We provide some sufficient conditions for the Lie triple higher derivation of a generalized matrix algebra to be of the standard form. Some applications to full matrix algebras are also supplied.

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1. INTRODUCTION

Let $A$ and $B$ be two unital algebras over a fixed unital 2–torsion free commutative ring $R$. A Morita context consists of $A, B$, two bimodules $\mathcal{M}_B$ and $\mathcal{B}_N$, and two bimodule homomorphisms called the pairings $\Phi_{MN} : \mathcal{M} \otimes_B \mathcal{N} \rightarrow \mathcal{A}$ and $\Psi_{MN} : \mathcal{N} \otimes_A \mathcal{M} \rightarrow \mathcal{B}$ satisfying the following commutative diagrams:

\[
\begin{array}{ccc}
\mathcal{M} \otimes_B \mathcal{N} \otimes_A \mathcal{M} & \xrightarrow{\Phi_{MN} \otimes I_M} & \mathcal{A} \otimes_A \mathcal{M} \\
\downarrow I_M \otimes \Psi_{N,M} & & \downarrow \cong \\
\mathcal{M} \otimes_B \mathcal{B} & \xrightarrow{\cong} & \mathcal{M},
\end{array}
\]

and

\[
\begin{array}{ccc}
\mathcal{N} \otimes_A \mathcal{M} \otimes_B \mathcal{N} & \xrightarrow{\Psi_{N,M} \otimes I_N} & \mathcal{B} \otimes_B \mathcal{N} \\
\downarrow I_N \otimes \Phi_{M,N} & & \downarrow \cong \\
\mathcal{N} \otimes_A \mathcal{A} & \xrightarrow{\cong} & \mathcal{N}.
\end{array}
\]

If $(A, B, \mathcal{M}, \mathcal{N}, \Phi_{MN}, \Psi_{N,M})$ is a Morita context then the set

\[
\mathcal{G} = \left( \begin{array}{c}
\mathcal{A} \\
\mathcal{M} \\
\mathcal{N} \\
\mathcal{B}
\end{array} \right) = \left\{ \begin{pmatrix} a & m \\ n & b \end{pmatrix} \mid a \in \mathcal{A}, m \in \mathcal{M}, n \in \mathcal{N}, b \in \mathcal{B} \right\}
\]

is an algebra under the usual matrix operations. Such an $R$-algebra is called a generalized matrix algebra. We further assume that $\mathcal{M}$ is faithful as an $(A, B)$-bimodule. If $\mathcal{N} = 0$, then $\mathcal{G}$ becomes a triangular algebra and will be

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denoted by \( \text{Tri}(\mathcal{A}, \mathcal{M}, \mathcal{B}) \). The main examples of generalized matrix algebras are triangular algebras and full matrix algebras.

The center of \( \mathcal{G} \) is

\[ Z(\mathcal{G}) = \{ a \oplus b : am = mb, na = bn \quad \text{for all} \quad m \in \mathcal{M}, n \in \mathcal{N} \}, \]

where \( (a \oplus b) = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \in \mathcal{G} \).

The identity element of \( \mathcal{G} \) is \( I = \begin{pmatrix} 1_A & 0 \\ 0 & 1_B \end{pmatrix} \), where \( 1_A \) and \( 1_B \) are the identities of \( \mathcal{A} \) and \( \mathcal{B} \), respectively.

Our mean by \( P_1 \) and \( P_2 \) are the nontrivial idempotents \( P_1 = \begin{pmatrix} 1_A & 0 \\ 0 & 0 \end{pmatrix} \) and \( P_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1_B \end{pmatrix} \).

We will use the following notations:

\[ \mathcal{G}_{11} = P_1 \mathcal{G} P_1, \mathcal{G}_{12} = P_1 \mathcal{G} P_2, \mathcal{G}_{21} = P_2 \mathcal{G} P_1, \mathcal{G}_{22} = P_2 \mathcal{G} P_2. \]

Then the generalized matrix algebra \( \mathcal{G} \) can be written as

\[ \mathcal{G} = \mathcal{G}_{11} + \mathcal{G}_{12} + \mathcal{G}_{21} + \mathcal{G}_{22}, \]

where \( \mathcal{G}_{11} \) and \( \mathcal{G}_{22} \) are subalgebras of \( \mathcal{G} \) which are isomorphic to \( \mathcal{A} \) and \( \mathcal{B} \), respectively. \( \mathcal{G}_{12} \) is a \((\mathcal{G}_{11}, \mathcal{G}_{22})\)–bimodule which is isomorphic to the \((\mathcal{A}, \mathcal{B})\)–bimodule \( \mathcal{M} \). Similarly, \( \mathcal{G}_{21} \) is a \((\mathcal{G}_{22}, \mathcal{G}_{11})\)–bimodule which is isomorphic to the \((\mathcal{B}, \mathcal{A})\)–bimodule \( \mathcal{N} \). Also \( \pi_A(Z(\mathcal{G})) \) and \( \pi_B(Z(\mathcal{G})) \) are isomorphic to \( P_1 Z(\mathcal{G}) P_1 \) and \( P_2 Z(\mathcal{G}) P_2 \), respectively.

The Lie and Lie triple products on an algebra \( \mathcal{A} \) are defined by \([A, B] = AB - BA\) and \([A, B, C] = [[A, B], C]\), respectively, for all \( A, B, C \in \mathcal{A} \). A linear map \( d : \mathcal{A} \to \mathcal{A} \) is called a derivation if \( d(AB) = d(A)B + Ad(B) \) for all \( A, B \in \mathcal{A} \). A linear map \( L : \mathcal{A} \to \mathcal{A} \) is called a Lie derivation if \( L([A, B]) = [L(A), B] + [A, L(B)] \) for all \( A, B \in \mathcal{A} \). More generally, a linear map \( L : \mathcal{A} \to \mathcal{A} \) is called a Lie derivation if

\[ L([A, B, C]) = [L(A), B, C] + [A, L(B), C] + [A, B, L(C)], \]

for all \( A, B, C \in \mathcal{A} \).

