can be imbedded as the first element of an  $A$ -normalized univalent subordination chain  $f(z, t)$  such that  $\{e^{-tA}f(\cdot, t)\}_{t \geq 0}$  is a normal family and  $f(\cdot, \log M) = e^{A \log M} \text{id}_{B^n}$ . We present a density result related to the family  $\tilde{\mathcal{R}}_{\log M}(\text{id}_{B^n}, \mathcal{N}_A)$ , which involves the subset  $\text{ex} \mathcal{N}_A$  of  $\mathcal{N}_A$  consisting of extreme points. These results are generalizations to  $\mathbb{C}^n$  of well known results due to Loewner, Pommerenke and Roth. We are also concerned with extreme points and support points associated with compact families generated by extension operators.

# Mathematical challenge to a new phase of materials science based on discrete geometric analysis

Motoko Kotani\*, A. Hirata L. J. Kang<sup>1</sup>, T. Fujita, B. Klumov, K. Matsue, A. R. Yavari, M. W. Chen

Material is a complex system governed by multiple scale structures. It is therefore important to understand relations between microscopic structures and macroscopic properties of a material. In the talk, we first discuss how the standard realization of a crystal gives a natural frame work to bridge micro-macro dynamics in the case of periodic system. Then we will show a new approach to understand a micro-middle range order of metallic glasses by using computational topology. This is a joint work with materials scientists at AIMR Tohoku University. SESSION: Articulated Systems: Combinatorics, Geometry and Kinematics

## A T(1)-type theorem for entangled multilinear Calderon-Zygmund operators

Vjekoslav Kovac\*, Christoph Thiele

We study singular multilinear forms

$$\Lambda((F_{i,j})_{(i,j) \in E})$$

acting on  $|E|$  two-dimensional functions  $F_{i,j}$  and defined by the expression

$$\int_{\mathbb{R}^{m+n}} K(x_1, \dots, x_m, y_1, \dots, y_n) \prod_{(i,j) \in E} F_{i,j}(x_i, y_j) dx_1 \dots dx_m dy_1 \dots dy_n.$$

Here  $K$  denotes a kernel that is singular along the “diagonal”  $x_1 = \dots = x_m, y_1 = \dots = y_n$  and we also assume that it satisfies perfect dyadic conditions. Our main result is a criterion for  $L^p$ -boundedness of these forms. In analogy with existing results in the literature we formulate the testing conditions as the dyadic weak boundedness property,

$$|\Lambda(\underbrace{\mathbf{1}_Q, \dots, \mathbf{1}_Q}_{|E|})| \leq C_1 |Q| \text{ for each dyadic square } Q,$$

and T(1)-type conditions for the adjoints,

$$\|T_{i,j}(\underbrace{\mathbf{1}_{\mathbb{R}^2}, \dots, \mathbf{1}_{\mathbb{R}^2}}_{|E|-1})\|_{\text{BMO}} \leq C_2 \text{ for each } (i,j) \in E.$$

These conditions imply that the  $L^p$  estimate

$$|\Lambda((F_{i,j})_{(i,j) \in E})| \leq C \prod_{(i,j) \in E} \|F_{i,j}\|_{L^{p_{i,j}}}$$

holds in a non-empty range of exponents  $p_{i,j}$  that depends on the combinatorial structure of  $\Lambda$ . The study of this class of multilinear forms is motivated by problems around two-dimensional generalizations of the bilinear Hilbert transform and questions about convergence of bilinear ergodic averages.

## Steenbrink vanishing extended

Sándor Kovács

A new notion, the DB index, a measure of how far a singularity of a pair is from being Du Bois, is introduced and used to generalize vanishing theorems of Steenbrink (1985), Greb-Kebekus-Kovcs-Peternell (2011), and the author (2011) with simpler and more natural proofs than the originals. An argument used in one of these proofs also yields an additional theorem connecting various pushforwards that lie outside of the range of the validity of the above vanishing theorems. This new vanishing theorem implies the Lipman-Zariski conjecture in a large class of cases including, but not limited to all the cases previously known.

## Gradient estimates of harmonic functions for Levy processes

Tadeusz Kulczycki

We prove gradient estimates of transition densities for some Lévy processes. Using this we obtain gradient estimates of harmonic functions with respect to some Lévy processes.

# Artinian Gorenstein algebras with linear resolutions

Andrew Kustin\*, Sabine El Khoury

For each pair of positive integers  $n, d$ , we construct a complex  $X$  of modules over a polynomial ring  $R$  (with integer coefficients). Let  $P$  be the polynomial ring  $k[x_1, \dots, x_d]$ , where  $k$  is a field. If  $I$  is a homogeneous ideal of  $P$ , generated by forms of degree  $n$ , and  $P/I$  is an Artinian Gorenstein algebra with a linear resolution, then there exists a homomorphism  $R \rightarrow P$ ; so that  $P \otimes_R X$  is a minimal homogeneous resolution of  $P/I$  by free  $P$ -modules. The construction of  $X$  is equivariant and explicit. We know the differentials of  $X$  as well as the modules. On the other hand, we do not know the homology of  $X$ ; nor do we know the properties of the modules that comprise  $X$ . Nonetheless, there is an ideal  $\tilde{I}$  of  $R$  and an element  $\delta$  of  $R$  so that  $\tilde{I}R_\delta$  is a Gorenstein ideal of  $R_\delta$  and  $X_\delta$  is a resolution of  $R_\delta/\tilde{I}R_\delta$  by projective  $R_\delta$ -modules. The talk is about joint work with Sabine El Khoury.

## Feedback stabilization for fluid dynamics equations

Cătălin-George Lefter

We investigate control and feedback stabilization to nonconstant trajectories for fluid dynamics models such as Navier-Stokes or magnetohydrodynamics equations.

## On the $L^\infty$ -uniqueness of diffusion operators on Riemannian manifolds Ludovic Dan Lemle

Let  $M$  be a complete non-compact Riemannian  $n$ -dimensional manifold,  $n \geq 2$ , with volume measure  $dx$ ,  $\nabla$  be the gradient operator on  $M$ ,  $\Delta$  be the Laplace-Beltrami operator on  $M$  and denote by  $C_0^\infty(M)$  the space of infinitely differentiable functions with compact support on  $M$ . As we known, in a series of important works, Bakry and his collaborators have developed some very good methods to solve problems related to diffusion operators on Riemannian manifolds. These methods use the *curvature-dimension inequality* and *Ricci curvature of a diffusion operator*. Consider the symmetric diffusion operator

$$\mathcal{A}^V f = \Delta f + \langle \nabla \phi, \nabla f \rangle - Vf, \quad f \in C_0^\infty(M)$$

where  $\phi \in C^\infty(M)$  is such that  $\nabla \phi \in L_{loc}^\infty(M, e^{-\phi(x)} dx)$  and  $V : M \rightarrow \mathbb{R}$  is a potential such that  $V \in L_{loc}^\infty(M, e^{-\phi(x)} dx)$ ,  $V \geq 0$ . Using an interplay between the methods of geometric analysis developed by Bakry and Emery and stochastic analysis of diffusion operators developed by Stroock and Varadhan, our goal is to prove the  $L^\infty(M, e^{-\phi(x)} dx)$ -uniqueness of the operator  $\mathcal{A}^V$  in the context of Bakry-Emery's Ricci curvature. As a consequence, is obtained the  $L^1(M, e^{-\phi(x)} dx)$ -uniqueness of weak solution for the Fokker-Planck-Kolmogorov equation associated with  $\mathcal{A}^V$ .

## A sharp aperture-weighted estimate for square functions

Andrei Lerner

Let  $S_\alpha(f)$  be the square function defined by means of the cone in  $\mathbb{R}_+^{n+1}$  of aperture  $\alpha$ . Let  $[w]_{A_p}$  denote the  $A_p$  characteristic of the weight  $w$ . We discuss an estimate for  $\|S_\alpha\|_{L^p(w)}$  that is sharp both with respect to  $[w]_{A_p}$  and  $\alpha$ .

## Pointwise convergence of the Fourier series near $L^1$

Victor Lie

A celebrated result of Carleson states that if  $f \in L^2([0, 1])$  then its Fourier series converges pointwise (almost everywhere) to  $f$ . Hunt showed that the same result holds for any function in  $L^p[0, 1]$  as long as  $p > 1$  respectively. More precisely, one is led to the following: **Question.** What is the largest Banach rearrangement invariant function space  $X$  in  $L^1[0, 1]$  for which one has that  $f \in X$  implies that the Fourier Series of  $f$  converges pointwise almost everywhere? Our talk is intended as a survey on the main results/progress made in the attempt of answering the above question.

# On the category of weak bialgebras

Esperanza López-Centella\*, Gabriella Böhm and José Gómez-Torrecillas

The aim of this talk is to describe weak (Hopf) bialgebras [4] as (Hopf) bimonoids in appropriate duoidal (also known as 2-monoidal) categories [2]. This interpretation is used to define a category  $\mathbf{wba}$  of weak bialgebras over a given field. As an application, the “free vector space” functor from the category of small categories with finitely many objects to  $\mathbf{wba}$  is shown to possess a right adjoint, given by taking (certain) group-like elements. This adjunction is proven to restrict to the full subcategories of groupoids and of weak Hopf algebras, respectively. As a corollary, we obtain equivalences between the category of small categories with finitely many objects and the category of pointed cosemisimple weak bialgebras; and between the category of small groupoids with finitely many objects and the category of pointed cosemisimple weak Hopf algebras. This extends the well-known relation between groups and pointed cosemisimple Hopf algebras, see e.g. [1]. Joint work with Gabriella Böhm and José Gómez-Torrecillas [3].

*Keywords:* weak bialgebra; duoidal category; groupoid; weak Hopf algebra; Hopf monoid.

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## Mathematical modeling of the dense avalanche onset

Oana Lupaşcu\*, Ioan R. Ionescu

Our main goal is to study the safety factor (of limit load) problem related to the shallow flow of a visco-plastic fluid/solid with heterogeneous thickness over a plane slope. The first objective is to find the appropriate functional space of the problem and to prove the existence of an onset velocity field. The second objective is to propose a numerical strategy to solve the limit load problem and to characterize the flow onset. We introduce an optimization problem (called the limit load or safety factor problem) to study the link between the yield limit, the external forces and the thickness distributions for which the shallow flow of a visco-plastic fluid/solid does, or does not occur. This optimization problem is reconsidered in the space of bounded deformations functions and the velocity boundary conditions are relaxed. We prove that the initial optimization problem is not changed and the reformulated safety factor problem has at least a solution, modeling the onset of the flow. We develop a DVDS-type numerical technique to solve the safety factor problem through shape optimization problem. The proposed numerical method makes use of a Fourier level set description of the subdomain and of a genetic algorithm in solving the non convex and non-smooth global optimization problem. The numerical approach is illustrated with some numerical simulations involving a Bingham circular dome, a Druker-Prager square dome on a plane slope and a thick Bingham fluid over an obstacle.

## Associated primes of local cohomology modules: a survey

Gennady Lyubeznik

Given a commutative Noetherian ring  $R$ , an ideal  $I$  of  $R$  and a finitely generated  $R$ -module  $M$ , is the set of the associated primes of a local cohomology module of  $M$  with support in  $I$  finite? In general the answer is negative but in the case that  $R$  is regular and  $M=R$  the answer is in many cases (for example in the case that  $R$  is a finitely generated algebra over a field) known to be positive. I am going to survey the work on this question including the very recent result (joint with Bhargav Bhatt, Manuel Blickle, Anurag Singh and Wenliang Zhang) to the effect that the answer is positive if  $R$  is the polynomial ring in finitely many variables over the integers and  $M=R$ . The methods used in dealing with this question involve a fascinating interplay of the Frobenius morphism in characteristic  $p \neq 0$  and the theory of  $D$ -modules in characteristic 0.

# The lower order terms of the holomorphic Morse inequalities

George Marinescu

The holomorphic Morse inequalities of Demailly show that the dimension of the cohomology groups of increasing tensor powers of a holomorphic line bundle grows polynomially, with the leading term given by a curvature integral. The question appears (formulated also by Demailly) to determine the lower order terms. To tackle the problem one can consider the Bergman kernel expansion, which by integration yields Morse inequalities. In joint work with X. Ma, we showed that the Bergman kernel has an asymptotic expansion if the curvature is everywhere non-degenerate. In a recent paper with C.-Y. Hsiao we gave a general condition which insures the existence of the asymptotics, namely the existence of a “small” spectral gap of the Kodaira-Laplacian. An upshot of our general result is for example that the Bergman kernel of a semi-positive line bundle on a compact Kähler manifold admits an asymptotic expansion on the set where the curvature is non-degenerate.

## Existence for nonlinear diffusion equations via a variational principle

Gabriela Marinoschi

We give existence results for a nonlinear diffusion equation with a monotonically increasing multivalued time-dependent nonlinearity derived from a convex potential. The results state that the solution of the nonlinear equation can be retrieved as the null minimizer of an appropriate minimization problem for a convex functional involving the potential and its conjugate. We treat both the case of a continuous potential and of a singular one.

## Homogenization for rigid suspensions with random velocity-dependent interfacial forces

Florian Maris\*, Yuliya Gorb, Bogdan Vernescu

We study suspensions of solid particles in a viscous incompressible fluid in the presence of highly oscillatory velocity-dependent surface forces. The flow at a small Reynolds number is modeled by the Stokes equations coupled with the motion of rigid particles arranged in a periodic array. The objective is to perform homogenization for the given suspension and obtain an equivalent description of a homogeneous (effective) medium, the macroscopic effect of the interfacial forces and the effective viscosity are determined using the analysis on a periodicity cell. In particular, the solutions  $\cong_{\omega}^{\epsilon}$  to a family of problems corresponding to the size of microstructure  $\epsilon$  and describing suspensions of rigid particles with random surface forces imposed on the interface, converge  $H^1$ -weakly as  $\epsilon \rightarrow 0$  a.s. to a solution of the so-called homogenized problem with constant coefficients. It is also shown that there is a corrector to a homogenized solution that yields a strong  $H^1$ -convergence.

## Graphs, groups, homology

Daniel Matei\*, Enrique Artal Bartolo, Jose I. Cogolludo-Agustin

A finite simplicial graph determines a cell subcomplex of the torus. The fundamental group of this complex is the so-called right-angled Artin group associated to the graph. It is the group generated by the vertices of the graph so that two generators commute if and only if there is an edge joining them. We are interested in the homology of the normal subgroups of an Artin group that are kernels of surjections onto the infinite cyclic group.

## Asymptotics of longest common subsequences of random strings and convex geometry

Heinrich Matzinger\*, Raphael Hauser, Ionel Popescu, Saba Amsalu

We consider two independent random strings  $X = X_1X_2\dots X_n$  and  $Y = Y_1Y_2\dots Y_n$  written in a finite alphabet. Let  $L_n$  denote the length of the Longest Common Subsequence (LCS) of the two strings  $X$  and  $Y$ . Every Common Subsequence of  $X$  and  $Y$  corresponds to an alignment with gaps of  $X$  and  $Y$ . For this we align the letters of the Common Subsequence with each other and leave the others out. We will also consider a generalization of the LCS called Optimal Alignment Score. For a long time the order of the fluctuation of  $L_n$  was an open question. We show how for several important situations, this question can be solved using convex geometry properties of the set of empirical distributions of the aligned letter pairs of all alignment with gaps. Note that LCS can be reformulated as a Last Passage Percolation (LPP) problem with correlations. For general (LPP) the question of the asymptotic fluctuation order had been open for many decades. For some LPP-related models like the Longest Increasing Subsequence the problem of the asymptotic fluctuation and distribution was solved by BaikDeiftJohansson (1999). It is however interesting that for our model under consideration the order of the fluctuation is totally different, despite our model also being a LPP-model! LCS and Optimal Alignments are main tools in modern genetic to identify similar genes in DNA-sequences. Instead of considering only one score such as LCS to determine relatedness we propose to use the empirical distribution of the aligned letter pairs along the optimal alignment. We prove that this empirical distribution asymptotically converges with probability one if we chose the scoring function at random.

Daniel Maxin

Sterilizing pathogens occur naturally in many animal populations. They can also be artificially engineered as an effective and humane method of pest control population. The opposite, enhancing effect, provided by parasites that actually promote mating success are, in comparison, rarely observed in nature. In this presentation I will discuss several recent results from the analysis of one and two-sex models that incorporate these disease induced effects on the reproduction. In particular I will discuss the possibility that a sterilizing effect may increase the survival fitness of the infected individual (by lowering the cost associated with reproduction) which leads to an even better pest control effectiveness. Sterilizing pathogens may also trigger an extreme Allee effect in the target population with host extinction and disease clearance as the only possible outcomes. Finally, using adaptive dynamics, I will describe several evolutionary scenarios that may explain why pathogen reproduction enhancement is rare in Nature.