Let us recall the concepts of higher derivations, Lie higher derivations and Lie triple higher derivations. Let \( L = (L_n)_{n \in \mathbb{N}} \) (here \( \mathbb{N} \) denotes the nonzero integers) be a sequence of linear maps on \( \mathcal{A} \) such that \( L_0 = \text{id}_\mathcal{A} \). Then \( L \) is said to be

1. a higher derivation if \( L_n(AB) = \sum_{i+j=n} L_i(A)L_j(B) \) for all \( A, B \in \mathcal{A}, n \in \mathbb{N} \);
2. a Lie higher derivation if \( L_n([A, B]) = \sum_{i+j=n} [L_i(A), L_j(B)] \) for all \( A, B \in \mathcal{A}, n \in \mathbb{N} \);
(3) a Lie triple higher derivation if

\[ L_n([A, B, C]) = \sum_{i+j+k=n} [L_i(A), L_j(B), L_k(C)], \]

for all \( A, B, C \in \mathcal{A}, n \in \mathbb{N}. \)

It is easy to check that a higher derivations is a Lie higher derivation and that a Lie higher derivation is a Lie triple higher derivation. However, the converse is not true, in general.

Many authors have studied Lie triple derivation on some algebras such as triangular algebras, von Neumann algebras, nest algebras and generalized matrix algebras (see, [1, 4, 5, 9, 10, 12, 13, 16] and the references therein). Lie triple derivations on triangular algebras were intensively studied by Xiao and Wei [16]. In recent years, some results on maps of triangular algebras have been extended to generalized matrix algebras (see [2, 7, 8, 17]). Xiao and Wei [17] gave some new characterizations on commuting mappings of generalized matrix algebras. Li and Wei [7] studied Lie derivations and semi-centralizing mappings of generalized matrix algebras. Du and Wang [2] gave a description of Lie derivations on generalized matrix algebras. This result was developed by Wang and Wang [14] by showing that under some conditions every multiplicative Lie \( n \)-derivations on a generalized matrix algebra is in standard form.

In [19] Wei and Xia studied the construction of higher derivations on a triangular algebra. In [15] they also showed that any Jordan higher derivation on a triangular algebra is a higher derivation. Fei and Chen in [18] studied the additive Lie higher derivations on nest algebras. Qi [11] proved that under some conditions every Lie higher derivation on a triangular algebra is proper. In [6] Li et al. studied Jordan derivations and antiderivations of a generalized matrix algebra.

The main aim of this paper is to show that under mild assumptions, every Lie triple higher derivation on a generalized matrix algebra is proper. That is, it can presented as a sum of a higher derivation and a sequence of center valued linear mappings vanishing on commutators. We prove the main result by an induction process. The paper is organized as follows. In Section 2 we prove our result for the case \( n = 1 \). Section 3 is devoted to the proof of our main result. As an application, we investigate the Lie triple higher derivations on a full matrix algebra.

2. LIE TRIPLE HIGHER DERIVATIONS ON A GENERALIZED MATRIX ALGEBRA

In this section, we present the following result which is the main aim of this paper. This generalizes (the case \( n = 3 \) of) the main result of [14] for a
Lie triple higher derivation.

**Theorem 2.1.** Let $\mathcal{G} = \left( \begin{array}{cc} A & M \\ N & B \end{array} \right)$ be a generalized matrix algebra with faithful $M$ and $L = \{L_n\}_{n \in \mathbb{N}}$ be a Lie triple derivation on $\mathcal{G}$. Suppose that $\mathcal{G}$ satisfies the following conditions:

(i) $Z(A) = \pi_A((Z(\mathcal{G})))$ and $Z(B) = \pi_B(Z(\mathcal{G}))$.

(ii) If $A \in \mathcal{A}$ and $[A, A] \subseteq Z(\mathcal{A})$ then $A \in Z(\mathcal{A})$. The same implication holds for $\mathcal{B}$.

(iii) either $\mathcal{A}$ or $\mathcal{B}$ does not contain nonzero central ideals.

(iv) If $\mathcal{N} \neq 0$, then for each $M \in \mathcal{M}$, the condition $MN = 0 = NM$ implies $M = 0$.

(v) For each $N \in \mathcal{N}$ the condition $MN = 0 = NM$ implies $N = 0$.

Then $L$ is of standard form. More precisely, there exist a sequence of higher derivations $d_n$ and a sequence of linear mappings $h_n : \mathcal{G} \to Z(\mathcal{G})$ vanishing on triple commutators of $\mathcal{G}$ such that $L_n(X) = d_n(X) + h_n(X)$ for all $X \in \mathcal{G}$, $n \in \mathbb{N}$.

The proof of Theorem 2.1 proceeds by induction on $n$. In this section we provide the proof, for the case $n = 1$, through several claims. Indeed, we show that for every lie triple derivation $L$ on $\mathcal{G}$ there exist a derivation $d$ on $\mathcal{G}$ and a linear mapping $h : \mathcal{G} \to Z(\mathcal{G})$ vanishing on triple commutators such that $L(X) = d(X) + h(X)$ for all $X \in \mathcal{G}$. This provides a different proof for (the case $n = 3$ of) the main result of [14]. We complete the proof in Section 3.

**Proof.** We present our proof through the following claims.

**Claim 1.** $L(I) \in Z(\mathcal{G})$.

**Proof.** Since $[[I, X], Y] = 0$ for all $X, Y \in \mathcal{G}$, we get $0 = L([[X, Y], I]) = [[L(I), X], Y]$ for all $X, Y \in \mathcal{G}$. Thus, $[L(I), X] \in Z(\mathcal{G})$, by (ii) $L(I) \in Z(\mathcal{G})$.

**Claim 2.** $P_1L(P_1)P_1 + P_2L(P_1)P_2 \in Z(\mathcal{G})$, $L(A_{ij}) \in \mathcal{G}_{ij}$ for all $1 \leq i \neq j \leq 2$.

**Proof.** Since $A_{21} = [[A_{21}, P_1], P_1]$ for any $A_{21} \in \mathcal{G}_{21}$, we get

$L(A_{21}) = L([[A_{21}, P_1], P_1])$

$= [[L(A_{21}), P_1], P_1] + [[A_{21}, L(P_1)], P_1] + [[A_{21}, P_1], L(P_1)]$

$= L(A_{21})P_1 - 2P_1L(A_{21})P_1 + A_{21}L(P_1)P_1 - L(P_1)A_{21} + P_1L(P_1)A_{21} + A_{21}L(P_1) - L(P_1)A_{21}$.

Multiplying $P_2$ and $P_1$ from the left and the right in the above equality, respectively, we obtain $2A_{21}L(P_1)P_1 - 2P_2L(P_1)A_{21} = 0$. Since $R$ is 2-torsion free we have

$A_{21}L(P_1)P_1 - P_2L(P_1)A_{21} = 0$. 

That is, $A_{21}P_1L(P_1)P_1 = P_2L(P_1)P_2A_{21}$, and this follows $P_1L(P_1)P_1 + P_2L(P_1)P_2 \in Z(\mathcal{G})$.