## On the second nilpotent quotient of higher homotopy groups for hypersolvable arrangements

Anca Măcinic\*, Daniel Matei, Ștefan Papadima

We examine the first non-vanishing higher homotopy group,  $\pi_p$ , of the complement of a hypersolvable, non-supersolvable, complex hyperplane arrangement, as a module over the group ring of the fundamental group,  $\mathbb{Z}\pi_1$ . We show that the second nilpotent  $I$ -adic quotient of  $\pi_p$  is determined by the combinatorics of the arrangement, and we give a combinatorial formula for the second associated graded piece,  $\text{gr}_1^1 \pi_p$ . We relate the torsion of this graded piece to the dimensions of the minimal generating systems of the Orlik–Solomon ideal of the arrangement  $\mathcal{A}$  in degree  $p + 2$ , for various field coefficients. When  $\mathcal{A}$  is associated to a finite simple graph, we show that  $\text{gr}_1^1 \pi_p$  is torsion-free, with rank explicitly computable from the graph.

## A new family of palindromic independence polynomials

Eugen Mândrescu

Let  $\alpha(G)$  be the size of a maximum independent set in a graph  $G$ , and  $s_k$  be the number of independent sets of cardinality  $k$  in  $G$ . The *independence polynomial* of  $G$  is

$$I(G; x) = s_0 + s_1x + s_2x^2 + \dots + s_\alpha x^\alpha, \alpha = \alpha(G), \text{ see [1].}$$

For a survey on independence polynomials of graphs see [2]. If  $s_{\alpha-i} = s_i$  holds for every  $i \in \{0, 1, \dots, \lfloor \alpha/2 \rfloor\}$ , then  $I(G; x)$  is *palindromic (symmetric)*. An algebraic technique taking care of symmetric independence polynomials may be found in [3]. The *corona* of the graphs  $G$  and  $H$  is the graph  $G \circ H$  obtained by joining each vertex of  $G$  to all the vertices of a copy of  $H$ . In this research we show that if  $H = K_r - e$ ,  $r \geq 2$ , then  $I(G \circ H; x)$  is palindromic and unimodal, with a unique mode. This finding generalizes results from [4] and [5], claiming that  $I(G \circ (K_2 - e); x) = I(G \circ 2K_1; x)$  is palindromic and unimodal for every graph  $G$ . (joint work with Vadim E. Levit)

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## ACC and termination

James McKernan

We will talk about ACC for the log canonical threshold and its connections to termination.

# A special case of restricted integer partitions

Mircea Merca

In a recent paper, we introduced a special case of integer partitions with restrictions in order to produce the fastest algorithm for generating integer partitions. In this talk we present some new properties of these partitions. As a corollary we obtain Euler's recurrence relation for the partition function  $p(n)$ .

## Local well-posedness for quasilinear Schrödinger equations

Jason Metcalfe

This talk will focus on recent joint works with J. Marzuola and D. Tataru. The focus is on low regularity local well-posedness for quasilinear Schrödinger equations. Due to the Mizohata condition, it does not suffice to work within Sobolev spaces, and the approach is to include a summability over cubes condition. The main estimates, such as local smoothing estimates, are then adapted to this framework.

## On a geometric inequality for submanifolds in Kähler manifolds

Ion Mihai

The normal scalar curvature conjecture, also known as the DDVV conjecture, was stated by De Smet, Dillen, Verstraelen and Vrancken in [Arch. Math. (Brno) **35** (1999), 115–128]. It was proven recently by Lu [J. Funct. Anal. **261** (2011), 1284–1308] and by Ge and Tang [Pacific J. Math. **237** (2008), 87–95], independently. We obtain the DDVV inequality, also known as generalized Wintgen inequality, for Lagrangian submanifolds in complex space forms. Some applications are given. For instance, we derive that a Lagrangian submanifold satisfying the equality case identically is a Chen submanifold. Also we state such an inequality for slant submanifolds in complex space forms.

2000 *Mathematics Subject Classification*: 53C40, 53C25.

*Keywords*. Wintgen inequality, DDVV conjecture, complex space form, invariant submanifold, Lagrangian submanifold, slant submanifold.

## PDE's involving a variable exponent Grushin-type operator

Mihai Mihăilescu

We define a Grushin-type operator with a variable exponent growth and establish existence results for an equation involving such an operator. A suitable function space setting is introduced. Regarding the tools used in proving the existence of solutions for the equation analysed here, they rely on the critical point theory combined with adequate variational techniques. This is a joint work with G. Morosanu (Central European University) and D. Stancu-Dumitru (Institute of Mathematics of the Romanian Academy).

## The extension problem for Hopf algebras

Gigel Militaru

For two given Hopf algebras  $A$  and  $H$  we classify all Hopf algebras  $E$  that fit into a sequence  $A \hookrightarrow E \rightarrow H$  such that  $E \rightarrow H$  splits in the category of coalgebras and  $A = E^{\text{co}(H)}$ . Equivalently, we classify all crossed products of Hopf algebras  $A \# H$ . The classification is up to an isomorphism of Hopf algebras that stabilizes  $A$  and co-stabilizes  $H$  by a cohomological type object  $\mathcal{H}^2(H, A)$  constructed explicitly. All crossed products  $A \# H_4 := A_{(a|g, x)}$  are explicitly described by generators and relations and classified: these quantum groups  $A_{(a|g, x)}$  are parameterized by the set  $\mathcal{ZP}(A)$  of all central primitive elements of  $A$ . Several examples are worked out in detail: in particular, over a field of characteristic  $p \geq 3$  an infinite family of non-isomorphic quantum groups of dimension  $4p$  is constructed. For the cyclic group  $C_n$ , all crossed products  $H_4 \# k[C_n]$  are explicitly described and classified in four possible ways. They are  $4n$ -dimensional quantum groups  $H_{4n, \lambda, t}$ , associated to all pairs  $(\lambda, t)$  consisting of an arbitrary unitary map  $t : C_n \rightarrow C_2$  and an  $n$ -th root  $\lambda$  of  $\pm 1$  – the choice of the sign  $\pm$  being dictated by the signature of  $t$ . The groups of automorphisms of these quantum groups are described. Joint work with A.L. Agore and C.G. Bontea.

# On Green function of subordinate Brownian motions

**Ante Mimica**

We present sharp estimates of Green function for a class of subordinate Brownian motions, which includes stable and geometric stable processes.

## Asymptotic freeness of orthogonally and unitarily invariant ensembles

**James A. Mingo\*, Mihai Popa, Emily Redelmeier**

There has been a strong relation between unitary invariance and asymptotic freeness ever since Voiculescu's 1991 paper on asymptotic freeness. At the first order level there is very little difference between the case of orthogonally and unitarily invariant ensembles. Above this level the transpose plays a significant role in the orthogonal case, something which isn't seen in the unitary case. This means one has to consider ensembles  $\{A_N\}$  in which there is joint limit distribution for words in  $A_N$  and  $A_N^t$ , i.e. a limit  $t$ -distribution. When one has an ensemble which has a limit  $t$ -distribution and is also unitarily invariant one gets the surprising result that  $A_N$  and  $A_N^t$  become free. In particular this applies to ensembles of Haar distributed random unitary operators.

## Analysis of diffusion models with Dirichlet marginal distribution

**Oana Mocioalca\*, Omar De la Cruz, Lu Chen**

Diffusion-type models that can fit the given marginal distribution and autocorrelation function have been found to have a wide application in, for example, Biology and finance. We propose a method of choosing the diffusion model with Dirichlet marginal distribution and a given linear drift, hence the autocorrelation function is exponentially decreasing. Also, more complicated models will be discussed.

## New weighted estimates for bilinear fractional integrals

**Kabe Moen**

We will discuss some new weighted estimates for a family of bilinear fractional integral operators that parallels the bilinear Hilbert transform. Our results are of interest because they hold when the target Lebesgue space has exponent less than one.

## Bioimpedance cardiometry - mathematical and numerical modelling

**Alexandru Morega\*, Mihaela Morega**

We present a mathematical model and numerical simulation results simulation, in the finite element method (FEM) technique, for the hemodynamic of the aorta, the change in the electrical conductivity of the blood, and the electrical field problem in the Electro Cardiometry (ECM) technique, a subset of the thoracic electrical bioimpedance (TEB) procedures. We are concerned with the direct problem of ECM-TEB, which consists in assessing the sensitivity of TEB to the flow dynamics. Realistic 3D computational domains produced by medical image based reconstruction techniques may be needed as anatomy plays a key role in investigating the ECM-TEB problem. The available analytic formulae for the electrical conductivity of the blood are difficult (if possible) to apply when modeling the aorta hemodynamic using an anatomically realistic computational domain. This difficulty may be solved by using an equivalent electrical conductivity based on averaging techniques applied to analytic results that outline the sensitivity of TEB to the blood flow dynamics.

## A ode approximation of the Laplace operator in a square

**Umberto Mosco**

We construct a one-parameter family of ODEs that converge to the Laplace equation in a square. The model applies to the evolution of one-dimensional dynamical boundaries infiltrating a two-dimensional surface.

# Wave operators for elliptic operators with fractal singularities

Umberto Mosco

We construct a sequence of uniformly elliptic operators with singular discontinuous coefficients on fractal fibers and prove the convergence of the associated wave operators.

## Boundary stabilization of the phase field system by finite-dimensional feedback controllers

Ionuț Munteanu

We design here a finite-dimensional stabilizing feedback Dirichlet boundary controller for the zero steady-state solution to the phase field system. The feedback controller is easily manageable from computational point of view since it is expressed in terms of the eigenfunctions  $\{\phi_j\}_{j=1}^N$ ,  $N \in \mathbb{N}$ , corresponding to the eigenvalues  $\{\lambda_j\}_{j=1}^N$  of the Laplace operator. The stabilizing algorithm, we develop here, is applicable under the assumption that the system  $\left\{ \frac{\partial \phi_j}{\partial \mathbf{n}} \right\}_{j=1}^N$  is linearly independent on the part of the boundary where the control is applied.

## Scattering for inter-critical NLS

Jason Murphy

We consider a class of defocusing power-type nonlinear Schrödinger equations for which the power of the nonlinearity lies between the mass- and energy-critical exponents. Adapting techniques developed to treat the mass- and energy-critical problems, we show that any solution that remains bounded in the critical Sobolev norm must be global and scatter. In particular, we use a concentration-compactness approach to reduce the problem to the preclusion of almost periodic solutions. Key ingredients in this final step include a long-time Strichartz estimate à la Dodson and a frequency-localized interaction Morawetz inequality.

## Factorizable completely positive maps and the Connes embedding problem

Magdalena Mușat\*, Uffe Haagerup

The class of factorizable completely positive maps (originating in work of C. Anantharaman-Delaroche) has gained particular significance in quantum information theory in connection with the settling (in the negative) of the *asymptotic quantum Birkhoff conjecture*. More precisely, in joint work with Uffe Haagerup we proved earlier that every non-factorizable unital completely positive and trace-preserving map on  $M_n(\mathbb{C})$ ,  $n \geq 3$ , provides a counterexample for the conjecture. We will explain a recently established connection to the Connes embedding problem in terms of a newly formulated asymptotic property of factorizable maps.

## Generalization of Mitchell's lemma. Applications

Constantin Năstăsescu\*, Septimiu Crivei, Laura Năstăsescu

The well-known Gabriel-Popescu theorem states that a Grothendieck category is a quotient category of a category of modules. We will present a generalization of a lemma due to B. Mitchell (A quick proof of the Gabriel-Popescu theorem, J. Pure Appl. Algebra (1981)). This generalization allows us to give a short proof to a theorem of Ulmer and to prove a generalization of the Gabriel-Popescu theorem.

## On the behavior of the candidate solutions of a genetic algorithm. Case study

Elena Nechita\*, Gloria Cerasela Crișan

Genetics Algorithms (GA) are part of a large field known as Evolutionary Algorithms (EA) which proved to be, during the last decades, strong tools for search and optimization. Based on the evolutionary principles and on the apparatus of probability theory to formalize the optimization processes, GA variants have been intensively analyzed in order to improve their performance.

This talk considers a function of two variables and approaches the search of its maxima with a canonic genetic algorithm: binary, fixed length coding of the potential solutions, single point crossover, bit flip mutation, and fitness-proportional selection. A study of the distribution of the discrete variable whose values are the elements of the search space is performed. Moreover, the search process is discussed with respect to the evolution of similarities between candidate solutions.

The robustness of the genetic algorithm is tested by varying its parameters for the designed case study, while the pattern identified in the behavior of the candidate solutions will be subject to validation in new experiments.

*Keywords:* Genetic Algorithms, optimization, distribution, robustness.

# On confinement and stochastic particles

Irina Nenciu

We will present new results concerning the essential self-adjointness of Schrödinger-type operators on arbitrary manifolds, and make connections with the question of confinement for stochastic particles. The results and techniques used are based on joint work with G. Nenciu.

# On long-time asymptotics for certain perturbations of the Toda lattice

Irina Nenciu

We will present the result of numerical simulations aimed at understanding the long time asymptotics for certain small perturbations of the (completely integrable) Toda lattice. We use the scattering theory associated with the usual Toda lattice, and investigate the evolution of the scattering data under the perturbed lattice. This is based on joint work with D. Bilman.

# Cocycle deformations of operator algebras

Sergey Neshveyev

Given a  $C^*$ -algebra  $A$  with a left action of a locally compact quantum group  $G$  on it and a unitary 2-cocycle  $\Omega$  on  $\hat{G}$ , we define a deformation  $A_\Omega$  of  $A$ . The construction behaves well under certain additional technical assumptions on  $\Omega$ , the most important of which is regularity, meaning that  $C_0(G)_\Omega \rtimes G$  is isomorphic to the algebra of compact operators on some Hilbert space. In particular, then  $A_\Omega$  is stably isomorphic to the iterated twisted crossed product  $\hat{G}^{\text{op}} \rtimes_\Omega G \rtimes A$ . Also, in good situations, the  $C^*$ -algebra  $A_\Omega$  carries a left action of the deformed quantum group  $G_\Omega$  and we have an isomorphism  $G_\Omega \rtimes A_\Omega \cong G \rtimes A$ . When  $G$  is a genuine locally compact group, we show that the action of  $G$  on  $C_0(G)_\Omega = C_r^*(\hat{G}; \Omega)$  is always integrable. Stronger assumptions of properness and saturation of the action imply regularity. We compare the construction with various previously known partial cases, such as Rieffel's deformation and deformations by cocycles on the duals of some solvable Lie groups recently constructed by Bieliavsky and Gayral. (Joint work with J. Bhowmick, A. Sangha and L. Tuset)

# Local index and torsion of $n$ -tuples of commuting operators

Ryszard Nest

Given  $n$ -tuple commuting of commuting operators, one can associate to it (under certain conditions) its local indices and determinant. We will give formulas for the behaviour of both under holomorphic functional calculus and sketch the proofs.

# Optimal convergence rates for the finite element method on three dimensional polyhedra

Victor Nistor\*, Constantin Băcuță, Ludmil Zikatanov

I describe a sequence of graded mesh refinements of a polyhedral domain that yields " $h^m$ "-optimal rates of convergence for the classical Finite Element Method.

# Short time asymptotics of Green functions and option pricing

Victor Nistor\*, Wen Cheng, Anna Mazzucato

We provide a new short time asymptotic for the Green function of a time dependent parabolic problem using parabolic rescaling and the Campbell-Baker-Hausdorff formula. Then we provide applications to option pricing for short and long dated options.

# On trilinear oscillatory integrals

**Diogo Oliveira e Silva\***, **Michael Christ**

We examine a certain class of trilinear integral operators which incorporate oscillatory factors  $e^{iP}$ , where  $P$  is a real-valued polynomial, and prove smallness of such integrals in the presence of rapid oscillations. Aiming at a direct analysis of the oscillatory integrals in question, we use a variety of tools which include sublevel set estimates, higher dimensional versions of van der Corput's lemma and corresponding multilinear analogues.

## The Buchberger resolution

**Anda Olteanu**

In this talk we define the Buchberger resolution which is a graded free resolution of a monomial ideal in a polynomial ring. In general, this resolution is not minimal. We give a complete characterization of the cases when the Buchberger resolution is minimal.

## Near field versus far field active cloaking

**Daniel Onofrei\***, **Richard Albanese**

In this talk we will introduce the general concept of active cloaking and introduce two current ideas for active radar/sonar cloaking. We will present the first method based on scattering cancellation in the far field and compare it with the novel approach where the near field of active antennas is controlled and used for the cancellation of the previously detected incoming field. We will highlight the main advantages of the last method and point to some of the current implementation challenges.

## Bounded and $L^p$ -weak solutions for the nonlinear equation of a branching type process

**Andrei-George Oprina\***, **Lucian Beznea**

We present existence results for the bounded solutions of the nonlinear equation  $\Delta u + c \sum_{k=1}^{\infty} q_k u^k = f$ , and for the associate Dirichlet problem with bounded boundary data;  $c$  is a bounded positive function on  $E$  generating a killing kernel and the Laplace operator may be replaced by the generator of a Borel right process with state space  $E$ . We also show the existence of the weak solution in  $L^p$  spaces. It is emphasized the classical connection between this equation and the discrete branching processes: the solutions of the nonlinear equation are produced from the harmonic functions with respect to the (linear) generator of a discrete branching type process with state space the set  $S$  of all finite configurations of  $E$ . Our approach is based on probabilistic and analytic potential theoretical methods, used on both spaces  $E$  and  $S$ , like the Revuz formula for the continuous additive functionals.

## Stochastic additive functionals with applications

**Adina Oprisan\***, **Andrzej Korzeniowski**

I consider a family of stochastic additive functionals of Markov processes switched by jump Markov processes subject to the effect of small random perturbations. Random perturbations arising in connection with averaging principle as well as with those leading to diffusion processes are discussed. I will present asymptotic properties of the type of law of large numbers, central limit theorems, almost sure central limit theorems and large deviations when the parameter characterizing the smallness of random perturbations converges to zero. Examples and applications such as stability and the problem of exit from a domain of attraction are included.

# Intersections of smooth and fractal sets

**Eyvindur Palsson\***, **Michael Gage**, **Allan Greenleaf**, **Alex Iosevich**, **Jonathan Pakianathan**

Classical intersection theory provides us with much information about the intersection of two smooth  $k$ -dimensional surfaces in Euclidean space. We will present an example that shows that the size of the intersection in general has no upper bound and contrast that with a theorem that gives an upper bound on the expected size of the intersection. One can also study intersections in a fractal setting. For example a classical theorem due to Mattila says that if  $A, B \subset \mathbb{R}^d$  of Hausdorff dimension  $s_A, s_B$  respectively, with  $s_A + s_B \geq d$ ,  $s_B > \frac{d+1}{2}$  and  $\dim_{\mathcal{H}}(A \times B) = s_A + s_B \geq d$ , then

$$\dim_{\mathcal{H}}(A \cap (z + B)) \leq s_A + s_B - d$$

for almost every  $z \in \mathbb{R}^d$ , in the sense of Lebesgue measure. We will present a mechanism for studying intersections of three or more sets in scenarios that cannot be reduced to the pairwise case such as in Mattila's theorem. These results are obtained through geometric and Fourier analytic methods.

## Twisted Hilbert transforms vs Kakeya sets of directions

**Javier Parcet\***, **Keith M. Rogers**

Given a discrete group  $G$  and an orthogonal action  $\gamma : G \rightarrow O(n)$  we study  $L_p$  convergence of Fourier integrals which are frequency supported on the semidirect product  $\mathbb{R}^n \rtimes_{\gamma} G$ . Given a unit  $u \in \mathbb{R}^n$  and  $1 < p \neq 2 < \infty$ , our main result shows that the twisted (directional) Hilbert transform  $H_u \rtimes_{\gamma} id_G$  is  $L_p$ -bounded iff the orbit  $\mathcal{O}_{\gamma}(u)$  is finite. This is in sharp contrast with twisted Riesz transforms  $R_u \rtimes_{\gamma} id_G$ , which are always bounded. Our result characterizes Fourier summability in  $L_p$  for this class of groups. We also extend de Leeuw's compactification theorem to this setting and obtain stronger estimates for functions with "lacunary" frequency support.

## Paracanonical systems of varieties of maximal Albanese dimension

**Rita Pardini\***, **Margarida Mendes Lopes**, **Gian Pietro Pirola**

Let  $X$  be a smooth complex projective variety of irregularity  $q > 0$ , and let  $H$  be an irreducible family of effective divisors of  $X$  that dominates a component of the group  $\text{Pic}(X)$ : given a divisor  $D$  algebraically equivalent to the elements of  $H$ , we give a cohomological criterion to ensure that  $D$  belong to  $H$ . By applying this criterion to the study of the main paracanonical system of a variety of general type with generically finite Albanese map, we are able to refine results due to Beauville in the case of surfaces and to Lazarsfeld and Popa in higher dimension. In particular, if the dimension of  $X$  is  $> 2$  we obtain an unexpected inequality between the numerical invariants of  $X$ , under the assumption that  $X$  has generically finite Albanese map and does not have fibrations of a certain type.

## On Baer modules

**Jae Keol Park\***, **S. Tariq Rizvi**

A right  $R$ -module  $M$  is called a Baer module if  $\ell_S(N) = Se$  with  $e^2 = e \in S$  for all  $N \leq M$ , where  $S = \text{End}(M_R)$ . When  $V_R$  is a nonsingular cyclic module over a commutative ring  $R$ , we discuss Baer intermediate modules between  $V_R$  and its injective hull  $E(V_R)$ . As application, it is shown that every nonsingular cyclic module over a commutative ring has the Baer module hull. Several examples related to our results on Baer modules are provided.

## Convex approximations of analytic functions

**Nicolae Pascu**

The univalence of an analytic function is an important problem of the Geometric function theory, and there are many sufficient conditions for univalence in the literature. If a function is not univalent, then, in practical problems, it is of interest to find a best approximation of it by a univalent function. We introduce a method for constructing the best convex univalent approximation of an analytic function, in the  $L^2$  sense. The construction is based on solving a certain semi-infinite quadratic programming problem, which may be of independent interest, and it is suitable for numerical implementation.

# Linear sofic groups and algebras

**Liviu Păunescu**

In a joint work with Goulmira Arzhantseva, we introduce linear sofic groups and linear sofic algebras. A group is linear sofic if and only if its group algebra is linear sofic. Linear soficity for groups is a priori weaker than soficity but stronger than weak soficity. We shall discuss problems in proving that linear sofic groups are sofic or that they satisfy Kaplansky's direct finiteness conjecture.

## A convex structure on sofic embeddings

**Liviu Păunescu**

Sofic groups were introduced by Gromov in 1999. A group is sofic if it can be approximated in a certain sense by finite objects. The sofic property of a group proved to be quite fruitful in various areas like geometric group theory, dynamical systems or operator algebras. A theorem by Elek and Szabo states that a group is amenable iff it has essentially only one sofic approximation. For non-amenable groups the space of sofic approximations is non separable in its natural topology. We shall try to understand this space by introducing a convex like structure in the sense of Nate Brown. I will discuss know examples and results about extreme points of this convex structure.

## Moment problems: a framework for relaxation

**Pablo Pedregal**

By examining a simple example in dimension one consisting in finding the convexification of a coercive integrand, we describe a mechanism to calculate relaxed variational principles under differential constraints as in typical situations in optimal design in conductivity. This formulation is written in terms of moments of several orders for probability measures associated with underlying gradient Young measures occurring in a natural way in this type of problems. It leads to semidefinite mathematical programming problems, and/or conic programming problems. This procedure furnishes a potential way of moving on to more complicated problems for mixtures of non-linear, conducting materials, and even to the case of mixtures of linear-elastic materials. The talk will focus on describing the methodology, and apply it to known situations.

## Rubio de Francia's extrapolation meets Yano's extrapolation: sharp exponents without examples in weighted estimates

**Carlos Perez**

It is well know that the so called iteration technique of Rubio de Francia can be used to prove the factorization and the extrapolation theorem for  $A_p$  weights. Using this iteration method we found a very general method of showing when appropriate weighted  $L^p$  bounds for operators are really sharp. A natural condition appears in the scenario which is related to the blow up of the  $L^p$  norm of the operator as  $p$  gets close to 1. This is in turn related to the classical well known extrapolation theorem due to Yano.

## Optimal power for an elliptic equation related to some Caffarelli-Kohn-Nirenberg inequalities

**Mayte Perez-Llanos**

In this talk we analyze the following elliptic problem related to some Caffarelli-Kohn-Nirenberg inequalities:

$$-\operatorname{div}(|x|^{-2\gamma}\nabla u) - \lambda \frac{u}{|x|^{2(\gamma+1)}} = |\nabla u|^p |x|^{-\gamma p} + cf, \quad u > 0 \text{ in } \Omega, \quad u|_{\partial\Omega} \equiv 0,$$

where  $\Omega \subset \mathbb{R}^N$  is a domain containing the origin,  $N \geq 3$ , and  $c, \lambda, \gamma, p$  are positive constants verifying  $0 < \lambda \leq \Lambda_{N,\gamma} = \left(\frac{N-2(\gamma+1)}{2}\right)^2$ ,  $-\infty < \gamma < \frac{N-2}{2}$  and  $p > 0$ . Our study concerns to existence of solutions to the former problem. More precisely, first we determine a critical value for the power  $p$ , in the sense that, beyond this value it does not exist any positive supersolution to our problem, not even in a very weak sense. In addition we show existence of solutions for all the values of  $p > 0$  below this threshold, with the restriction  $\gamma > -\frac{N(1-p)+2}{2-p}$ . The existence of solutions for  $0 < p < 1$  and  $\gamma \leq -\frac{N(1-p)+2}{2-p}$  is an open question.

# Exponential decay of the power spectrum and finite dimensionality for solutions of the three dimensional primitive equations

**Mădălina Petcu**

In this talk we are interested in estimating the number of modes, volumes and points in the physical domain, sufficient to describe well the asymptotic behavior of the solution of the three dimensional primitive equations. We also study the exponential decay of the spatial power spectrum of the three dimensional primitive equations. We recall that the three dimensional primitive equations are modelling the motion of the oceans and of the atmosphere. The question of determining how many points in the physical domain can describe well the asymptotic behavior of the whole flow is coming from practice. Indeed, in many practical situations, the experimental data are collected from a finite number of points in the physical domain and we are interested to know how many such points we need to use in order to have a relevant information for the flow.

## CRRA utility maximization under dynamic risk constraint

**Traian Pirvu**

The problem of optimal investment with CRRA (constant, relative risk aversion) preferences, subject to dynamic risk constraints on trading strategies, is the main focus of this talk. Several works in the literature, which deal either with optimal trading under static risk constraints or with VaR-based dynamic risk constraints, are extended. The market model considered is continuous in time and incomplete, and the prices of financial assets are modeled by Ito processes. The dynamic risk constraints, which are time and state dependent, are generated by a general class of risk measures. Optimal trading strategies are characterized by a quadratic BSDE. Within the class of time consistent distortion risk measures, a three-fund separation result is established. Numerical results emphasize the effects of imposing risk constraints on trading.

## On the structure of fusion categories with few irreducible degrees

**Julia Yael Plavnik\*, Sonia Natale**

In this talk we shall consider the general problem of understanding the structure of a fusion category  $\mathcal{C}$  after the knowledge of the set  $\text{c.d.}(\mathcal{C})$  of Frobenius-Perron dimensions of its simple objects. For a finite group  $G$ , the knowledge of the set  $\text{c.d.}(G) = \text{c.d.}(\mathbf{k}G)$  gives in some cases substantial information about the structure of  $G$ . It is known, for instance, that if  $\text{c.d.}(G)$  is at most 3, then  $G$  is solvable. We shall show various structural results regarding nilpotency and solvability, in the sense introduced by Etingof, Gelaki, Nikshych and Ostrik, of certain classes of integral fusion categories and semisimple Hopf algebras under restrictions on the set of its irreducible degrees. We shall study separately the the odd and even-dimensional cases.

## Scattering for a cubic-quintic nonlinear Schrödinger equation on $\mathbb{R}^3$

**Oana Pocovnicu\*, Rowan Killip, Tadahiro Oh, Monica Vişan**

In this talk, we consider the cubic-quintic nonlinear Schrödinger equation (NLS) on  $\mathbb{R}^3$  whose nonlinearity is given by the sum of a defocusing quintic power and a focusing cubic power. Notice that the quintic nonlinearity is energy-critical on  $\mathbb{R}^3$ , while the cubic one is energy-subcritical. We consider the issue of scattering for the cubic-quintic NLS. Previously, Tao, Vişan, and Zhang proved global well-posedness in  $H^1$  and scattering for solutions with small mass. The main result in this talk is scattering for solutions that are situated below a branch of (rescaled) solitons in the mass-energy plane. Our result shows that while all solutions exist globally in time, the region of scattering is bounded by (rescaled) solitons, namely the situation is very different from the energy-critical defocusing/focusing quintic NLS. Our proof combines variational arguments and concentration-compactness and rigidity arguments.

## Degenerate PDEs, martingale and mimicking problems

**Camelia Pop\*, Paul Feehan**

We solve four intertwined problems, motivated by mathematical finance, concerning degenerate-parabolic partial differential operators and degenerate diffusion processes. First, we consider a parabolic partial differential equation on a half-space whose coefficients are suitably Hölder continuous and allowed to grow linearly in the spatial variable and which becomes degenerate along the boundary of the half-space. We establish existence and uniqueness of solutions in weighted Hölder spaces which incorporate both the degeneracy at the boundary and the unboundedness of the coefficients. Second, we show that the martingale problem associated with a degenerate elliptic differential operator with unbounded, locally Hölder continuous coefficients on a half-space is well-posed in the sense of Stroock and Varadhan. Third,

we prove existence, uniqueness, and the strong Markov property for weak solutions to a stochastic differential equation with degenerate diffusion and unbounded coefficients with suitable Hölder continuity properties. Fourth, for an Itô process with degenerate diffusion and unbounded but appropriately regular coefficients, we prove existence of a strong Markov process, unique in the sense of probability law, whose one-dimensional marginal probability distributions match those of the given Itô process.

## Exponential splitting for nonautonomous linear discrete-time systems in Banach spaces

Ioan-Lucian Popa\*, Mihail Megan

In this talk we consider some concepts of exponential splitting for nonautonomous linear discrete-time systems. These concepts are generalizations of some well-known concepts of (uniform and nonuniform) exponential dichotomies. Connections between these concepts are presented and some illustrating examples prove that these are distinct.

### Some special classes of infinite matrices

Nicolae Popa

There are plenty of Banach spaces of infinite matrices. We mention the classical Banach space of infinite matrices defining bounded linear operators on  $\ell_2$ , denoted by  $B(\ell_2)$ , endowed with operator norm, the space of all Schur multipliers  $M(\ell_2)$ , with its natural norm, as well as some matrix versions of classical Lebesgue spaces, which will be introduced further. In what follows we present a simple method for such a construction, which works in many spaces of infinite matrices. The idea of this method is inspired by classical Fourier analysis, namely we consider the diagonals of an infinite matrix as the analogue of Fourier coefficients of some periodic function (distribution). We intend now to use different criteria for membership functions to some classical Banach spaces (see [2,1]) in order to find their extension to arbitrary infinite matrices. Next, let

$$E_k = \begin{pmatrix} 0 & 0 & \dots & 0 & 0 & 0 & \dots \\ 0 & 0 & \dots & 0 & 0 & \dots & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \dots \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots \\ 0 & 0 & \dots & 0 & 1 & 1 & \dots \\ 0 & 0 & \dots & 0 & 1 & 1 & \dots \end{pmatrix},$$

where the number of null rows and columns in  $E_k$  is  $k$ . We recall that by  $*$  we mean the usual Schur product. Let  $X$  be a Banach space of infinite matrices and  $TX$  be its subspace of Toeplitz matrices. If  $TX$ , endowed with  $X$ -norm, is a Banach space isometrically isomorphic to a Banach space of periodic functions (distributions) on  $[0, 1]$ , we denote by  $\tilde{X}$  the latter. We denote by  $(X, X)$  the space of all Schur multipliers on  $X$ , that is the space of all infinite matrices  $A$  such that  $A * B \in X$ , for any  $B \in X$ . Under the natural norm  $\|A\| = \sup_{\|B\|_X \leq 1} \|A * B\|_X$  the space  $(X, X)$  becomes a Banach space.

**Theorem** *Let  $X$  be a Banach space of infinite matrices and  $\tilde{X}$  as above. If  $E_k \in (X, X)$  for all  $k \geq 0$ , and*

$$\|f_1\|_{\tilde{X}} + \sum_{k=1}^{\infty} \|f_k - f_{k+1}\|_{\tilde{X}} \|E_k\|_{(X, X)} < \infty,$$

then the matrix  $A = \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \\ \vdots \end{pmatrix} \in X$ .

The following result extends Theorem 7.2.2-(4) [1].