By multiplying $P_2$ from the left and right in the first equality, we obtain $P_2L(A_{21})P_2 = 0$. Similarly by multiplying $P_1$ from both sides in the first equality, we get $P_1L(A_{21})P_1 = 0$. So we have

$$L(A_{21}) = P_1L(A_{21})P_2 + P_2L(A_{21})P_1.$$ 

For any $A_{21}, B_{21}, C_{21} \in G_{21}$, $[A_{21}, B_{21}] = 0$. Applying the above equality we have

$$0 = L[[A_{21}, B_{21}], C_{21}]$$

$$= [L(A_{21}), B_{21}], C_{21}] + [[A_{21}, L(B_{21})], C_{21}] + [[A_{21}, B_{21}], C_{21}]$$

$$= [[P_1L(A_{21})P_2, B_{21}], C_{21}] + [[A_{21}, P_1L(B_{21})P_2], C_{21}].$$

This leads to

$$[P_1L(A_{21})P_2, B_{21}] + [A_{21}, P_1L(B_{21})P_2] \in Z(\mathcal{G}).$$

One can easily checked that $L(-P_2) \in Z(\mathcal{G})$, so we have

$$0 = L[[A_{21}, -P_2], B_{21}]$$

$$= [L(A_{21}), -P_2], B_{21}] + [[A_{21}, L(-P_2)], B_{21}] + [[A_{21}, -P_2], B_{21}]$$

$$= [[P_1L(A_{21})P_2, -P_2], B_{21}] + [[A_{21}, P_1L(-P_2)P_2], B_{21}].$$

This leads to

$$[-P_1L(A_{21})P_2, B_{21}] + [A_{21}, P_1L(B_{21})P_2] \in Z(\mathcal{G}).$$

Applying the last two equalities we obtain $2[P_1L(A_{21})P_2, B_{21}] \in Z(\mathcal{G})$ that implies

$$[P_1L(A_{21})P_2, B_{21}] \in Z(\mathcal{G}) \text{ for all } A_{21}, B_{21} \in G_{21}.$$ 

It follows that $P_1L(A_{21})P_2B_{21} - B_{21}P_1L(A_{21})P_2 \in Z(\mathcal{G})$.

Thus, $P_1L(A_{21})P_2\mathcal{N} \subseteq Z(\mathcal{A})$ and $\mathcal{N}P_1L(A_{21})P_2 \in Z(\mathcal{B})$.

Since $P_1L(A_{21})P_2\mathcal{N}$ is central ideal of $\mathcal{A}$, by Condition(iii), $P_1L(A_{21})P_2\mathcal{N} = 0$, so $\mathcal{N}P_1L(A_{21})P_2 = 0$. Applying (v) $P_1L(A_{21})P_2 = 0$. Thus, $L(A_{21}) = P_2L(A_{21})P_1$ and $L(A_{21}) \in \mathcal{G}_{21}$.

Using (iv), by the same argument one can prove $L(A_{12}) = P_1L(A_{12})P_2$ and $L(A_{12}) \in \mathcal{G}_{12}$. 

**Claim 3.** $P_2L(A)P_2 \in P_2Z(\mathcal{G})P_2$ and $P_1L(B)P_1 \in P_1Z(\mathcal{G})P_1$, for all $A \in \mathcal{G}_{11}$, $B \in \mathcal{G}_{22}$.

**Proof.** For $A \in \mathcal{G}_{11}$, $B \in \mathcal{G}_{22}$ and $X \in \mathcal{G}$, $[[A, B], X] = 0$. It follows that $[[L(A), B], X] + [[A, L(B)], X] = 0$. So $[L(A), B] + [A, L(B)] \in Z(\mathcal{G})$. 

Consequently, \( [P_2L(A)P_2, B] \in Z(\mathcal{B}) \) for all \( B \in \mathcal{G}_{22} \) and \( [A, P_1L(B)P_1] \in Z(\mathcal{A}) \) for all \( A \in \mathcal{G}_{11} \). By (i) and (ii), \( P_2L(A)P_2 \in P_2Z(\mathcal{G})P_2 \) and \( P_1L(B)P_1 \in P_1Z(\mathcal{G})P_1 \).

**Claim 4.**

1. For every \( A \in \mathcal{G}_{11} \), \( P_1L(A)P_2 = AL(P_1)P_2 \) and \( P_2L(A)P_1 = P_2L(P_1)A \).
2. For every \( B \in \mathcal{G}_{22} \), \( P_1L(B)P_2 = P_1L(P_2)B \) and \( P_2L(B)P_1 = BL(P_2)P_1 \).

**Proof.** Let \( A \in \mathcal{G}_{11} \), then

\[
0 = L([A, P_2], P_1] = [L(A), P_2], P_1] + [A, L(P_2), P_1]
\]

\[
= -P_2L(A)P_1 - P_1L(A)P_2 + AL(P_2)P_1 - L(P_2)A - AL(P_2) + P_1L(P_2)A.
\]

Multiplying \( P_1 \) and \( P_2 \) from the left and the right in the above equality, respectively, we arrive to \( P_1L(A)P_2 + AL(P_2)P_2 = 0 \) which implies that \( P_1L(A)P_2 = AL(P_1)P_2 \).

Multiplying \( P_2 \) and \( P_1 \) from the left and the right in that equality, respectively, we get \( P_2L(A)P_1 + P_2L(P_2)A = 0 \) which leads to \( P_2L(A)P_1 = P_2L(P_1)A \).

Claim 5. \( L(\mathcal{G}_{ii}) \subseteq \mathcal{G}_{ii} + \mathcal{G}_{12} + \mathcal{G}_{21} \) for \( i = 1, 2 \).

**Proof.** Let \( \varphi \) be the algebraic isomorphism from \( \pi_A(Z(\mathcal{G})) \) to \( \pi_B(Z(\mathcal{G})) \), whose existence is guaranteed by the faithfulness of \( \mathcal{M} \), such that \( AM = M\varphi(A) \) for all \( A \in \pi_A(Z(\mathcal{G})) \) and for all \( M \in \mathcal{M} \). Applying Claim 3, for every \( A \in \mathcal{G}_{11} \) and \( B \in \mathcal{G}_{22} \) we have

\[
L(A) = (P_1L(A)P_1 - \varphi^{-1}(P_2L(A)P_2) + P_1L(A)P_2 + P_2L(A)P_1
+(\varphi^{-1}(P_2L(A)P_2 + P_2L(A)P_2).
\]

and

\[
L(B) = (P_1L(B)P_1 + \varphi(P_2L(B)P_2) + P_1L(B)P_2 + P_2L(B)P_1
+(P_2L(B)P_2 - \varphi^{-1}(P_2L(B)P_2).
\]

Thus, we conclude that \( L(\mathcal{G}_{ii}) \subseteq \mathcal{G}_{ii} + \mathcal{G}_{12} + \mathcal{G}_{21} \) for \( i = 1, 2 \).

For any \( A_{11} \in \mathcal{G}_{11} \) and \( B_{22} \in \mathcal{G}_{22} \) we define \( h_1(A_{11}) = P_2L(A_{11})P_2 \) and \( h_2(B_{22}) = P_1L(B_{22})P_1 \). By Condition (i) and Claim 2, \( h_1 : \mathcal{A} \rightarrow Z(\mathcal{B}) \) and \( h_2 : \mathcal{B} \rightarrow Z(\mathcal{A}) \) are linear mappings such that \( h_1([[X_{11}, Y_{11}], Z_{11}]) = 0 \) for all \( X_{11}, Y_{11}, Z_{11} \in \mathcal{G}_{11} \) and \( h_2([[X_{22}, Y_{22}], Z_{22}]) = 0 \) for all \( X_{22}, Y_{22}, Z_{22} \in \mathcal{G}_{22} \). By Claim 3, for any \( A_{11} \in \mathcal{G}_{11} \) we have

\[
L(A_{11}) = P_1L(A_{11})P_1 + P_1L(A_{11})P_2 + P_2L(A_{11})P_1 + P_2L(A_{11})P_2
= A_{11}L(P_1)P_2 + L(A_{11})P_1 + P_2L(P_1)A_{11} + h_1(A_{11})P_2.
\]

But \( [[A_{11}, S_{12}], P_2] = A_{11}A_{12} \) for all \( S_{12} \in \mathcal{G}_{12} \). So
\[ L(A_{11}S_{12}) = P_1[L(A_{11}), S_{12}]P_2 + [A_{11}, L(S_{12})]P_2 + [A_{11}, S_{12}], L(P_2)P_2 \]
\[ = P_1L(A_{11})S_{12} - S_{12}L(A_{11})P_2 + A_{11}L(S_{12})P_2 \]
\[ = L(A_{11})S_{12} + A_{11}L(S_{12}) - h_1(A_{11})S_{12}. \]

For any \( B_{11} \in \mathcal{G}_{11} \) by the above equation we have
\[ L(A_{11}B_{11}S_{12}) = L(A_{11}B_{11})S_{12} + A_{11}B_{11}L(S_{12})P_2 - h_1(A_{11})B_{11}S_{12}. \]

On the other hand,
\[ L(A_{11}B_{11}S_{12}) = L(A_{11})B_{11}S_{12} + A_{11}L(B_{11})S_{12} - h_1(A_{11})B_{11}S_{12} \]
\[ = L(A_{11})B_{11}S_{12} + A_{11}(L(B_{11})S_{12} + B_{11}L(S_{12})) \]
\[ - h_1(A_{11})B_{11}S_{12} - h_1(B_{11})A_{11}S_{12}. \]

Comparing the above two equations, we see that
\[ (L(A_{11}B_{11}) - L(A_{11})B_{11} - A_{11}L(B_{11}) + h_1(A_{11})B_{11} + h_1(B_{11})A_{11})S_{12} = 0, \]
for all \( S_{12} \in \mathcal{G}_{12} \). Thus,
\[ (L(A_{11}B_{11}) - L(A_{11})B_{11} - A_{11}L(B_{11}) + h_1(A_{11})B_{11} + h_1(B_{11})A_{11})P_1 \mathcal{G}P_2 = 0. \]

Since \( M \) is faithful, it implies that
\[ (L(A_{11}B_{11}) - L(A_{11})B_{11} - A_{11}L(B_{11}) + h_1(A_{11})B_{11} + h_1(B_{11})A_{11})P_1 = 0. \]

By using Claim 4, for any \( A_{22} \in \mathcal{G}_{22} \) we have
\[ L(A_{22}) = P_1L(A_{22})P_1 + P_1L(A_{22})P_1 + P_2L(A_{22})P_1 + P_2L(A_{22})P_2 \]
\[ = L(A_{11})B_{11}S_{12} + A_{11}(L(B_{11})S_{12} + B_{11}L(S_{12})) \]
\[ = P_1h_2(A_{22}) + P_1L(P_2)A_{22} + A_{22}L(P_2)P_1 + P_2L(A_{22}). \]

But \([S_{12}, A_{22}], P_2] = S_{12}A_{22} \) for all \( S_{12} \in \mathcal{G}_{12} \).
Again by Claim 4 we obtain
\[ L(S_{12}A_{22}) = L(S_{12})A_{22} + S_{12}L(A_{22}) - S_{12}h_2(A_{22}). \]

For any \( B_{22} \in \mathcal{G}_{22} \), by the above equation, we have
\[ L(S_{12}A_{22}B_{22}) = L(S_{12})A_{22}B_{22} + S_{12}L(A_{22}B_{22})P_2 - S_{12}h_2(A_{22})B_{22} \]
\[ + S_{12}A_{22}L(B_{22}) - S_{12}h_2(B_{22})A_{22} - S_{12}A_{22}h_2(B_{22}). \]

Comparing the above two equations, we see that
\[ S_{12}(L(A_{22}B_{22}) - L(A_{22})B_{22} + A_{22}L(B_{22}) - h_2(A_{22}B_{22}) - h_2(A_{22})B_{22} - h_2(B_{22})A_{22}) = 0, \]

for all \( S_{12} \in \mathcal{G}_{12} \). Thus,

\[ P_1G P_2(L(A_{22}B_{22}) - L(A_{22})B_{22} + A_{22}L(B_{22}) - h_2(A_{22}B_{22}) - h_2(A_{22})B_{22} - h_2(B_{22})A_{22}) = 0. \]

Since \( \mathcal{M} \) is faithful, this implies that

\[ P_2(L(A_{22}B_{22}) - L(A_{22})B_{22} + A_{22}L(B_{22}) - h_2(A_{22}B_{22}) - h_2(A_{22})B_{22} - h_2(B_{22})A_{22}) = 0. \]

Now for any \( A_{11} \in \mathcal{G}_{11} \) and \( A_{22} \in \mathcal{G}_{22} \) we define \( h(X) = h_1(A_{11}) + h_2(A_{22}) \) and \( d(X) = L(X) - h(X) \), where \( X = A_{11} + A_{12} + A_{21} + A_{22} \in \mathcal{G} \).