**Corollary**

Let  $X = B(\ell_2)$ ,  $\tilde{X} = L^\infty(0, 1)$  and  $A = \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \\ \vdots \end{pmatrix}$ , where

$$f_1(x) = \sum_{n=1}^{\infty} \frac{1}{n} \sin 2\pi nx, \quad x \in [0, 1],$$

$$f_k(x) = \sum_{n=1}^{\infty} a_n^k \frac{1}{n} \sin 2\pi nx, \quad x \in [0, 1], \quad k = 2, 3, \dots,$$

$0 \leq a_n^k \leq 1$  for all  $k, n$ ;  $(1 - a_n^2) \downarrow_n 0$  and  $(a_n^k - a_n^{k+1}) \downarrow_n 0$ , for  $k = 2, 3, \dots$ . Then  $A \in B(\ell_2)$  and

$$\|A\|_{B(\ell_2)} \leq C(2 - \lim_{k \rightarrow \infty} a_1^k),$$

where  $C > 0$  depends only on the sequence  $(a_1^k)_{k \geq 1}$ .

**Corollary** Let  $X \in M(\ell_2)$ ,  $\tilde{X} = M[0, 1]$  and  $A = \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \\ \vdots \end{pmatrix}$ , where

$$f_k(x) = \frac{1}{2}a_0^k + \sum_{n=1}^{\infty} a_n^k \cos 2\pi n x, \quad x \in [0, 1] \text{ and } k = 1, 2, \dots$$

Moreover, let  $(a_n^1)_{n \geq 0}$  be a quasiconvex sequence with  $\lim_n a_n^1 = 0$  and

$$M_0 := \sum_{n=0}^{\infty} (n+1) |\Delta^2 a_n^1|.$$

Let  $(a_n^k - a_n^{k+1})_n$  be a family, indexed on  $k$ , of quasiconvex sequences converging to zero. Let

$$M_k = \sum_{n=1}^{\infty} (n+1) |\Delta^2 (a_n^k - a_n^{k+1})|, \quad k = 1, 2, \dots$$

If  $\sum_{k=0}^{\infty} M_k < \infty$ , it follows that  $A \in L^1(\ell_2)$ .

Here  $M(\ell_2)$  is the usual Banach space of Schur multipliers.

## References

- [1] R. Edwards, *Fourier Series: a modern introduction*, vol. I, Springer Verlag, New-York, 1979.
- [2] A. Zygmund, *Trigonometric series*, Cambridge University Press, Cambridge, 1959.

# Characteristic functions and operator models on noncommutative polydomains

**Gelu Popescu**

We introduce a class of noncommutative polydomains in  $B(H)^n$  and characterize the  $n$ -tuples of operators which admit characteristic functions. The characteristic function is constructed explicitly as an artifact of the noncommutative Berezin kernel associated with the polydomain, and it turns out to be a complete unitary invariant for the class of completely non-coisometric tuples, which are determined. An operator model theory along the lines of Sz.-Nagy–Foias theory of contractions is also presented.

# Shy and fixed distance couplings on Riemannian manifolds

**Ionel Popescu\*, Mihai N. Pascu**

We show that on any Riemannian manifold with the Ricci curvature non-negative we can construct a coupling of two Brownian motions which are staying fixed distance for all times. We show a more general version of this for the case of Ricci bounded from below by  $k$ . In the terminology of Burdzy, Kendall and others, a shy coupling is a coupling in which the Brownian motions do not couple in finite time with positive probability. What we construct here is a strong version of shy couplings on Riemannian manifolds. On the other hand, this can be put in contrast with some results of von Renesse and K. T. Sturm which give some form of couplings for which the distance between the motions is bounded above by a deterministic exponential function in terms of the lower bound on the Ricci curvature.

Mihai Putinar

Due to recent works orchestrated by the late Julius Borcea, the half-century old Sendov conjecture (about the location of critical points of a complex polynomial) can be interpreted in terms of rank-one perturbations of normal matrices. A few related observations of operator theory and potential theory flavor will be presented.

## Approximation numbers of composition operators on the Dirichlet space

Herve Queffelec\*, P. Lefevre, D. Li, L. Rodriguez-Piazza

In a recent joint work (2012) with D.Li and L.Rodriguez-Piazza, we studied the approximation numbers of composition operators on the Hardy space  $H^2$  of the unit disk  $\mathbb{D}$ . One application was an optimal improvement of a theorem first proved by E.Gallardo-Gutierrez and M.Gonzalez (2003), and then improved by O.Elfallah, K.Kellay, M.Shanbankhah, H.Youssfi (2011), on the thinness of the set of contact points for a given symbol  $\varphi$  acting on  $H^2$  :

$$E_\varphi = \{u \in \partial\mathbb{D} ; |\varphi(u)| = 1\}.$$

This application used our estimates on approximation numbers of  $C_\varphi$  as well as the notion of peak set for the disk algebra  $A(\mathbb{D})$  (through an old result of Fatou). In a joint work under progress with P.Lefevre, D.Li and L.Rodriguez-Piazza, which will be the subject of the talk, we perform a similar study for composition operators on the Dirichlet space  $\mathcal{D}$  of the unit disk  $\mathbb{D}$ . The situation here is more intricate in several respects: first (due to a theorem of Beurling), one has to switch from zero Lebesgue measure to zero logarithmic capacity. Secondly, not all analytic self-maps  $\varphi$  of the disk generate a bounded composition operator. Finally, the notion of peak set is more restrictive and delicate to handle. One essential fact is that a compact set  $K \subset \partial\mathbb{D}$  of logarithmic capacity zero is a peak set for some  $\varphi \in \mathcal{D} \cap A(\mathbb{D})$  such that  $C_\varphi : \mathcal{D} \rightarrow \mathcal{D}$ . We also have reasonable (although still incomplete) information on approximation numbers of composition operators acting on this new space. Both facts allowed us to improve the results obtained by the previously quoted authors in the case of the Dirichlet space. The improvement turns out to be optimal in this new situation as well.

## A Hopf algebra freeness result revisited

David Radford

The affine (Hopf) algebra of the special linear group  $SL(2\ell, k)$  for  $\ell \geq 1$  is not a free module over one of its Hopf subalgebras. In this talk we give a very simple proof of this fact, where there are mild restrictions on the field  $k$ . The proof boils down to showing that a certain module is not cyclic.

## Wellposedness for the Westervelt equation with nonlinear damping

Petronela Radu\*, Barbara Kaltenbacher

In this talk I will present recent results on wellposedness for the Westervelt equation with acoustic-acoustic and elastic-acoustic coupling. The Westervelt equation is widely used in nonlinear acoustics for the simulation of high intensity focused ultrasound (HIFU) which has a wide range of technical and medical applications from lithotripsy or thermotherapy to ultrasound cleaning or welding and sonochemistry. The model proposed shows that we could relax the regularity for the solutions we are seeking by appropriately using the nonlinear damping in the equation.

## Semi-parabolic Hénon maps

Remus Radu\*, Raluca Tănase

Consider the parameter space  $\mathcal{P}_\lambda \subset \mathbb{C}^2$  of complex Hénon maps  $H_{c,a}(x, y) = (x^2 + c + ay, ax)$  which have a fixed point with one eigenvalue a root of unity  $\lambda$ ; these are called semi-parabolic Hénon maps. Unlike hyperbolic transformations which exhibit structural stability, semi-parabolic transformations are not generally expected to be structurally stable. Inside the parabola  $\mathcal{P}_\lambda$ , we look at those Hénon maps that are small perturbations of a quadratic polynomial  $p$  with a parabolic fixed point of multiplier  $\lambda$ . We get an open disk of parameters for which the semi-parabolic Hénon map is structurally stable on the Julia sets  $J$  and  $J^+$ . The set  $J^+$  can be understood as an inductive limit of  $J_p \times \mathbb{D}$  under an appropriate solenoidal map, where  $J_p$  is the Julia set of the polynomial  $p$ . The set  $J$  is homeomorphic to a solenoid with identifications.

**Claudiu Raicu\*, Jerzy Weyman, Emily Witt**

I will report on joint work with Jerzy Weyman and Emily Witt on computing the GL-equivariant description of the local cohomology modules with support in the ideal of maximal minors of a generic matrix, as well as of those with support in the ideal of  $2n \times 2n$  Pfaffians of a  $(2n + 1) \times (2n + 1)$  generic skew-symmetric matrix. I will explain the main tools that we employ in our study, namely (1) the BGG correspondence between the homology groups of linear complexes over a polynomial ring and minimal free resolutions over the exterior algebra, and (2) the exterior algebra analogue of the geometric technique for computing syzygies. Our work is based on that of Akin, Buchsbaum and Weyman, who describe the minimal free resolutions of the powers of the ideal of maximal minors of a generic matrix, and on the work of Boffi and Sánchez who obtain the corresponding result for Pfaffians.

## Random matrices and light propagation through opaque materials

**N. Raj Rao**

We consider the scientific problem of when light can be completely transmitted (i.e., with no reflection) through "opaque" materials such as paper, egg shells or chicken breasts. Here, we have a scattering matrix associated with the material, which describes how incident plane waves couple into outgoing plane waves; it is the random locations of the physical scatterers that produces the random scattering matrix. This matrix is very different from any of the classical ensembles in random matrix theory. Exploiting the physical properties of this matrix leads to a remarkable connection between the properties of the scattering matrices of each tiny slice of a medium and the scattering matrix of the composite medium and brings into focus the role of free probability. Using high-precision numerics on physically accurate system models to flesh out this connection reveals the surprising effectiveness of theoretical predictions made using free probability. We highlight these connections and discuss application-motivated extensions of free probability.

## Neural network function – density, geometry and dynamics

**Anca Rădulescu**

The study of dynamic networks has been the focus of great interest in recent research. Many natural systems are organized as networks, in which the nodes (be they cells, individuals or web servers) interact in a time-dependent fashion. It has been hypothesized that there are two key conditions for optimal function in such networks: a well-balanced adjacency matrix (the underlying graph should appropriately combine robust features and random edges) and well-balanced connection strengths (driving optimal dynamics in the system). The object of our study is relating connectivity to the temporal behavior of the network. We consider an  $n$ -dimensional dynamical system, in which variables are nonlinear oscillators, coupled according to a connectivity scheme that obeys certain constraints, but also incorporates random aspects. We study how the phase space dynamics and bifurcations of the system change when perturbing the underlying adjacency graph. Understanding the effects of configuration on coupled dynamics is of great importance for a wide variety of applications. Recent studies have used a combination of dynamical systems and graph theoretical approaches to investigate general organizational principles of brain networks. With nodes and edges defined according to modality appropriate scales, these studies support certain generic topological properties of the human brain architecture, such as modularity, small-worldness, the existence of hubs and other connectivity density patterns. We will illustrate how these properties may be used in the future to better understand neural processes, or even as biomarkers for behavioral traits or neuropsychiatric conditions.

## Stability issues in viscoelastic flows

**Michael Renardy**

Unlike the Newtonian case, it is in general not known whether stability of non-Newtonian flows can be inferred from linearized spectra. The lecture will review recent work on rigorous criteria for stability of non-Newtonian flows. These criteria are based on supplementing spectral information with short wave asymptotics, using a technique originally developed for the Euler equations.

# Skew Calabi-Yau algebras and homological identities

Manuel Reyes\*, Daniel Rogalski, James J. Zhang

A skew Calabi-Yau algebra is a generalization of a Calabi-Yau algebra which allows for a non-trivial Nakayama automorphism. I will discuss homological identities related to the Nakayama automorphism, with some applications. These identities show (i) how the Nakayama automorphism of a smash product algebra  $A\#H$  is related to the Nakayama automorphisms of a graded skew Calabi-Yau algebra  $A$  and a finite-dimensional Hopf algebra  $H$  that acts on it; and (ii) that Nakayama automorphism of a skew Calabi-Yau algebra  $A$  has trivial homological determinant in case  $A$  is noetherian, connected graded, and Koszul.

## On a module theoretic analogue of a left Rickart ring

S. Tariq Rizvi\*, Gangyong Lee, Cosmin Roman

The notion of a Rickart ring was introduced by Maeda and Hattori and developed by many authors including Kaplansky who also introduced the stronger notion of a Baer ring. A ring  $R$  is called right Rickart (Baer) if the right annihilator of any single element (any nonempty subset) of  $R$  is a right direct summand of  $R$ . Left sided notions are defined similarly. While the notion of a Baer ring is always left-right symmetric, this does not hold true for the Rickart property of rings. Using the endomorphism of a module, we recently introduced the notion of a Rickart module extending the right Rickart property of a ring. Let  $M_R$  be any right  $R$ -module and  $S = \text{End}_R(M)$ .  $M$  is called a *Rickart module* if the right annihilator in  $M$  of any single element of  $S$  is a direct summand of  $M$ , i.e.,  $r_M(\varphi) = \text{Ker}\varphi \leq^{\oplus} M$  for every  $\varphi \in S$ . Now we extend the notion of a left Rickart ring and provide a module theoretic analogue as follows:  $M_R$  is called an  $\mathcal{L}$ -Rickart module if the left annihilator in  $S$  of any single element of  $M$  is generated by an idempotent in  $S$  as a left ideal. Equivalently, for any  $m \in M$ ,  $\mathbf{I}_S(m) = Se$  for some  $e^2 = e \in S$ . Note that  $\mathbf{I}_S(m) = \mathbf{I}_S(mR)$ . In this work we will present results which connect this notion to other existing notions and provide examples that illustrate and delimit our results.

## Solutions to a full model for thermoviscoelastic materials

Elisabetta Rocca\*, Riccarda Rossi

In this talk, we analyze a PDE system arising in the modeling of phase transition and damage phenomena in thermoviscoelastic materials. The resulting evolution equations in the unknowns (absolute temperature),  $u$  (displacement), and (phase/damage parameter) are strongly nonlinearly coupled. Moreover, the momentum equation for  $u$  contains phase-dependent elliptic operators, That is why, we have to resort to a suitable weak solvability notion for the analysis of the problem: it consists of the weak formulations of the heat and momentum equation, and, for the phase/damage parameter, of a generalization of the principle of virtual powers.

## Directional maximal operators and lacunarity in higher dimensions

Keith Rogers

I will present a notion of lacunarity in higher dimensions for which the associated directional maximal operators are bounded in  $L^p(\mathbb{R}^n)$  with  $p > 1$ . This extends the two-dimensional work of Strömberg, Córdoba-Fefferman, Nagel-Stein-Wainger, Sjögren-Sjölin and Alfonseca-Soria-Vargas. Bateman proved that their notion of lacunarity, which is easy to describe using the order on the circle, is in some sense the correct one in that directional maximal operators in two dimensions are bounded if and only if the associated directions are lacunary (of finite order). Our higher dimensional notion derives from a localisation principle: the maximal operator associated to a set of directions can be controlled by the maximal operator associated to a subset of the directions. If time permits, I will also explain how this can be used to give an (albeit less descriptive) characterisation of the directions which give rise to bounded maximal operators in higher dimensions. This is joint work with Javier Parcet.

## Central sequence $C^*$ -algebras and absorption of the Jiang-Su algebra

Mikael Rørdam\*, Eberhard Kirchberg

We extend the seminal work of Matui and Sato, that a simple nuclear separable  $C^*$ -algebra with finitely many extremal tracial states absorbs the Jiang-Su algebra if and only if it has strict comparison, to the case where the extreme boundary of the trace simplex is compact and of finite topological dimension. We can also relax the comparison condition to much weaker one. Our results are obtained through a systematic study of the central sequence  $C^*$ -algebra.

# An optimal matching problem for the euclidean distance

**Julio Daniel Rossi**

We deal with an optimal matching problem, that is, we want to transport two measures to a given place, where they will match, minimizing the total transport cost that in our case is given by the sum of the Euclidean distance that each measure is transported. We show that such a problem has a solution. Furthermore we perform a method to approximate the solution of the problem taking limit as  $p \rightarrow \infty$  in a system of PDE's of  $p$ -Laplacian type. (joint work with J. Mazon and J. Toledo)

## On generalized stochastic variational inequalities

**Eduard Rotenstein\*, Anouar Gassous, Aurel Răscanu**

The talk proves for the beginning the existence and the uniqueness of a solution for the following multivalued deterministic variational inequality with oblique subgradients, considered in a non-convex domain:

$$\begin{cases} x'(t) + H(x(t)) \partial^- \varphi(x(t)) \ni g(t), & a.e. t \in [0, T] \\ x(0) = x_0 \in Dom(\varphi), \end{cases}$$

where  $\partial^- \varphi$  is the Fréchet subdifferential of a semiconvex function  $\varphi$  and the matrix application  $x \mapsto H(x)$  is a Lipschitz mapping. The presence of the oblique reflection brought by the term  $H(x) \partial^- \varphi(x)$  leads to the use of different techniques comparing to the cases of standard reflection in non-convex domains or oblique reflection in convex domains. The cases of a generalized Skorohod problem and of a stochastic variational inequality with oblique reflection, considered in a non-convex domain are also envisaged.

## Doubly commuting quotient modules

**Jaydeb Sarkar**

In this talk we will discuss a complete characterization of the doubly commuting quotient modules of the Hardy module over the polydisc. We prove that for a large class of submodules, essential doubly commutativity is a two variables phenomenon.

## New properties of certain general integral operators on analytic functions

**Grigore Ştefan Sălăgean**

Some univalence, starlikeness and convexity properties of certain general integral operators on analytic functions are established. Some of this operators were considered before by D. Breaz, N. Breaz, B. A. Frasin etc.

## Some considerations regarding the implications of fuzzy logic in the qualitative mathematical modeling of some aspects surrounded by uncertainty and ambiguity within managerial decisional process

**Lucian Sârb**

It is well known that the purpose of all organizations or companies in the contemporary competitive and globalized economic environment can be summarized in six words: to work more efficiently and performant. Therefore, the business processes are under the imperative of taking the best decisions, otherwise they will not be able to be performed in an efficient manner, given the increasingly more competitive global environmental specific to information society. More than that, aspects such the uncertainty met within different decisional circumstances are becoming more frequently, fact that conducts towards certain situations surrounded by ambiguity for the decisional human factors from organizations. Thats why, the informatization or the use of some expert systems in enterprises primarily aims to achieve this target: more rapid, more reliable and more efficient decisions. Most of this decisions imply taking into account more decisional alternatives evaluated through linguistic assessments, not by real numbers, depending by certain selection criteria and by their weight of importance, as well by the potential performance of these decisional alternatives with respect to each selection criteria pursued. Moreover, in this circumstances, the reasoning of decisional factors often tends to be a subjective one, fact revealed by itself abstract thinking of the human being., materialized often through linguistic values. In this context, the decisional process needs to be modelled by a qualitative mathematical tool and maybe the most suitable one for this fact is the fuzzy logic through its related fuzzy sets. Therefore, the main topic of this talk is to develop a fuzzy methodology or algorithm for modelling the multicriterial decisional processes within different decisional circumstances surrounded by uncertainty, depending of its level of compatibility with the decisional conditions concerned. In this framework, embedding this algorithm

within a fuzzy software tool created in Java programming language, in order to provide more accurate, reliable, objective and quickly results, represents a very useful leverage for all managers from the world in dealing with the uncertainty and ambiguity decisional situations and with their reasoning marked by subjectivity.

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*Keywords:* fuzzy logic, multicriteria decision making process, uncertainty, subjectivity.

## Some remarks on braided pointed Hopf algebras

**Hans-Juergen Schneider**

I will discuss some general results on braided pointed Hopf algebras with applications to Nichols algebras of braided vector spaces. The special case of Nichols algebras of Yetter-Drinfeld modules over group algebras is important for the classification of pointed Hopf algebras.

## Small data global well-posedness and consequences of convolution inequalities

**Tobias Schottdorf**

We summarize a few “sleek” small data global existence results for quadratic nonlinear dispersive equations. These are obtained in the absence of resonance, which manifests itself through a geometric condition on the characteristic hypersurfaces involved in the nonlinear interaction. We complement this with a negative result in the presence of (mildly) degenerate resonance, manifested through lack of differentiability of the solution operator. This part will make use of convolution estimates and their dependence on transversality.

## Stillman’s question on bounding projective dimension

**Alexandra Seceleanu**

Hilbert’s Syzygy Theorem is a classical result bounding the projective dimension of any homogeneous ideals in a given polynomial ring. More recently, M. Stillman asked whether the projective dimension of homogeneous ideals could be bounded purely in terms of the degrees of a minimal set of generators, without knowledge of the ambient polynomial ring. This and the similar question for regularity are still open and have attracted a great deal of interest because of their relevance from both a theoretical and a computational perspective. I will discuss some approaches and recent progress on these and related questions.

## A SIR epidemic model structured by immunological variables

**Laurențiu Segă**

Standard mathematical models for analyzing the spread of a disease are usually either epidemiological or immunological. The former are mostly ODE-based models that use classes like susceptibles, recovered, infectives, latently infected, and others to describe the evolution of an epidemic in a population. Some of them also use structure variables, such as size or age. The latter describe the evolution of the immune system/pathogen in the infected host—evolution that usually results in death, recovery or chronic infection. There is valuable insight to be gained from combining these two types of models, as that may lead to a better understanding of the severity of an epidemic. We propose a new type of model that combines the two by using variables of immunological nature as structure variables for epidemiological models. We prove the well-posedness of the proposed model under some restrictions and conclude with a look at a practical application of the model.

## Poincaré series of modules over compressed Gorenstein local rings

**Liana Segă**

Given positive integers  $e$  and  $s$  we consider Gorenstein Artinian local rings  $R$  whose maximal ideal  $\mathfrak{m}$  satisfies  $\mathfrak{m}^s \neq 0 = \mathfrak{m}^{s+1}$  and  $\text{rank}_{R/\mathfrak{m}}(\mathfrak{m}/\mathfrak{m}^2) = e$ . We say that  $R$  is a *compressed Gorenstein local ring* when it has maximal length among such rings. It is known that generic Gorenstein Artinian algebras are compressed. If  $s \neq 3$ , we prove that the Poincaré series of all finitely generated modules over a compressed Gorenstein local ring are rational, sharing a common denominator. A formula for the denominator is given. When  $s$  is even this formula depends only on the integers  $e$  and  $s$ . Note that for  $s = 3$  examples of compressed Gorenstein local rings with transcendental Poincaré series exist, due to Bøgvad.

# Some properties of the indicators of Hopf algebras

Kenichi Shimizu

Let  $H$  be a finite-dimensional Hopf algebra over a field  $k$ . The  $n$ -th indicator  $\nu_n(H)$ , introduced by Kashina, Montgomery and Ng [KMN], is defined to be the trace of  $h \mapsto S(h_{(1)} \cdots h_{(n-1)})$  ( $h \in H$ ), where  $S$  is the antipode of  $H$  and  $h_{(1)} \otimes \cdots \otimes h_{(m)} \in H^{\otimes m}$  is the iterated comultiplication of  $h$  in Sweedler's notation. In [KMN], they showed that  $\nu_n(H)$  is an invariant of the  $k$ -linear monoidal category of left  $H$ -modules for all  $n \geq 1$ . I will talk about my recent investigation [S] of properties  $\nu_n$ 's. In particular, I give a new description of  $\nu_n(H)$  and, as an application, show that  $\nu_n(H)$  is a cyclotomic integer for all  $n \geq 1$ . Also obtained are formulas of  $\nu_n$  of  $H^{\text{op}}$  and the Drinfeld double  $D(H)$ : If  $k$  is the field of complex numbers, then

$$\nu_n(H^{\text{op}}) = \overline{\nu_n(H)} \quad \text{and} \quad \nu_n(D(H)) = |\nu_n(H)|^2,$$

where the latter formula has been conjectured in [KMN] and proved in the case where  $H$  is semisimple. If time permits, I will mention an extension of  $\nu_n(H)$  for not necessarily positive  $n$ .  $\nu_n(H)$  is shown to be an invariant of the  $k$ -linear monoidal category of left  $H$ -modules for all  $n$ . Among  $\nu_n$ 's for  $n \leq 0$ , only  $\nu_0$  and  $\nu_{-1}$  are interesting since  $\nu_{-n}$  for  $n \geq 2$  can be expressed by  $\nu_n$  and  $\nu_{-1}$ . I will give interpretations of  $\nu_0$  and  $\nu_{-1}$  in the framework of finite tensor categories, while the category-theoretical meaning of  $\nu_n$  for  $n \geq 2$  is unclear.

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# Improvements of NIST statistical test suite

Emil Simion

In this talk we propose an improvement of the statistical decision, regarding the randomness, proposed by National Institute of Standards and Technologies (NIST) in the guideline Statistical Test Suite (STS) Special Publication (SP) 800-22, on computing the second order error (the probability of acceptance a false hypothesis). Also we present several generalizations of these statistical tests to non-uniform distributions.

# Propagation of IPW in monoclinic crystals subject to initial fields

Olivian Simionescu-Panait

Inhomogeneous plane waves (IPW) arise in many areas of mechanics of continua, including Rayleigh, Love and Stoneley waves from the linear elasticity theory, TE and TM waves from electromagnetism, or viscoelastic waves. The use of complex vectors (called *bivectors*) leads to a direct formulation of the condition of propagation for this kind of waves. We present here the conditions of propagation for inhomogeneous plane waves in monoclinic crystals subject to initial electromechanical fields. We obtain the components of the electroacoustic tensor for the class 2, resp.  $m$ , of the monoclinic system. For particular isotropic and anisotropic directional bivectors we derive and analyze the decomposition of the propagation condition. In the first particular case we show that the specific coefficients are similar to those from the problem of guided waves propagation in monoclinic crystals subject to a bias.

# Local cohomology of polynomial rings over the integers

Anurag Singh

We will discuss a recent result, joint with Bhatt, Blickle, Lyubeznik, and Zhang, that the local cohomology of a polynomial ring over the ring of integers has finitely many associated prime ideals.

# Martin kernels for cones

Bartłomiej Siudeja\*, Krzysztof Bogdan, Andrzej Stos

Martin kernels for cones are homogeneous harmonic functions on cones with zero boundary conditions and a point mass at infinity. We give precise asymptotics for the homogeneity constant for Martin kernels for fractional Laplacians on narrow cones.

# On numerical discretization for the motion of a self-propelled deformable structure in a viscous incompressible fluid

**Loredana Smaranda**

We focus on a numerical method for the discretization of an initial and boundary value problem that models the self-propelled motion of one deformable solid in a bidimensional viscous incompressible fluid. In the model, we suppose that the solid is subjected to a known deformation field representing the action of the aquatic organism muscles. The governing equations consist of the Navier-Stokes equations for the fluid, coupled to Newton's laws for the solid. The numerical method we propose is based on a global weak formulation, where the nonlinear term in the Navier-Stokes model is discretized using the characteristic function. Since the formulation is global in space, this characteristic function is extended in an appropriated manner inside of the creature, taking into account its deformation. We first concentrate our attention in the semi-discretization in time and we prove the stability and the convergence of the scheme. The numerical method is consistent enough with the motion of the creature and for this reason, the discretization in space variable is successfully implemented using finite element method.

## A classification of differential invariants for multivariate cryptosystems

**Daniel Smith**

We derive a discrete geometric result, a classification of differential symmetries for certain multivariate functions over finite fields. As an application, we prove differential security in our model for a well-known family of multivariate cryptosystems.

## Local wellposedness of Chern-Simons-Schrödinger

**Paul Smith**

The Chern-Simons-Schrödinger model in two spatial dimensions is a covariant NLS-type problem and is  $L^2$  critical. We prove that, with respect to the heat gauge, this system is locally well-posed for initial data that is small in  $H^s$ ,  $s > 0$ . This work is joint with Baoping Liu and Daniel Tataru.

## Some results on finitely presented algebras

**Agata Smoktunowicz**

We look at some properties of finitely presented algebras. We show that such algebras contain noncommutative free subalgebras, and that they can be homomorphically mapped onto prime algebras with linear growth, under mild assumptions on the number of the defining relations of each degree. By a result of Small and Warfield, the structure of algebras with linear growth is well understood; they showed that it is close to the polynomial ring over the base field. Subsequently Small, Stafford and Warfield showed that affine semiprime algebras satisfy a polynomial identity. We will show how our results in conjunction with the Small, Stafford and Warfield theorem can be used to solve special cases of some open problems in noncommutative ring theory. In particular our results imply that finitely presented algebras with a reduced number of relations are neither Jacobson radical nor nil, but it is not known if the Jacobson radical of such algebras is nil. We also recall some old and new open questions about finitely presented algebras, nil rings and Jacobson radical rings.

## The Moore-Penrose inverse- theory and computational aspects

**Alicja Smoktunowicz**

E. Moore introduced the general reciprocal in 1920. This was later rediscovered by R. Penrose in 1955. Nowadays the general reciprocal is called the Moore-Penrose inverse. The Moore-Penrose inverse is a powerful tool in the least squares problem, statistics, control, image processing, pattern recognition and other practical applications. Given an  $m \times n$  complex matrix  $A$ , there exists a unique  $n \times m$  matrix  $X$ , called the Moore-Penrose inverse, such that  $AXA = X$ ,  $XAX = X$ ,  $AX = (AX)^H$  and  $XA = (XA)^H$ , where  $A^H$  is the conjugate transpose. We present a comparison of certain direct methods for computing the Moore-Penrose inverse. We study the numerical stability and algebraic complexity of the considered algorithms. It is a joint work with Iwona Wróbel.

# Involutory quasi-Hopf algebras and Ribbon quasi-Hopf algebras

Yorck Sommerhäuser

According to a definition by D. Bulacu and B. Torrecillas, a quasi-Hopf algebra is called involutory if the square of its antipode is given by conjugation with a certain element built from the evaluation element and the coevaluation element. In analogy with the Hopf algebra case, they conjectured that a semisimple cosemisimple quasi-Hopf algebra is involutory in this sense. An a priori different question was raised by D. Bulacu about work of Y. Zhu and the speaker: He asked whether a semisimple quasi-Hopf algebra over a field of characteristic zero always becomes a ribbon Hopf algebra if the ribbon element is defined via a certain formula that involves the Drinfel'd element, the evaluation element, and the coevaluation element, in analogy with the fact that for Hopf algebras, the inverse Drinfel'd element is a ribbon element in this situation. As it turns out, these two questions are related. In the talk, we explain how they are related, and answer them both. The talk is based on a recent article (Adv. Math. 236 (2013), 158-223) written jointly with Y. Zhu.

## On nonlinear Schrödinger type equations with nonlinear damping

Christof Sparber\*, Paolo Antonelli, Remi Carles

We consider a class of nonlinear Schrödinger type equations augmented by nonlinear damping terms. This class of equations is purely dispersive, but no longer Hamiltonian. Local and Global well-posedness results (in the energy space) under various assumptions on the involved parameters will be presented. In addition, the long time behavior and, in particular, the possible extinction of solutions will be studied.

## Torsion in kernels of induced maps on divisor class groups

Sandra Spiroff\*, Sean Sather-Wagstaff

We discuss torsion elements in the kernel of the map on divisor class groups of excellent local normal domains  $A$  and  $A/I$ , when  $I$  is an ideal of finite projective dimension. The motivation for this work is a result of Griffith-Weston which applies when  $I$  is principal. In particular, under certain hypotheses we show that for any positive integer  $e > 1$  which represents a unit in  $A$ , the kernel of  $\text{Cl}(A) \rightarrow \text{Cl}(A/I)$  contains no element of order  $e$ .

## Kokotsakis meshes and flexible quad meshes

Hellmuth Stachel

A Kokotsakis mesh is a polyhedral structure consisting of an  $n$ -sided central polygon which is surrounded by a belt of polygons. We are interested in meshes which are continuously flexible when each involved facet is assumed to be a rigid body which can rotate relatively to each neighbor facet about the common edge. Kokotsakis meshes are of fundamental importance for the following: 1) The flexible versions in the case of a central triangle ( $n = 3$ ) are directly connected with Bricard's Octahedra. Due to recent results of G. Nawratil almost all non-trivial self-motions of planar Stewart-Gough-Platforms are related to these octahedra. 2) A quadrangular mesh ('quad mesh' by short) is a simply connected subset of a polyhedral surface consisting of planar quadrangles, edges and vertices in the Euclidean 3-space. A quad mesh is continuously flexible if in any generic pose all included Kokotsakis meshes are continuously flexible. A remarkable example of a flexible quad mesh arises when there is an initial flat pose where the convex quadrangles form a tessellation with the symmetry group  $p2$ . In the generic case the complete tessellation is an example of a flexible periodic framework with the property that the symmetry group of each flexion remains isomorphic to that of the initial flat pose. In special cases an  $m \times n$  quad mesh out of the tessellation can admit a pose where the mesh forms a tiling on a cylinder of revolution; after wrapping around a cylinder two opposite boundaries fit exactly together - apart from a shift. When in such a closing pose the boundaries are glued together along their overlap then the mesh is infinitesimally rigid.

## Estimates for nonlocal analogues of the wave equation

John Stalker

I will discuss some integrodifferential equations of which the simplest is

$$u_{tt} + \mu u = J * u$$

where  $J \in L^1(\mathbb{R}^{1+n})$  is nonnegative and  $\mu = \int J$ . This equation is very far from satisfying Huygens' principle. In fact, the support of any nonzero solution is all of  $\mathbb{R}^{1+n}$ . It can, nonetheless, be thought of as an analogue of the wave equation and satisfies some of the same estimates. It even exhibits a sort of "almost finite speed of propagation".