Applying Claims 1, 2 and 5 we arrive to the next claim.

**Claim 6.** \( d(I) = 0, \) \( d(\mathcal{G}_{ij}) \subseteq \mathcal{G}_{ij} \), \( d(P_1) \in \mathcal{G}_{12} \), \( d(P_2) \in \mathcal{G}_{21} \) and \( d(\mathcal{G}_{ii}) \subseteq \mathcal{G}_{ii} \oplus \mathcal{G}_{12} + \mathcal{G}_{21} \) for \( 1 \leq i \neq j \leq 2 \).

**Claim 7.** \( d \) is a derivation.

We divide the proof into the following three steps:

**Step 1.** \( d(A_{ii}B_{ii}) = d(A_{ii})B_{ii} + A_{ii}d(B_{ii}) \) for all \( A_{ii}, B_{ii} \in \mathcal{G}_{ii}, i = 1, 2 \).

*Proof.* We check it for \( i = 1 \).

\[
\begin{align*}
d(A_{11}B_{11}) & = L(A_{11}B_{11}) - h(A_{11}B_{11}) \\
& = (L(A_{11})B_{11} + A_{11}L(B_{11}) - h_1(A_{11})B_{11} - h_1(B_{11})A_{11})P_1 \\
& = (L(A_{11}) - h_1(A_{11}))B_{11} + A_{11}(L(B_{11}) - h_1(B_{11}))P_1 \\
& = d(A_{11})B_{11} + A_{11}d(B_{11}).
\end{align*}
\]

**Step 2.** For every \( A_{ij}, B_{ij} \in \mathcal{G}_{ij} \),

1. \( d(A_{ii}B_{ij}) = d(A_{ii})B_{ij} + A_{ii}d(B_{ij}). \)
2. \( d(B_{ij}A_{jj}) = d(B_{ij})A_{jj} + B_{ij}d(A_{jj}). \)

*Proof.* Note that \( A_{11}B_{12} = [A_{11}, B_{12}], P_2 \) holds true for all \( A_{11} \in \mathcal{G}_{11} \) and \( B_{12} \in \mathcal{G}_{12} \).

Therefore,

\[
\begin{align*}
d(A_{11}B_{12}) & = L([A_{11}, B_{12}], P_2) \\
& = [[L(A_{11}), B_{12}], P_2] + [[A_{11}, L(B_{12})], P_2] + [[A_{11}, B_{12}], L(P_2)] \\
& = [[d(A_{11}), B_{12}], P_2] + [[A_{11}, d(B_{12})], P_2] + [[A_{11}, B_{12}], d(P_2)] \\
& = d(A_{11})B_{12} + A_{11}d(B_{12}).
\end{align*}
\]

Also \( B_{12}A_{22} = [B_{12}, A_{22}], P_2 \) holds for all \( B_{12} \in \mathcal{G}_{12} \) and \( A_{22} \in \mathcal{G}_{22} \).
We then have
\[
d(B_{12}A_{22}) = L([[B_{12}, A_{22}], P_2])
\]
\[
= [[L(B_{12}), A_{22}], P_2] + [[B_{12}, L(A_{22})], P_2] + [[B_{12}, A_{22}], L(P_2)]
\]
\[
= [[d(B_{12}), A_{22}], P_2] + [[B_{12}, d(A_{22})], P_2] + [[B_{12}, A_{22}], d(P_2)]
\]
\[
= d(B_{12})A_{22} + B_{12}d(A_{22}).
\]

Similarly as
\[
A_{22}B_{21} = [[A_{22}, B_{21}], P_1]
\]
holds for all \( A_{22} \in G_{22} \) and \( B_{21} \in G_{21} \), we get
\[
d(A_{22}B_{21}) = L([[A_{22}, B_{21}], P_1])
\]
\[
= [[L(A_{22}), B_{21}], P_1] + [[A_{22}, L(B_{21})], P_1] + [[B_{21}, A_{22}], L(P_1)]
\]
\[
= [[d(A_{22}), B_{21}], P_1] + [[A_{22}, d(B_{21})], P_1] + [[B_{21}, A_{22}], d(P_1)]
\]
\[
= d(A_{22})B_{21} + A_{22}d(B_{21}).
\]

As \( B_{21}A_{11} = [[B_{21}, A_{11}], P_1] \) holds for all \( B_{21} \in G_{21} \) and \( A_{11} \in G_{11} \),
\[
d(B_{21}A_{11}) = L([[B_{21}, A_{11}], P_1])
\]
\[
= [[L(B_{21}), A_{11}], P_1] + [[B_{21}, L(A_{11})], P_1] + [[B_{21}, A_{11}], L(P_1)]
\]
\[
= [[d(B_{21}), A_{11}], P_1] + [[B_{21}, d(A_{11})], P_1] + [[B_{21}, A_{11}], d(P_1)]
\]
\[
= d(B_{21})A_{11} + B_{21}d(A_{11}).
\]

**Step 3.** For every \( A_{ij}, B_{ij} \in G_{ij} \),

(1) \( d(A_{ij}B_{ji}) = d(A_{ij})B_{ji} + A_{ij}d(B_{ji}) \).

(2) \( d(B_{ji}A_{ij}) = d(B_{ji})A_{ij} + B_{ji}d(A_{ij}) \).

**Proof.** Note that \( [A_{12}, B_{21}] = [[A_{12}, P_2], B_{21}] \) holds for all \( A_{12} \in G_{12} \) and \( B_{21} \in G_{21} \). It follows that
\[
h_1([A_{12}, B_{21}]) = L([[A_{12}, P_2], B_{21}]) - d([[A_{12}, P_2], B_{21}])
\]
\[
= [[L(A_{12}), P_2], B_{21}] + [[A_{12}, L(P_2)], B_{21}] + [[A_{12}, P_2], L(B_{21})]
\]
\[
+ d(A_{12}B_{21} - B_{21}A_{12})
\]
\[
= [[d(A_{12}), P_2], B_{21}] + [[A_{12}, d(P_2)], B_{21}] + [[A_{12}, P_2], d(B_{21})]
\]
\[
+ d(A_{12}B_{21}) - d(B_{21}A_{12})
\]
\[
= d(A_{12})B_{21} - P_2d(A_{12})B_{21} - B_{21}d(A_{12})P_2 + B_{21}d(A_{12})
\]
\[
+ A_{12}d(B_{21}) - d(B_{21})A_{12} - d(A_{12}B_{21}) + d(B_{21}A_{12}),
\]
which in turn implies that
\[
\mathcal{K} = (d(A_{12})B_{21} + A_{12}d(B_{21}) - d(A_{12}B_{21})) + (d(B_{21}A_{12}) - d(B_{21})A_{12} - B_{21}d(A_{12}))
\]
lies in \( P_2Z(G)P_2 \).
By the condition (i), we have
\[ K = (d(A_{12}B_{21} + A_{12}d(B_{21}) - d(A_{12}B_{21})) + (d(B_{21}A_{12}) - d(B_{21})A_{12}
\quad - B_{21}d(A_{12})) = Z(B). \]