# On the CI property of the tangent cone of a toric ring

Dumitru Stamate

We study the complete intersection(CI) property for the tangent cone ( $grK[S]$ ) of a toric ring  $K[S]$  associated to a semigroup  $S$ . Fix a semigroup  $S$  generated by the nonnegative integers  $n_1$ .

## Some inequalities about the norms of generalized Gaussian Wick products

Aurel Stan

Recently Da Pelo and Lanconelli introduced a family  $\{\diamond_t\}_{0 \leq t \leq 2}$  of commutative and associative products. They are defined as follows:

$$f \diamond_t g = \Gamma\left(\frac{1}{\sqrt{t}}I\right) \left[ \Gamma(\sqrt{t}I)f \cdot \Gamma(\sqrt{t}I)g \right],$$

for random variables  $f$  and  $g$  measurable with respect to the sigma-algebra given by a Gaussian probability measure, where  $\Gamma(cI)$  denotes the second quantization operator of  $c$  times the identity, for every constant  $c$ . It turns out that this product is related to the way of choosing the sample points  $s_i^*$  in the subintervals  $[s_{i-1}, s_i]$  of a partition of the interval  $[a, b]$  when defining the stochastic integral  $\int_a^b X_s dB_s$ . Thus for  $t = 0$ ,  $f \diamond_0 g = \lim_{t \rightarrow 0} f \diamond_t g$  is the classic Wick product of  $f$  and  $g$ , and it corresponds to choosing the sample points at the beginning of the sub-intervals of each partition, that means to the Itô integral. For  $t = 1$ , the sample points become the midpoints of the subintervals, that means we obtain the Stratonovich integral. For  $t = 2$ , the sample points become the right-end points of the subintervals. The purpose of this talk is to present some inequalities about the  $L^p$ -norms of random variables of the form  $f \diamond_t g$ .

## Anisotropic variable exponent PDEs

Denisa Stancu-Dumitru

We will present some nonlinear elliptic problems involving an anisotropic operator with variable exponents which allows a distinct behavior of partial derivatives in various directions. In the particular case when the variable exponents are all equal, this operator has similar properties with the  $p(\cdot)$ -Laplacian. For these problems, we investigate the existence and multiplicity of solutions. The proofs of our results combine the critical point theory and different variational methods.

## Asymptotic analysis of a fluid-thin structure interaction problem

Ruxandra Stavre\*, Grigory Panasenکو

In the present talk we consider the interaction "viscous fluid-thin plate" when the thickness of the plate,  $\varepsilon$ , tends to zero, while the density and the Young's modulus are of different orders with respect to  $\varepsilon$ . We discuss the conditions when this model is an asymptotic derivation of a previous problem studied by the authors in [1]. The paper [1] deals with the viscous fluid-elastic membrane interaction problem, where the membrane is described by Sophie Germain's hyperbolic equation of the fourth order in the space variable. This equation is a limit model for the elasticity equation set in a thin domain with a given force at the lateral boundary (see [2], Ch. 3). We prove that, with some assumptions on the data, the leading term of the asymptotic expansion satisfies the equations of our previous article. We apply the partial asymptotic decomposition method when a part of the plate is described by a 1D model while the other part is modeled by the 2D elasticity equations, with appropriate junction conditions.

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## Metric trees and Lang's origami universal molecules

Ileana Streinu

In a seminal paper from 1996, Robert Lang introduced the Universal Molecule method for designing folded origami bases with an underlying metric tree structure. We show that, conversely, every metric tree can be realized as a Lang origami base.

# Sparsity and pebble games in combinatorial rigidity

Ileana Streinu

A famous open problem is to give a combinatorial characterization for generic minimally rigid frameworks made from fixed-length bars connected by rotatable joints. The question can be traced back to the work of James Clerk Maxwell in 1864, who gave a necessary condition expressible as a special kind of graph sparsity; it has then been shown in 1970 by Laman that the condition is also sufficient for 2D rigidity. Beyond this and a few other isolated cases, very little is known, especially in higher dimensions. Pebble games are a family of very simple and efficient algorithms for answering sparsity questions, and for calculating rigidity and flexibility parameters when Maxwell's sparsity applies. In this talk, I will present these varied facets of combinatorial rigidity and conclude with some recent results.

## Numerical extremal radial solutions for Dirichlet problems with mean curvature operator in Minkowski space

Călin Șerban

We are concerned with numerical approximation of extremal (minimal/maximal) classical radial solutions for the nonlinear Dirichlet problem associated with the mean extrinsic curvature operator in Minkowski space:

$$\operatorname{div} \left( \frac{\nabla v}{\sqrt{1 - |\nabla v|^2}} \right) + g(|x|, v) = 0 \quad \text{in } \mathcal{B}(R), \quad v = 0 \quad \text{on } \partial\mathcal{B}(R),$$

where  $R > 0$ ,  $\mathcal{B}(R) = \{x \in \mathbb{R}^N : |x| < R\}$ .

## On Koszul rings and corings

Dragoș Ștefan

Koszul algebras, introduced by Priddy in 1970, play an important role in many fields of Mathematics, including Algebraic Topology, Algebraic Geometry, Representation Theory, Combinatorics, etc. The main aim of the talk is to survey the most important properties and applications of some rather recent generalizations of these graded algebras. A special attention will be paid to Koszul rings and their Koszul dual corings. Our approach is based on Koszul pairs, a new tool that helps us to investigate the Hochschild (co)homology of Koszul rings. Several examples of Koszul pairs, related to twisted tensor products of algebras and to certain braided bialgebras in the category of bimodules over a semisimple ring, will be given.

## Gain-sparsity and the rigidity of symmetric frameworks in the plane

Shin-ichi Tanigawa

Borcea and Streinu introduced an extension of rigidity matrices for analyzing the rigidity of infinite periodic frameworks, and this extension was used to prove a combinatorial characterization of the symmetry-forced rigidity of periodic frameworks by Malestein and Theran. Similar matrices, called orbit rigidity matrices, were proposed by Schulze and Whiteley for finite symmetric frameworks in order to detect finite motions that preserve the underlying symmetry. In this talk, I will show combinatorial necessary and sufficient conditions for orbit rigidity matrices to have the maximum rank. Specifically, we show that the row matroids of orbit rigidity matrices belong to a certain class of combinatorial matroids, called gain-sparsity matroids, on group-labeled graphs. Our result implies a combinatorial characterization of the symmetry-forced rigidity of generic finite frameworks under certain point group symmetries in the plane. In other words, we can decide whether a symmetric framework has a symmetry-preserving finite flex under the assumption that joints are placed as generic as possible subject to the underlying symmetry. I will also show that the idea of using orbit rigidity matrices can be extended to the analysis of the infinitesimal rigidity of symmetric frameworks. This result leads to a combinatorial characterization of the infinitesimal rigidity of generic frameworks with the reflection symmetry or the rotation symmetry of order two or three. The talk mainly reports on recent joint works with Tibor Jordan, Viktoria Kaszanitzky and Bernd Schulze.

# Henon maps and discrete groups

Raluca Tănase

Consider the standard family of complex Hénon maps  $H(x, y) = (p(x) - ay, x)$ , where  $p(x)$  is a quadratic polynomial and  $a$  is a complex parameter. Let  $\mathbb{D}$  be the open unit disk in the complex plane and  $U^+$  the set of points that escape to infinity under forward iterations of the Hénon map. The analytic structure of the escaping set  $U^+$  is well understood from previous work of J. Hubbard and R. Oberste-Vorth as a quotient of  $(\mathbb{C} - \mathbb{D}) \times \mathbb{C}$  by a discrete group of automorphisms  $\Gamma$  isomorphic to  $\mathbb{Z}[1/2]/\mathbb{Z}$ . On the other hand, the boundary  $J^+$  of  $U^+$  is a complicated fractal object, on which the Hénon map behaves chaotically. We will show how to extend the group action to  $\mathbb{S}^1 \times \mathbb{C}$ , in order to represent the set  $J^+$  as a quotient of  $\mathbb{S}^1 \times \mathbb{C}/\Gamma$  by an equivalence relation. We will analyze this extension for Hénon maps that are small perturbations of hyperbolic polynomials with connected Julia sets.

## Energy critical Maxwell-Klein-Gordon and Yang-Mills flows

Daniel Tătaru\*, Joachim Krieger, Jacob Sterbenz

The Maxwell-Klein-Gordon and Yang-Mills flows are nonlinear versions of the linear wave equations, which arise from geometric interpretations of the scalar wave, respectively the Maxwell Lagrangian. The 4+1 dimensional case is of special interest, as the conserved energy is invariant with respect to the scaling of the problem. I will provide an overview of recent small data global well-posedness results for these problems.

## $L^p$ theory for outer measures and applications

Christoph Thiele\*, Yen Do

Outer measures are subadditive set functions: the outer measure of a disjoint union of sets is less than or equal to the sum of the outer measures of each set. If the sets are Caratheodory measurable, then equality holds. Measurable sets and measurable functions are cornerstones for classical Lebesgue theory and the theory of  $L^p$  spaces. In contrast, we consider outer measure spaces which do not give rise to non-trivial measurable sets. Nevertheless, one succeeds in defining reasonable "outer  $L^p$  spaces" on these outer measure spaces. This is a new point of view on some classical concepts, examples include Carleson measures, tent spaces, potential theory, capacity. In particular we describe how this theory describes results in time frequency analysis such as boundedness of the bilinear Hilbert transform and almost everywhere convergence of Fourier series.

## Bifurcation values and regularity at infinity of polynomial maps

Mihai Tibăr

Let  $f : \mathbb{C}^n \rightarrow \mathbb{C}^p$ , for  $n > p > 0$  be a non-constant polynomial mapping. It is well-known that  $f$  is a locally trivial topological fibration over the complement of the *bifurcation set*  $B(f)$ , also called *the set of atypical values*. The atypical values may come from the critical values but also from the asymptotic behaviour of the fibres. One can easily see this in the example  $f(x, y) = x + x^2y$ , where the value  $0 \in \mathbb{C}$  is not critical but there is no trivial fibration over any neighbourhood of  $0 \in \mathbb{C}$ . A complete characterization of  $B(f) \setminus \text{Sing}f$  is yet an open problem; actually this is available in the case  $n = 2$  and  $p = 1$  only. Several other questions subsist, mostly related to the Jacobian Conjecture. One has imagined various ways to approximate  $B(f)$ , essentially through the use of *regularity conditions* at infinity. We shall give an overview on recent results and discuss a number of regularity conditions at infinity, in their analytic and topological aspects.

## Generalized Dini theorems for nets of functions on arbitrary sets

Vlad Timofte\*, Aida Timofte

We characterize the uniform convergence of pointwise monotonic nets of real functions defined on an arbitrary set, with no topological structure. The conditions equivalent to the uniform convergence trivially hold in the particular setting of the classical Dini theorem. Our vector-valued generalization characterizes the uniform convergence of pointwise monotonic nets of functions with relatively compact range in a Hausdorff locally convex ordered space. For such nets of continuous functions on a compact space, we get the equivalence between the strong-uniform and weak-pointwise convergences; this also merges the abstract Dini-Weston theorem on monotonic nets from a Hausdorff locally convex ordered space. Our results remove any topological requirements on the domain of the functions and put compactness in the right place: the target space.

# The Strauss conjecture on black holes

Mihai Tohăneanu\*, H. Lindblad, J. Metcalfe, C. Sogge, C. Wang

The Strauss conjecture for the Minkowski spacetime in three dimensions states that the semilinear equation

$$\square u = |u|^p, \quad u(0) = \epsilon f, \quad \partial_t u(0) = \epsilon g$$

has a global solution for all  $f$  and  $g$  smooth, compactly supported and  $\epsilon$  small enough if  $p > 1 + \sqrt{2}$ . We prove a similar result in the context of Schwarzschild and Kerr with small angular momentum black holes.

## Elliptic surfaces and Zariski pairs for conic-line arrangements

Hiro-o Tokunaga

A pair  $(B_1, B_2)$  of reduced plane curves  $B_i$  ( $i = 1, 2$ ) of degree  $n$  in  $\mathbb{P}^2 = \mathbb{P}^2(\mathbb{C})$  (the base field of this article is always the field of complex numbers  $\mathbb{C}$ ) is called a Zariski pair of degree  $n$  if it satisfies the following conditions:

- (i)  $B_i$  ( $i = 1, 2$ ) are curves of degree  $n$  such that the combinatorial type of  $B_1$  is the same as that of  $B_2$ . There exist tubular neighborhoods  $T_i(B_i)$  of  $B_i$  ( $i = 1, 2$ ) such that  $(T_1(B_1), B_1)$  is homeomorphic to  $(T_2(B_2), B_2)$ .
- (ii)  $(\mathbb{P}^2, B_1)$  is not homeomorphic to  $(\mathbb{P}^2, B_2)$ .

In this talk, we consider Zariski pairs for  $(B_1, B_2)$  for "conic-line arrangements" and "conic arrangements." We give examples of Zariski pairs  $(B_1, B_2)$  for "conic-line arrangements" satisfying the following conditions:

- (i)  $\deg B_1 = \deg B_2 = 7$ .
- (ii) Irreducible components of  $B_i$  ( $i = 1, 2$ ) are lines and conics.
- (iii) Singularities of  $B_i$  ( $i = 1, 2$ ) are nodes, tacnodes and ordinary triple points.

We also consider Zariski  $N$ -plets for conic arrangements. In order to construct plane curves as above, we make use of the geometry of sections and bisections of certain rational elliptic surfaces. Also the elementary arithmetic on the Mordell-Weil group of such elliptic surfaces play key roles to distinguish the topology of pairs via the existence problem of dihedral covers with given reduced curves as the branch loci.

Part of this work is a joint work with Shinzo Bannai.

## Kernel density estimators for random fields satisfying an interlaced mixing condition

Cristina Tone

Density estimation and kernel density estimation for random processes has generated a considerable amount of interest and has been studied intensively by numerous authors such as Rosenblatt [5, 6] and Parzen [4]. In the case of dependent observations, kernel-type estimators of probability density have been studied by Roussas [7, 8], Zhurbenko [10], Tran [9], Masry and Györfi [1], Miller [3, 2] etc. The extensive interest in kernel-type estimators of probability density is partly due to the fact that many useful stochastic processes, among them various time series models, satisfy a strong mixing property. For a sequence of strictly stationary random fields that are uniformly  $\rho'$ -mixing and satisfy a Lindeberg condition, a central limit theorem is obtained for sequences of "rectangular" sums from the given random fields. The "Lindeberg CLT" is then used to prove a CLT for some kernel estimators of probability density for some strictly stationary random fields satisfying  $\rho'$ -mixing, and whose probability density and joint densities are absolutely continuous. The significance of our results on kernel estimators of probability density consists in having the feature that the normalizing constants are (asymptotically) the same as in the independent and identically distributed (i.i.d) case. Nevertheless, this fact shows that those procedures for estimating probability density are in a strong sense robust against a nontrivial departure (as the  $\rho'$ -mixing in our case) from the standard i.i.d. context.

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## Carleman estimates and stabilization of hyperbolic systems in absence of geometric observability conditions

**Daniel Toundykov\*, Matthias Eller**

The sharp sufficient conditions for observability in control systems are formulated via geometric optics and are linked to the structure of closed geodesics in the underlying physical domain. However, the necessary unique continuation property for PDEs is an intrinsically weaker requirement and does not impose such restrictions. It is therefore often possible to stabilize an evolution system by placing feedback controls on subsets of the domain that fail to satisfy the geometric conditions. The price to pay is the necessity to work with smoother solutions and the stabilization rates obtained thereby are no longer exponential. In this work we present specialized Carleman estimates and a generalization of a pioneering strategy due to G. Lebeau and L. Robbiano to prove uniform stability (for strong solutions) of 1st-order hyperbolic systems without reliance on the geometric observability assumptions.