Using Claim 6 and multiplying \( B_{22} \) from the left in the above equality, we obtain
\[ B_{22}d(B_{21}A_{12}) - B_{22}d(B_{21})A_{12} - B_{22}B_{21}d(A_{12}) = B_{22}K. \]

Thus, we conclude that \( K \) is central ideal of \( B \). So by the condition (iii),
\[ K = d(A_{12}B_{21} + A_{12}d(B_{21}) - d(A_{12}B_{21})) + (d(B_{21}A_{12}) - d(B_{21})A_{12}
\quad - B_{21}d(A_{12}) = 0. \]

Therefore,
\[ d(A_{12})B_{21} + A_{12}d(B_{21}) - d(A_{12}B_{21}) = -(d(B_{21}A_{12}) - d(B_{21})A_{12}
\quad - B_{21}d(A_{12})) \in G_{11} \cap G_{22}. \]

And this implies that
\[ d(A_{12})B_{21} + A_{12}d(B_{21}) - d(A_{12}B_{21}) = 0 \]
\[ \text{and } d(B_{21}A_{12}) - d(B_{21})A_{12} - B_{21}d(A_{12}) = 0. \]

Let us choose arbitrary elements \( S, T \in G \). Suppose that \( S = S_{11} + S_{12} + S_{21} + S_{22} \) and \( T = T_{11} + T_{12} + T_{21} + T_{22} \), for any \( S_{ij}, T_{ij} \in G_{ij} \) with \( 1 \leq i \leq j \leq 2 \).

By steps 1, 2, 3 we obtain
\[
\begin{align*}
d(ST) &= d(S_{11}T_{11} + S_{11}T_{12} + S_{12}T_{21} + S_{12}T_{22} \\
&\quad + S_{21}T_{11} + S_{21}T_{12} + S_{22}T_{21} + S_{22}T_{22}) \\
&= d(S_{11})T_{11} + S_{11}d(T_{11}) + d(S_{11})T_{12} + S_{11}d(T_{12}) \\
&\quad + d(S_{12})T_{21} + S_{12}d(T_{21}) + d(S_{12})T_{22} + S_{12}d(T_{22}) \\
&\quad + d(S_{21})T_{11} + S_{21}d(T_{11}) + d(S_{21})T_{12} + S_{21}d(T_{12}) \\
&\quad + d(S_{22})T_{21} + S_{22}d(T_{21}) + d(S_{22})T_{22} + S_{22}d(T_{22}) \\
&= d(S)T + Sd(T).
\end{align*}
\]

Therefore \( d \) is a derivation, as claimed.

**Claim 8.** \( h \) vanishes at the triple commutator \([X, Y, Z]\) for all \( X, Y, Z \in G \).

**Proof.** Since \( h(X) \in Z(G) \), we have
\[ h([[X, Y], Z]) = L([[X, Y], Z]) - d([[X, Y], Z]) \]
\[ = [[L(X), Y], Z] + [[X, L(Y)], Z] + [[X, Y], L(Z)] - d([[X, Y], Z]) \]
\[ = [[d(X), Y], Z] + [[X, d(Y)], Z] + [[X, Y], L(Z)] - d([[X, Y], Z]) = 0. \]

The proof for \( n = 1 \) is now complete.  \( \square \)
3. PROOF OF THEOREM 2.1

In the previous section we have proved that for the Lie triple derivation $L_1$ on $\mathcal{G}$ there exist a derivation $d_1$ and a linear mapping $h_1 : \mathcal{G} \to Z(\mathcal{A})$ such that $h_1([X,Y,Z]) = 0$ and $L_1(X) = d_1(X) + h_1(X)$ for all $X, Y, Z \in \mathcal{G}$. Moreover, for every $A \in \mathcal{G}_{11}, B \in \mathcal{G}_{22}$, $d_1$ and $h_1$ have the following properties:

\[
P_1 L_1(P_1)P_1 + P_2 L_1(P_1)P_2 \in Z(\mathcal{G});
\]
\[
P_2 L_1(A)P_2 \in P_2 Z(\mathcal{G})P_2, \quad P_1 L_1(B)P_1 \in P_1 Z(\mathcal{G})P_1;
\]
\[
P_1 L_1(A)P_2 = AL_1(P_1)P_2, \quad P_2 L_1(A)P_1 = P_2 L_1(P_1)A;
\]
\[
P_1 L_1(B)P_2 = P_1 L_1(P_2)B, \quad L_1(P_1) \in \mathcal{G}_{12}; L_1(P_2) \in \mathcal{G}_{21},
\]
\[
P_2 L_1(B)P_1 = BL_1(P_2)P_1;
\]
\[
L_1(\mathcal{G}_{12}) \subseteq \mathcal{G}_{12}, L_1(\mathcal{G}_{21}) \subseteq \mathcal{G}_{21}; L_1(\mathcal{G}_{ii}) \subseteq \mathcal{G}_{ii} + \mathcal{G}_{12} + \mathcal{G}_{21}, \quad (i = 1, 2).
\]

Now suppose that the conclusion holds for all $m < n \in \mathbb{N}$. That is, there exist linear maps $d_m : \mathcal{G} \to \mathcal{G}$ and $h_m : \mathcal{G} \to Z(\mathcal{G})$ such that $L_m(X) = d_m(X) + h_m(X)$, $h_m([X,Y,Z]) = 0$ and $d_m(XY) = \sum_{i+j=m} d_i(X)d_j(Y)$ for all $X, Y, Z \in \mathcal{G}$. Moreover, $L_m$ and $d_m$ have the following properties:

\[
P_m : \left\{
\begin{array}{l}
P_1 L_m(P_1)P_1 + P_2 L_m(P_1)P_2 \in Z(\mathcal{G});