## About the stochastic regularity of distorted Brownian motions

**Gerald Trutnau\*, Jiyong Shin**

Let  $\Omega \subset \mathbb{R}^d$  some domain which is either open or closed. We consider a classical energy form

$$\mathcal{E}(f, g) = \frac{1}{2} \int_{\Omega} \nabla f \cdot \nabla g \, d\mu, \quad f, g \in D,$$

where  $d\mu = \rho \, dx$ , with  $\rho$  locally integrable and  $\rho > 0$   $dx$ -a.e, and  $D$  is a set of test functions with compact support. Additional regularity assumptions on  $\rho$  ensure the *closability* of the form  $(\mathcal{E}, D)$  on  $L^2(\Omega, \mu)$ . These can be quite mild. For instance, a Hamza type condition on  $\rho$  is in general sufficient. The closability is important because it implies the existence of an associated process  $\mathbb{M} = ((X_t)_{t \geq 0}, (P_x)_{x \in \Omega})$ , which by general Dirichlet form theory can be decomposed as

$$X_t = x + B_t + \int_0^t \frac{\nabla \rho}{2\rho}(X_s) ds, \quad \mathbb{P}_x\text{-a.s for } x \in \Omega \setminus N, \tag{1}$$

where a possible additional local time on the boundary of  $\Omega$  may occur in (1) in case of Neumann boundary conditions and existing boundary. Here  $N$  is a set of zero capacity, which is not hit by  $\mathbb{M}$ . In this sense one can regard  $\Omega \setminus N$  as natural state space of the process.  $\mathbb{M}$  is known as the *distorted Brownian motion* since the nineteen seventies, and  $\frac{\nabla \rho}{2\rho} = \frac{1}{2} \nabla(\log \rho)$  is the so-called *logarithmic derivative*. Under the mild closability assumptions, the logarithmic derivative can be highly singular on the set  $\{\rho = 0\}$ . The main drawback of equation (1) is that  $N$  is in general abstract and not explicitly known. In this talk we shall consider the question under which conditions on  $\rho$  (1) is valid for as much as possible explicitly determined  $x \in \Omega$ .

## On countably skewed Brownian motion with accumulation point

**Gerald Trutnau\*, Youssef Ouknine, Francesco Russo**

Countably skewed Brownian motion (CSBM) is a special case of distorted Brownian motion in dimension one. Existence and pathwise uniqueness of CSBM was presented by LeGall in 1984 in an abstract frame for some special cases and then explicitly presented by Takanobu in 1986 assuming a uniform, strictly positive distance between the skew reflection points. In this case CSBM is a semimartingale and conservative, i.e. without explosion in finite time. This is not the case when the sequence of skew reflection points has an accumulation point. In this case we present necessary and/or sufficient conditions for existence, pathwise uniqueness, non-explosion and for CSBM to be a semimartingale.

# *F*-Splitting invariants of monoid algebras

Kevin Tucker

The *F*-splitting ratio and dimension are invariants first defined by I. Aberbach and F. Enescu that govern the asymptotic number of splittings of the iterates of Frobenius for a local ring. In this talk, I will discuss joint work with M. Herring giving a combinatorial formula for the *F*-splitting ratio and dimension of monoid algebras. Our characterization builds up the work M. Von Korff in the normal setting, and realizes the *F*-splitting ratio as the (appropriately scaled) lattice volume of particular polytope. In particular, this makes it rather easy to produce many examples of local rings of various *F*-splitting dimensions.

## A global division theorem for test ideals in positive characteristic

Kevin Tucker\*, Karl Schwede

In characteristic zero, multiplier ideals are useful singularity invariants with a number of remarkable properties. For example, Ein and Lazarsfeld have proven a global division theorem for multiplier ideals, and then used it to show effective nullstellensatz bounds on smooth projective varieties. In similar settings in positive characteristic, Hara and Yoshida have introduced test ideals in analogy and have shown that they correspond to multiplier ideals under reduction to prime characteristic. In this talk, I will describe joint work with Schwede showing that test ideals satisfy a modified version of the global division theorem. Along the way, I will also give a uniform description of test and multiplier ideals that further strengthens the link between them.

## Stein method, Malliavin calculus and applications

Ciprian Tudor

The Stein method allows to measure the distance between the laws of two random variables. Recently, this method combined with the Malliavin calculus, led to several interesting results. We will present the basic facts related to this theory and we will give some recent applications to limit theorems for the solution to the heat equation.

## Orbital free entropy

Yoshimichi Ueda

I'll survey orbital free entropy initially introduced by Hiai, Miyamoto and myself and quite recently 'strengthened' by Biane and Dabrowski and also by myself.

## Haagerup property, weak amenability, and semi-direct products

Alain Valette

Haagerup property and weak amenability (à la Cowling-Haagerup) are weak forms of amenability which, notoriously, are not stable under semi-direct products. E.g.,  $\mathbb{Z}^2$  and  $SL_2(\mathbb{Z})$  have both properties, but  $\mathbb{Z}^2 \rtimes SL_2(\mathbb{Z})$  has none. We consider a discrete group  $G$  acting on  $\mathbb{R}^n$  through a linear representation  $\rho$ , and study the semi-direct product  $\mathbb{R}^n \rtimes G$ . Assume that  $G$  has the Haagerup property (resp. is weakly amenable). We prove that  $\mathbb{R}^n \rtimes G$  has the Haagerup property (resp. is weakly amenable) if and only if the closure of  $\rho(G)$  in  $GL_n(\mathbb{R})$  (for the usual topology) is amenable. This is joint work with Y. Cornuier.

## Spectral bound for the $L^p$ -distortion of finite graphs

Alain Valette\*, Pierre-Nicolas Jolissaint

We give a lower bound for the  $L^p$ -distortion  $c_p(X)$  of finite graphs  $X$ , depending on the first eigenvalue  $\lambda_1^{(p)}(X)$  of the  $p$ -Laplacian and the maximal displacement of permutations of vertices. For a  $k$ -regular vertex-transitive graph it takes the form  $c_p(X)^p \geq \text{diam}(X)^p \lambda_1^{(p)}(X) / 2^{p-1} k$ . This bound is optimal for expander families and, for  $p = 2$ , it gives the exact value for cycles and hypercubes. As a new application we give a non-trivial lower bound for the  $L^2$ -distortion of a family of Cayley graphs of  $SL_n(q)$  ( $q$  fixed,  $n \geq 2$ ) with respect to a standard two-element generating set.

# Generalised logarithmic bundles

Jean Valles

We introduce a family of vector bundles  $\mathcal{D}_r(\mathcal{A})$  of rank  $r \geq 2$  associated to a line arrangement  $\mathcal{A}$ . When  $r = 2$ , these bundles are called *logarithmic bundles*, or *derivation bundles* and they are studied in many old and recent papers. In particular, a Torelli theorem was given and partially proved by Dolgachev in 1993 (I complete the proof in 2000): *Two logarithmic bundles  $\mathcal{D}(\mathcal{A})$  and  $\mathcal{D}(\mathcal{A}')$  are isomorphic if and only if  $\mathcal{A} = \mathcal{A}'$  or if  $\mathcal{A}$  and  $\mathcal{A}'$  are tangent to the same smooth conic.* When  $r \geq 2$ , we propose a similar statement. Let us call  $Z_{\mathcal{A}}$  and  $Z_{\mathcal{A}'}$  the set of points that correspond, by the classical projective duality, to  $\mathcal{A}$  and  $\mathcal{A}'$ . Then we prove : **Theorem** : *Two generalised logarithmic bundles  $\mathcal{D}_r(\mathcal{A})$  and  $\mathcal{D}_r(\mathcal{A}')$  are isomorphic if and only if  $\mathcal{A} = \mathcal{A}'$  or if  $Z_{\mathcal{A}}$  and  $Z_{\mathcal{A}'}$  belong to the same curve of degree  $r$ .*

## Singularities of axisymmetric free surface flows with gravity

Eugen Varvaruca\*, Georg Weiss

We present some recent results on singularities of steady axisymmetric solutions of the Euler equations for a fluid, incompressible and with zero vorticity, acted on only by gravity, and which has a free surface. We use geometric methods to analyze the asymptotics of the velocity field and of the free surface at stagnation points as well as at points on the axis of symmetry. At points on the axis of symmetry which are not stagnation points, constant velocity motion is the only blow-up profile consistent with the invariant scaling of the equation. This suggests the presence of downward pointing cusps at those points. At stagnation points on the axis of symmetry, the unique blow-up profile consistent with the invariant scaling of the equation is the "Garabedian pointed bubble" solution with water above air. Thus at stagnation points on the axis of symmetry with no water above the stagnation point, the invariant scaling of the equation cannot be the right scaling. A fine analysis of the blow-up velocity enables us to identify the correct scaling in that case under the additional assumption that the free surface is described by an injective curve. This last result relies on a frequency formula in combination with a concentration compactness result for the axially symmetric Euler equations by Delort; while the concentration compactness result alone does not lead to strong convergence in general, we prove the convergence to be strong in our application.

## Normal extensions of subnormal linear relations via quaternionic Cayley transforms

Florian-Horia Vasilescu\*, Adrian Sandovici

A quaternionic Cayley transform for linear relations is introduced and some of its properties are exhibited. The role played by the linear relations whose quaternionic Cayley transforms are unitary operators is emphasized.

## Solitonic hierarchies for almost Kaehler Ricci flows and off-diagonal solutions in gravity

Sergiu Văcaru

We outline some selected topics on encoding exact solutions of the Ricci soliton and (modified) Einstein equations into solitonic hierarchies derived for nonholonomic curve flows with associated bi-Hamilton structure. The main idea is to adapt the geometric constructions to nonholonomic distributions on (semi) Riemannian manifolds and work with alternative connections (to the Levi-Civita one) which are also metric compatible and completely determined by the metric structure and/or the corresponding canonical almost symplectic form. The Ricci flow equations and, in particular, various modifications of the Einstein field equations can be formulated equivalently in such nonholonomic variables. The priority of the so-called canonical distinguished connection is that it allows us to decouple fundamental geometric/ physical equations and construct very general classes of generic off-diagonal solutions determined by corresponding types of generating and integration functions depending on all (spacetime) coordinates. Almost Kähler variables are more convenient for quantizing nonlinear systems following well defined methods of geometric / deformation quantization, or the A-brane formalism. Our main goal is to show how the nonholonomic flow evolution and related geometric equations can be derived from modified Perelman's functionals in such forms that there are generated solutions with prescribed nonlinear wave solitonic symmetries. As main results, we prove that the geometric and physical data for various classes of Ricci soliton, (modified) Einstein spaces and other important physical equations can be encoded into multi-component versions of the sine-Gordon, or modified Korteweg - de Vries equations and additional nonholonomic constraints. Finally, we provide examples of exact solutions in the nonholonomic Ricci flow / soliton and (modified) gravity theories.

# Spectral functionals for noncommutative almost Kaehler Ricci flows and Lorentzian star products

**Sergiu Văcaru**

The work concerns our research on noncommutative Lorentzian generalizations of the nonholonomic Ricci flow theory and some applications. We present an introduction to the geometry of nonholonomic distributions and almost Kähler models of (pseudo) Riemannian spaces. Then we provide an attempt to generalize such constructions to noncommutative geometry encoding real non-compact hyperbolic configurations and physically important Lorentzian spacetimes (extending the A. Connes approach with spectral triples and Dirac operators for Euclidean signatures). There are reformulated and generalized in (pseudo) almost symplectic variables our former constructions on nonholonomic Dirac operators and Perelman's spectral functionals for (non) commutative nonholonomic Ricci flows. We involve certain geometric ideas and methods due to M. Matsumoto and J. Kern on deriving nonholonomically induced Riemann–Cartan and almost Kähler geometries. For such models of (non)commutative geometry, the symplectic forms and connections are completely determined by corresponding fundamental metric and Finsler, or Lagrange–Hamilton, generating functions and similar constructions for conventional nonholonomic splitting on Lorentz manifolds and corresponding tangent bundles. The priority of almost symplectic geometric objects and variables is that together with the canonical almost Kähler – Dirac operator we can define a covariant (associative) star product. This allows us to formulate canonical deformation procedures of (non) commutative geometries and generalized Ricci flows into theories with Fedosov type or A–brane quantization (by E. Witten and S. Gukov). Finally, there are outlined some recent results and applications related to (modified) classical and quantum gravity models and considered explicit examples of exact solutions for (non) commutative Ricci solitons and generic off–diagonal Einstein spaces.

## Local rings of embedding codepth 3

**Oana Veliche**

A complete local ring of embedding codepth 3 has a minimal free resolution of length 3 over a regular local ring. Such resolutions carry a differential graded algebra structure, based on which one can classify local rings of embedding codepth 3. We give examples of algebra structures that have been conjectured not to occur. The talk is based on a joint work with Lars W. Christensen. The talk is based on a joint work with Lars W. Christensen.

## Partial representations and partial actions of Hopf algebras

**Joost Vercruysse**

Partial Representations and Partial Actions of Hopf Algebras. Partial group actions were introduced by Exel as a tool to endow certain classes of  $C^*$  algebras with a structure of a more general crossed product. The pure algebraic study of partial actions - and the related concept of partial representations - lead to a new research area with a strong link to groupoids and applications in diverse directions such as a partial Galois theory. This partial Galois theory was in turn generalized to the setting of corings and lead to the introduction of partial (co)actions of Hopf algebras. In this joint work with Marcelo Muniz Alves, Eliezer Batista and Stefaan Caenepeel, we further investigate how various aspects from the theory of partial group actions can be brought to the setting of Hopf algebras.

## Moser iteration for SPDE and applications to stochastic fluids

**Vlad Vicol**

We address the regularity of invariant measures for the stochastic Navier-Stokes equations in the infinite Reynolds number limit. One of the main steps in the analysis is obtaining a stochastic version of the DeGiorgi-Nash-Moser parabolic regularization, with bounds that are independent of the drift. This is joint work with Nathan Glatt-Holtz and Vladimir Sverak.

## Holder continuity for a drift-diffusion equation with pressure

**Vlad Vicol**

We address the persistence of Hölder continuity for weak solutions of the linear drift-diffusion equation with nonlocal pressure

$$u_t + b \cdot \nabla u - \Delta u = \nabla p, \quad \nabla \cdot u = 0$$

on  $[0, \infty) \times \mathbb{R}^n$ , with  $n \geq 2$ . The drift velocity  $b$  is assumed to be at the critical regularity level, with respect to the natural scaling of the equations. The proof draws on Campanato's characterization of Hölder spaces, and uses a maximum-principle-type argument by which we

control the growth in time of certain local averages of  $u$ . We provide an estimate that does not depend on any local smallness condition on the vector field  $b$ , but only on scale invariant quantities. This is joint work with Luis Silvestre.

## The sound of random graphs

**Balint Virag\***, **Charles Bordenave**, **Arnab Sen**

Infinite random graphs, such as Galton-Watson trees and percolation clusters may have real numbers that are eigenvalues with probability one, providing a consistent "sound". These numbers correspond to atoms in their density-of-states measure. When does the sound exist? When are there only finitely many atoms? When is the measure purely atomic? I will review many examples and show some elementary techniques for studying these problems, including some developed in joint works with Charles Bordenave and Arnab Sen. The last question is open for percolation clusters in  $Z^d$ ,  $d \geq 3$ , and for incipient Galton-Watson trees.

## On the periodicity of the asynchronous flows

**Serban Vlad**

The signals are (discrete time and real time) binary functions that model the behavior of the digital electrical signals from electrical engineering. The (regular, autonomous, time invariant) asynchronous systems are models of the asynchronous circuits making use of signals and Boolean functions  $\Phi : \{0, 1\}^n \rightarrow \{0, 1\}^n$  and they act in this framework similarly with the dynamical systems; the difference is that the coordinate functions  $\Phi_i, i \in \{1, \dots, n\}$  are computed independently on each other. The purpose of the talk is that of presenting some properties of periodicity of the (discrete time and real time) asynchronous flows.

## Martin boundary for subordinate Brownian motion

**Zoran Vondracek\***, **P. Kim**, **R. Song**

In this talk I will give a survey of some recent results on the Martin boundary of both bounded and unbounded sets with respect to a rather wide class of subordinate Brownian motions. The main tool in studying the Martin boundary is the boundary Harnack principle for nonnegative harmonic functions. I will describe a uniform version of this principle for the finite part of the boundary and infinity. Building upon these results, one can identify the finite part of the Martin boundary of the so called  $\kappa$ -fat set with the Euclidean boundary, and the infinite part by a single point. Finally, I will also discuss minimal thinness for subordinate Brownian motion in half-space.

## Evolution delay equations with nonlocal initial conditions

**Ioan Vrabie**

We consider a class of abstract nonlinear delay evolution equation of the form

$$\begin{cases} u'(t) \in Au(t) + f(t, u_t), & t \in \mathbb{R}_+, \\ u(t) = g(u)(t), & t \in [-\tau, 0], \end{cases}$$

which encompass various models of both diffusion and transport phenomena.

Here  $A : D(A) \subseteq X \rightsquigarrow X$  is an  $m$ -dissipative operator in the Banach space  $X$ ,  $\tau \geq 0$ ,  $f : \mathbb{R}_+ \times C([- \tau, 0]; \overline{D(A)}) \rightarrow X$  is jointly continuous and Lipschitz with respect to the second argument and  $g : C_b([- \tau, +\infty); \overline{D(A)}) \rightarrow C([- \tau, 0]; \overline{D(A)})$  is nonexpansive. We prove that if the resolvent of  $A$ , i.e.  $(I - A)^{-1}$  is compact, the nonlinear semigroup generated by  $A$  decays exponentially and the Lipschitz constant of  $f$  is not too big, then the unique  $C^0$ -solution of the problem above is almost periodic.