P_2 L_m(A)P_2 \in P_2 Z(\mathcal{G})P_2, \quad P_1 L_m(B)P_1 \in P_1 Z(\mathcal{G})P_1;

P_1 L_m(A)P_2 = AL_1(P_1)P_2, P_2 L_m(A)P_1 = P_2 L_1(P_1)A;

P_1 L_m(B)P_2 = P_1 L_m(P_2)B, L_n(P_1) \in \mathcal{G}_{12}, L_n(P_2) \in \mathcal{G}_{21},

P_2 L_m(B)P_1 = BL_1(P_2)P_1;

L_m(\mathcal{G}_{12}) \subseteq \mathcal{G}_{12}, L_m(\mathcal{G}_{21}) \subseteq \mathcal{G}_{21}, L_m(\mathcal{G}_{ii}) \subseteq \mathcal{G}_{ii} + \mathcal{G}_{12} + \mathcal{G}_{21}, \quad (i = 1, 2).
\end{array}\right.
\]

We are going to show that $L_n$ also satisfies the similar properties. We prove this through the following claims.

**Claim 1.** $P_1 L_n(P_1)P_1 + P_2 L_n(P_1)P_2 \in Z(\mathcal{G})$.

**Proof.** Since $A_{21} = [[[A_{21}, P_1], P_1]]$ for any $A_{21} \in \mathcal{G}_{21}$, we get

\[
L_n(A_{21}) = L([[A_{21}, P_1], P_1]) = \sum_{i+j+k=n} [L_i(A_{21}), L_j(P_1), L_k(P_1)]
\]

\[
= [[L_n(A_{21}), P_1], P_1] + [[n, L_{A_{21}}(P_1)], P_1] + [[A_{21}, P_1], L_n(P_1)]
\]

\[
= L_n(A_{21})P_1 - P_1 L(A_{21}) + A_{21} L_n(P_1)P_1 - L_n(P_1)A_{21} + P_1 L_n(P_1)A_{21}
\]

\[
+ A_{21} L_n(P_1) - L_n(P_1)A_{21}.
\]

Multiplying $P_2$ and $P_1$ from the left and the right in the above equality respectively, we obtain

\[
2A_{21} L_n(P_1)P_1 - 2P_2 L_n(P_1)A_{21} = 0.
\]
Since $R$ is 2–torsion free we have

$$A_{21}L_n(P_1)P_1 - P_2L_n(P_1)A_{21} = 0.$$  

That is,

$$P_1L_n(P_1)P_1 + P_2L(P_1)P_2 \in Z(G).$$

**Claim 2.** $P_2L_n(G_{ii})P_2 \subseteq P_2Z(G)P_2$ and $P_1L_n(G_{22})P_1 \subseteq P_1Z(G)P_1,$  
$L_n(G_{ij}) \subseteq G_{ij}$ for all $1 \leq i \neq j \leq 2.$

**Proof.** For every $A \in G_{11},$ $B \in G_{22},$ and $X \in G,$ $[[A, B], x] = 0.$ We thus have

$$0 = \sum_{i+j+k=n} [L_i(n), L_j(P_1), L_k(P_1)]$$

$$= [[L_n(A), B], X] + \sum_{i+j+k=n, i \neq 0, n} [L_i(n), L_j(P_1), L_k(P_1)] + [[A, L_n(B)], X].$$

Consequently,

$$[P_2L_n(A)P_2, B] \in Z(G_{22})$$  
and

$$[A, P_1L_n(B)P_1] \in Z(G_{11})$$  
for all $A \in G_{11}, B \in G_{22},$ and it follows that $P_2L_n(A)P_2 \in P_2Z(G)P_2$ and $P_1L_n(B)P_1 \in P_1Z(G)P_1.$

**Claim 3.**

1. $P_1L_n(A)P_2 = AL_n(P_1)P_2$ and $P_2L_n(A)P_1 = P_2L_n(P_1)A \quad (A \in G_{11}).$

2. $P_1L_n(B)P_2 = P_1L_n(P_2)B$ and $P_2L_n(B)P_1 = BL_n(P_2)P_1 \quad (B \in G_{22}).$

**Proof.** Let $A \in G_{11},$ then

$$0 = L_n([[A, P_2], P_1]) = [[L_n(A), P_2], P_1] + [[A, L_n(P_2), P_1] - P_2L_n(A)P_1$$
$$- P_1L_n(A)P_2 + AL_n(P_2)P_1 - L_n(P_2)A - AL(P_2) + P_1L_n(P_2)A.$$  

Multiplying $P_1$ and $P_2$ from the left and the right in the above equality, respectively, we obtain $P_1L_n(A)P_2 + AL_n(P_2)P_2 = 0,$ which implies that $P_1L_n(A)P_2 = AL_n(P_1)P_2.$

Similarly we can prove that $P_2L_n(A)P_1 = P_2L_n(P_1)A.$ Part (2) needs a similar argument.

The next claim needs a similar argument as for Claim 5 in Section 2.

**Claim 4.** $L_n(G_{ii}) \subseteq G_{ii} + G_{12} + G_{21}$ for $i = 1, 2.$

For every $A \in G_{11}$ and $B \in G_{22}$ we define $h_{n_1}(A) = P_2L_n(A)P_2$ and $h_{n_2}(B) = P_1L_n(B)P_1.$ By Claim 2, $h_{n_1} : G_{11} \rightarrow Z(G_{22})$ and $h_{n_2} : G_{22} \rightarrow Z(G_{11})$ are linear mappings such that $h_{n_1}([[X_{11}, Y_{11}], Z_{11}]) = 0$ for all $X_{11}, Y_{11}, Z_{11} \in G_{11}$ and $h_{n_2}([[X_{22}, Y_{22}], Z_{22}]) = 0$ for all $X_{22}, Y_{22}, Z_{22} \in G_{22}.$
One can easily checked that \( L(A_{11}B_{11}) = L(A_{11})B_{11} + A_{11}L(B_{11}) - h_1(A_{11}B_{11}) - h_1(B_{11})A_{11} \), and \( L(A_{22}B_{22}) = L(A_{22})B_{22} + A_{22}L(B_{22}) - h_1(A_{22}B_{22}) - h_1(B_{22})A_{22} \).

Now for \( X = A_{11} + A_{12} + A_{21} + A_{22} \in \mathcal{G} \), we define \( h_n(X) = h_1(A_{11}) + h_2(A_{22}) \) and \( d_n(X) = L_n(X) - h_n(X) \).

Applying Claims 1, 2 and 4 we have the following claim.