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# Relation degrees in positive characteristic

**Adela Vraciu**

We give a formula for the smallest degree of a non-Koszul relation on  $x_1^{d_1}, \dots, x_n^{d_n}, (x_1 + \dots + x_n)^{d_{n+1}}$  in a polynomial ring  $k[x_1, \dots, x_n]$  when the characteristic of the field  $k$  is a prime number. As applications, we study the weak Lefschetz property of monomial complete intersections in positive characteristic, and the diagonal F-threshold of diagonal hypersurfaces.

## Enumerative geometry of hyperplane arrangements

**Max Wakefield\*, Will Traves**

The moduli space of hyperplane arrangements has been studied mostly for the reason of checking whether certain algebraic or geometric properties are combinatorial. In this presentation we study the moduli space of various hyperplane arrangements from an enumerative geometry perspective. We will present formulas for solutions to some enumerative problems for families of hyperplane arrangements containing a certain number of fixed points. We also compute characteristic numbers for some families of generic arrangements, one of whose dual problems contains the braid arrangement  $A_3$ .

## Cohomology jump loci of quasi-projective varieties

**Botong Wang\*, Nero Budur**

Classical Hodge theory gives nontrivial restrictions on possible homotopy types of smooth complex projective varieties. In this talk, we will study the underlying topological space of a complex variety  $X$  by the space of rank one local system  $M_B(X)$ , and some canonical strata called cohomology jump loci. The main result is when  $X$  is a smooth quasi-projective variety, all the cohomology jump loci are unions of torsion translates of subtori.

## Cohomology jump loci of local systems

**Botong Wang\*, Nero Budur**

Cohomology jump loci of smooth complex quasi-projective varieties have a rigid arithmetic structure. This is a joint result with Botong Wang. We will also present a conjectural interpretation of cohomology support loci via Bernstein-Sato ideals which would immediately provide implemented algorithms to compute these loci. This interpretation is a generalization of the classical result of Malgrange and Kashiwara relating Milnor monodromy of hypersurfaces with classical Bernstein-Sato polynomials. Complements of hyperplane arrangements provide interesting formulas.

## Recovering information from the character tables of Hopf algebras- generalizing results for finite groups

**Sara Westreich\*, Miriam Cohen**

We study properties of semisimple Hopf algebras  $H$  over algebraically closed fields of characteristic 0 resulting from their generalized character tables. We show that just as normal subgroups of a finite group can be recovered from its character table so does the generalized character table of  $H$  reflect the normal left coideal subalgebras of  $H$ ,  $N$ . These  $N$  are the Hopf analogues of normal subgroups in the sense that they arise from Hopf quotients. Such  $N$  appear as Hopf algebraic analogues of (semi) kernels and as images of the Drinfeld map of quasitriangular Hopf algebras. We apply these ideas to prove Hopf analogues of known results in group theory. Among the rest we prove that columns of the character table are orthogonal and that all entries are algebraic integers. We prove a Burnside-Brauer theorem for almost cocommutative  $H$ . We also prove the Hopf algebras analogue of the following (Burnside) theorem: If  $G$  is a nonabelian simple group then  $\{1\}$  is the only conjugacy class of  $G$  which has prime power order.

# Composition of Haar paraproducts

Brett Wick

Paraproducts are important operators in harmonic analysis and there are well known characterizations of when an individual paraproduct is bounded. An interesting question is to characterize when the composition of two, potentially unbounded, paraproducts have a bounded composition. In this talk we will give necessary and sufficient conditions that characterize when the composition of certain compositions of Haar paraproducts are bounded.

## Torsion in tensor products of modules over local domains

Roger Wiegand\*, Olgur Celikbas, Srikanth Iyengar, Greg Piepmeyer

Let  $S = \mathbb{C}[x_1, \dots, x_n]_{(x_1, \dots, x_n)}$ , the local ring of rational functions defined at the origin, and let  $M$  and  $N$  be non-zero finitely generated  $S$ -modules. In 1961 Auslander proved that if  $M \otimes_S N$  is torsion-free, then both  $M$  and  $N$  must be torsion-free. A crucial step in the proof was his famous rigidity theorem: If  $\text{Tor}_i^S(M, N) = 0$  for some  $i$ , then  $\text{Tor}_j^S(M, N) = 0$  for all  $j \geq i$ . He showed also (still assuming that  $M \otimes_S N$  is torsion-free) that  $\text{Tor}_j^S(M, N) = 0$  for all  $j \geq 1$  and deduced from this the beautiful formula

$$\text{pd}_S(M \otimes_S N) = \text{pd}_S(M) + \text{pd}_S(N) \quad (\dagger)$$

where “pd” denotes projective dimension, that is, the length of a minimal free resolution. In the past twenty years, many of these ideas have been revisited, but for more general rings. In this talk I will concentrate on *complete intersections*, exemplified by integral domains of the form  $R = S/(f_1, \dots, f_c)$ , where  $f_1, \dots, f_c$  is an  $S$ -regular sequence. When  $c \geq 1$ ,  $R$ -modules typically have infinite projective dimension, but one can still seek a substitute for  $(\dagger)$ , taking a cue from the Auslander-Buchsbaum formula,  $\text{depth}(M) + \text{pd}_R(M) = \text{depth}(R)$ , valid whenever  $M$  has finite projective dimension. Then  $(\dagger)$  becomes the “depth formula”:

$$\text{depth}(M) + \text{depth}(N) = \text{depth}(R) + \text{depth}(M \otimes_R N) \quad (\ddagger)$$

Indeed, this holds whenever  $\text{Tor}_j^R(M, N) = 0$  for  $j \geq 1$ . Huneke and Wiegand proved this about 20 years ago and used it, along with a theorem on rigidity of Tor, to prove that if  $c = 1$  (the case of a *hypersurface*) and if  $\text{depth}(M \otimes_R N) = \text{depth}(R)$ , then either  $M$  or  $N$  must be a free module. In this talk I will discuss the case  $c \geq 2$  and give examples to show that the situation is rather different from that of hypersurfaces. The general theme, however, is the same: Good depth properties on the tensor product  $M \otimes_R N$  imply the vanishing of Tor and, in turn, good depth properties on the modules  $M$  and  $N$ . The vanishing of Hailong Dao’s  $\eta$ -pairing will play a crucial role in some of the results.

## Prime ideals in quotients of mixed power series-polynomial rings

Sylvia Wiegand\*, Ela Celikbas, Christina Eubanks-Turner

Let  $R$  be a commutative one-dimensional Noetherian domain and let  $x$  and  $y$  be indeterminates over  $R$ . For  $A = R[[x]][y]$  or  $R[y][[x]]$  and  $Q$  a height-one prime ideal of  $A$ , we describe the set  $\text{Spec}(A/Q)$  of prime ideals of  $A/Q$  as a partially ordered set ordered by inclusion. In case  $R$  is a countable one-dimensional Noetherian domain, we characterize those partially ordered sets that arise as  $\text{Spec}(A/Q)$ . These partially ordered sets can be realized for  $R = \mathbb{Z}$ , the ring of integers,

## Constructive Hironaka desingularization in large characteristic

Jaroslav Wlodarczyk\*, E. Bierstone, D. Grigoriev, P. Milman

We show that there exists a canonical resolution of singularities for algebraic varieties over positive characteristic which is very large with respect to the degree of generating polynomials of the varieties.

## Affine invariant harmonic analysis

Jim Wright

We survey some recent progress on the connections between problems in harmonic analysis and affine measure.

# Hopf subalgebras of finite depth

**Christopher Young\***, **Lars Kadison**, **Alberto Hernandez**

A Hopf subalgebra of a finite-dimensional Hopf algebra has depth controlled by the depth of its generalized quotient  $V$ , a module coalgebra having depth  $n$  if its  $(n + 1)$ -st tensor power divides a multiple of its  $n$ -th tensor power. If  $V$  is a semisimple module over a pointed Hopf algebra, or  $V$  is projective, or either Hopf algebra has finite representation type, then  $V$  has finite depth. The problem of finite depth for an arbitrary Hopf subalgebra is also equivalent to whether either Hopf algebra has finite depth in its smash product with  $V^*$ . We will discuss what else we know and do not know in this area.

## Quasi-Frobenius-Lusztig kernels for simple Lie algebras

**Yinhuo Zhang\***, **Gongxiang Liu**, **Fred Van Oystaeyen**

We construct the quasi-Frobenius-Lusztig kernel associated with a simple Lie algebra.

## Optimal placement of sensors, actuators and dampers for waves

**Enrique Zuazua**

In this lecture we address the problem of the optimal placement of sensors, actuators and dampers for wave equations. We first discuss the dissipative wave equation where, due to the non-selfadjoint nature of the generator of the dynamics, characterizing the decay rate of solutions as time tends to infinity needs to take into account both spectral properties and the propagation of bicharacteristic rays. We present the state of the art in what concerns the optimal placement of dampers. We then turn our attention to the conservative wave equation and the optimal placement of sensors and actuators, both fundamental problems from a control theoretical point of view, with many potential applications. Using Fourier series representations the problem can be recast as an optimal design one involving all the spectrum of the laplacian. We develop a complete theory allowing to distinguish, depending on the complexity of the data to be observed/controlled, cases in which the solution is a classical set constituted by a finite number of subdomains, from others in which the optimal set is of Cantor type or those when relaxation occurs. These results will be illustrated by numerical simulations. Most of the work presented in this lecture is part of ongoing research in collaboration with Y. Privat (ENS Cachan, Antenne de Bretagne, France) and E. Trélat (Université Pierre et Marie Curie (Paris 6), Laboratoire Jacques-Louis Lions, Paris, France).

## On a Simplified Model for the ERTBP

Mihai Bărbosu\*, Tiberiu Oproiu

This paper examines the planar movement of a mass point using Rein's model for the Elliptic Restricted Three-Body Problem (ERTBP). In this simplified model, the ERTBP admits a first integral analogous to Jacobi's integral from the circular problem. Here we determine the zero velocity curves that separate the real motion from the imaginary motion, and we study the linear stability of Lagrange's equilibrium points.

## Sufficient conditions for instability of equilibrium for circulatory and gyroscopic systems

Petre Birtea\*, Dan Comănescu, Ioan Cașu

We give a method which generates sufficient conditions for instability of equilibria for circulatory and gyroscopic conservative systems. The method is based on the Gramians of a set of vectors whose coordinates are powers of the roots of the characteristic polynomial for the studied systems. New instability results are obtained for general circulatory and gyroscopic conservative systems. We also apply this method for studying the instability of motion for a charged particle in a stationary electromagnetic field.

## Geometries Induced by Logarithmic Oscillations as Examples of Gromov Hyperbolic Spaces

Wladimir-Georges Boskoff

We explore the connection between the geometries generated by logarithmic oscillations and the class of metric spaces satisfying the condition of Gromov hyperbolicity. We start our discussion with the most fundamental examples, inspired from classical geometries, e.g. the Euclidean distance on the infinite strip or Hilbert's distance on the unit disk, and we continue our study with Barbilian's distance, which historically appeared as a natural extension of a model of hyperbolic geometry. We introduce a new metric, called the stabilizing metric, and study its properties. Continuing this study, we explore a class of extensions of this distance and show that, under some analytic conditions, in nitely many new examples of Gromov hyperbolic metric spaces can be constructed. Using similar procedures, we construct Vuorinen's stabilizing metric  $j_G$  and its extensions and we discuss their Gromov hyperbolicity.

## On the center of locally convex $C(K)$ -modules

Omer Gok

Let  $X$  be a locally convex  $C(K)$ -module. In this talk , we deal with the center of the dual space of  $X$ .

## Plane compacta as attractors of IFS's and Borsuk's conjecture

Valeriu Guțu

J. Hutchinson (1981) has shown that any *hyperbolic Iterated Function System (IFS)*, consisting of a finite collection of contractions in a complete metric space, possesses a unique invariant compact set, called the *attractor* of this IFS. The structure of attractors and their properties were studied by many authors. M. Hata (1985), M. Barnsley (1988), P. F. Duvall and L. S. Husch (1992), M. Kwieciński (1999), S. Crovisier and M. Rams (2006), M. J. Sanders (2009), M. Kulczycki and M. Nowak (2012), T. Banakh and M. Nowak (2013) and others have studied the possibility of compacta to be obtained as attractors, as well as some examples of compacta, which can not serve as attractor of any hyperbolic IFS. We study this problem from another side: *Which compacta can serve as attractors of hyperbolic IFS's?* In this connection a natural question arises: *Given a compact, what is the minimal number of contractions of a hyperbolic IFS (provided it exists) needed to obtain this compact as attractor?* This question is related also to Borsuk's conjecture [1]. We show that any finite union of convex compacta in  $\mathbb{R}^n$  can be represented as the attractor of a hyperbolic IFS. F. W. Levi [2] has proved that every convex compact set in the plane can be covered by three translates of its interior, excepting parallelograms, when four translates are needed. We show that any plane convex compact may be realized as attractor of an IFS, consisting of at most three contractions. V. Boltyansky [3] has proved that every plane convex compact with diameter  $d$ , which may be embedded uniquely in a figure of constant width  $d$ , may be covered by not less than three figures of smaller diameter. As a consequence, we show that if  $A$  is a plane convex compact with diameter  $d$ , which may be embedded uniquely in a figure of constant width  $d$ , then the minimal number of contractions of an IFS, having  $A$  is attractor, is equal to three. This work was supported by the HCSTD ASM Grants 11.817.08.41F and 12.839.08.06F.

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## A study of equivalent norms on Lorentz sequence spaces

Anca Marcoci

The problem of duality of Lorentz spaces has been studied by many authors. It is well known that some of the functionals which define Lorentz spaces are not a norm and this is in a strong connection with the duality. In this paper we study some quasi norms and equivalent norms in the discrete case.

## Function spaces techniques in Banach spaces of infinite matrices

Liviu Marcoci

The starting point of studying weak variants of classical linear operators on  $l^p$  spaces was the connection with harmonic analysis. In this paper we present some results proved essentially with function spaces techniques.

## An extension of Gehring's lemma to Orlicz spaces on metric measure spaces

Marcelina Mocanu

We present a self-improvement property for inequalities of reverse Jensen type in doubling metric measure spaces, generalizing Gehring's lemma in this setting. Applications to higher integrability of Orlicz-Sobolev minimizers of some convex variational integrals are given.

## Semisimple group algebras: primitive idempotents and minimal non-abelian left group codes

Gabriela Olteanu

In [2, 3, 5, 6], Jaspers, Olteanu, del Río and Van Gelder described a complete set of orthogonal primitive idempotents in each Wedderburn component of a semisimple group algebra  $FG$  for various classes of finite groups  $G$  using strong Shoda pairs and the description of the Wedderburn decomposition of  $QG$  given by Olivieri, del Río and Simón in [4]. Using the computation of primitive idempotents in finite group algebras, in [5] we provide algorithms to construct minimal left group codes for a large class of groups and fields. To illustrate our methods, we give alternative constructions to some best linear codes over  $F_2$  and  $F_3$ . Our methods to construct minimal left group codes were implemented using a programming language provided by the computer algebra system GAP and we included them in the GAP package Wedderga [1].

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Octav Olteanu

One uses  $L^1$  polynomial approximation on unbounded intervals in existence of the solution of multidimensional Markov moment problem in terms of quadratic forms, similarly to the one-dimensional case. Secondly, we illustrate the relationship between Markov moment problem and Mazur-Orlicz theorem by means of applications to concrete spaces. To this end, extension of linear operators with two constraints is used. Finally, earlier of our results on this subject are generalized or improved.

## Isac's Cones at Various Topologies

Vasile Postolică

The regretted professor George Isac, Department of Mathematics and Computer Science, Royal Military College of Canada, P.O. 17000, Kingston, Ontario, Canada, K7K 7B4 discovered and introduced for the study of Pareto type efficiency the notion of “nuclear cone” in [1], published it in [2] and called later on “supernormal cone”, since it appears stronger than the usual concept of “normal cone” considered the best for the investigations of ordered topological vector spaces. For the first time, we named these convex cones in separated locally convex spaces as “Isac's Cones” in [3], after the previous, long ago, acceptance on professor Isac's part. This research work is devoted to the study of this concept in comparison with varied topologies on the same linear space, in order to continue the investigations given in [4].

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## On the tensor products of linear relations on Hilbert spaces

Adrian Sandovici

Certain characterizations of the Friedrichs and the Krein von Neumann extensions of the tensor product of two nonnegative linear relations  $A$  and  $B$  in terms of the Friedrichs and the Krein von Neumann extensions of  $A$  and  $B$  are provided. Also, a characterization of the extremal extensions of the tensor product of  $A$  and  $B$  is also given.