**Claim 5.** \( d_n(I) = 0, \) \( d_n(\mathcal{G}_{12}) \subseteq \mathcal{G}_{12}, d_n(\mathcal{G}_{21}) \subseteq \mathcal{G}_{21}, d_n(\mathcal{P}_1) \in \mathcal{G}_{12}, \) \( d_n(\mathcal{P}_2) \in \mathcal{G}_{21} \) and \( d_n(\mathcal{G}_{ii}) \subseteq \mathcal{G}_{ii} + \mathcal{G}_{12} + \mathcal{G}_{21}, \) \( (i = 1, 2) \).

**Claim 6.** \( d_n(XY) = \sum_{t+l=n} d_t(X)d_l(Y) \) for all \( X, Y \in \mathcal{G} \).

_Proof._ It is enough to show that for every \( A_{ii}, B_{ii} \in \mathcal{G}_{ii}, A_{ij}, B_{ij} \in \mathcal{G}_{ij}, \)

1. \( d_n(A_{ii}B_{ij}) = \sum_{t+l=n} d_t(A_{ii})d_l(B_{ij}). \)
2. \( d_n(B_{ij}A_{jj}) = \sum_{t+l=n} d_t(B_{ij})d_l(A_{jj}). \)
3. \( d_n(A_{ii}B_{ii}) = \sum_{t+l=n} d_t(A_{ii})d_l(B_{ii}). \)

To these end, as \( A_{11}B_{12} = [[A_{11}, B_{12}], P_2] \) holds true for all \( A_{11} \in \mathcal{G}_{11} \) and \( B_{12} \in \mathcal{G}_{12} \), we get

\[
d_n(A_{11}B_{12}) = L_n([[A_{11}, B_{12}], P_2]) = \sum_{t+l+k=n} [[L_t(A_{11}), L_l(B_{12})], L_k(P_2)]
\]

\[
= \sum_{t+l+k=n} [d_t(A_{11}), d_l(B_{12})], d_k(P_2) = \sum_{t+l=n} d_t(A_{11})d_l(B_{12});
\]

indeed, \( d_i(A_{11}) \in \mathcal{G}_{11} + \mathcal{G}_{12} + \mathcal{G}_{21}, d_i(B_{12}) \in \mathcal{G}_{12} \) and \( d_i(P_2) \in \mathcal{G}_{21}, \) \( 1 \leq i \leq n. \)

We have also \( B_{12}A_{22} = [[B_{12}, A_{22}], P_2] \) for all \( B_{12} \in \mathcal{G}_{12} \) and \( A_{22} \in \mathcal{G}_{22}, \) and this follows that

\[
d_n(B_{12}A_{22}) = L_n([[B_{12}, A_{22}], P_2]) = \sum_{t+l+k=n} [[L_t(B_{12}), L_l(A_{22})], L_k(P_2)]
\]

\[
= \sum_{t+l+k=n} [d_t(B_{12}), d_l(A_{22})], d_k(P_2) = \sum_{t+l=n} d_t(B_{12})d_l(A_{22});
\]

indeed, \( d_i(A_{22}) \in \mathcal{G}_{22} + \mathcal{G}_{12} + \mathcal{G}_{21} \) and \( d_i(B_{12}) \in \mathcal{G}_{12} \) and \( d_i(P_2) \in \mathcal{G}_{21}, \) \( 1 \leq i \leq n. \)

Similar arguments can apply to complete the proofs of (1) and (2). Now we prove part (3) for \( i = 1. \) Indeed,

\[
d_n(A_{11}B_{11}) = L_n(A_{11}B_{11}) - h_n(A_{11}B_{11}) = (L_n(A_{11})B_{11} + A_{11}L_n(B_{11}) - h_n(A_{11})B_{11} - h_n(B_{11})A_{11})P_1 - (L_n(A_{11}) - h_1(A_{11}))B_{11} + A_{11}(L_n(B_{11}) - h_n(B_{11}))P_1 = d_n(A_{11})B_{11} + A_{11}d_n(B_{11}).
\]

**Claim 7.** \( h_n \) vanishes at commutators \( [X, Y, Z] \) for all \( X, Y, Z \in Z(\mathcal{G}). \)
Proof. Let $X, Y, Z \in G$ then

$$h_n([X,Y,Z]) = L_n([X,Y,Z]) - d_n([X,Y,Z])$$

$$= \sum_{i+j+k=n} [[L_i(X), L_j(Y)], L_k(Z)] - d_n(XYZ) + d_n(YXZ)$$

$$+ d_n(ZXY) - d_n(ZYX)$$

$$= \sum_{i+j+k=n} [[d_i(X) + h_i(X), d_j(Y) + h_j(Y)], d_k(Z) + h_k(Z)]$$

$$- \sum_{i+j+k=n} d_i(X)d_j(Y)d_k(Z) + \sum_{i+j+k=n} d_i(Y)d_j(X)d_k(Z)$$

$$+ \sum_{i+j+k=n} d_i(Z)d_j(X)d_k(Y) + \sum_{i+j+k=n} d_i(Z)d_j(Y)d_k(X)$$

$$= 0.$$

And this completes the proof of Theorem 2.1. □

The main example of generalized matrix algebras are full matrix algebras and triangular algebras. Let $A$ be a unital algebra and let $M_n(A)$ be the algebra of all $n \times n$ matrices with $n \geq 2$. Then $M_n(A)$ is called a full matrix algebra. A direct verification reveals that $M_n(A)$ can be presented as a generalized matrix algebra of the form

$$M_n(A) = \begin{pmatrix} A & M_{1 \times (n-1)}(A) \\ M_{(n-1) \times 1}(A) & M_{(n-1) \times (n-1)}(A) \end{pmatrix}.$$ 

It can be readily verified that $Z(M_{(n-1) \times (n-1)}(A)) = Z(A)I_{(n-1)}$. It has also known that $M(A)$ does not contain nonzero central ideal (see [2, Lemma 1]). That $M_n(A)$ satisfies the condition (ii) of Theorem 2.1 has been proved in [16]. The other requirements of Theorem 2.1 are also hold for $M_n(A)$. We thus arrive to the next consequence of Theorem 2.1.

**Corollary 3.1.** Every Lie triple higher derivation on the full matrix algebra $M_n(A)$ is in standard form.

4. CONCLUSION

We have shown that under mild assumptions every Lie triple higher derivation on a generalized matrix algebra is proper. To the best of our knowledge, however, the problem of finding those conditions such that a multiplicative Lie $n$-higher derivation on a generalized matrix algebra, is of standard form is a somewhat open question.
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