A REMARK ON THE EXISTENCE OF POSITIVE SOLUTIONS FOR A CLASS OF INFINITE SEMIPOSITONE PROBLEMS *VIA* SUB AND SUPERSOLUTIONS METHOD

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Using the method of sub-super solutions, we study the existence of positive solutions for a class of infinite semipositone problems involving nonlocal operator.

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1. INTRODUCTION

In this paper, we are interested in the existence of positive solutions for the p-Kirchhoff-type problem

(1.1)
$$\begin{cases} -M\left(\int_\Omega |\nabla u|^p\,dx\right)\Delta_p u = au^{p-1} - bu^\gamma - f(u) - \frac{c}{u^\alpha},\ x\in\Omega, & x \in \Omega, \\ u = 0, & x \in \partial\Omega, \end{cases}$$

where Δ_p denotes the p-Laplacian operator defined by $\Delta_p z = \mathrm{div}(|\nabla z|^{p-2}\nabla z)$, $p > 1$, $\gamma > p-1$, $\alpha \in (0,1)$, a, b and c are positive constants, Ω is a bounded domain in $\mathbb{R}^N$ with a smooth boundary $\partial\Omega$, and $f : [0,\infty) \to \mathbb{R}$ is a continuous function. This model describes the steady states of a logistic growth model with grazing and constant yield harvesting. It also describes the dynamics of the fish population with natural predation and constant yield harvesting.

We make the following assumptions:

(H_1) $f : [0,\infty) \to \mathbb{R}$ is a continuous function such that $\lim_{s\to\infty} f(s) = \infty$.

(H_2) There exist $A > 0$ and $\beta > p-1$ such that $f(s) \le As^\beta$, for all $s \ge 0$.

(H_3) $M : [0,\infty) \to \mathbb{R}$ is a continuous and increasing function such that $0 < M_0 \le M(t) \le M_\infty$ for all t.

In [16], the authors have studied the equation $-\Delta u = g(u) - (c/u^\alpha)$ with Dirichlet boundary conditions, where g is nonnegative and nondecreasing and $\lim_{u\to\infty} g(u) = \infty$. The case $g(u) := au - f(u)$ has been studied in [14], where $f(u) \ge au - N$ and $f(u) \le Au^\beta$ on $[0,\infty)$ for some N, $A > 0$, $\beta > 1$ and this g

may have a falling zero. Here u is the population density and $au^{p-1} - bu^{\gamma} - f(u)$ represents logistics growth. This model describes grazing of a fixed number of grazers on a logistically growing species (see [15, 20]). The herbivore density is assumed to be a constant which is a valid assumption for managed grazing systems and the rate of grazing is given by $\frac{c}{u^{\alpha}}$. At high levels of vegetation density this term saturates to c as the grazing population is a constant. This model has also been applied to describe the dynamics of fish populations (see [15, 23]). The diffusive logistic equation with constant yield harvesting, in the absence of grazing was studied in [22]. More recently, reaction-diffusion models have been used to describe spatiotemporal phenomena in disciplines other than ecology, such as physics, chemistry (see [24, 25]). In this paper, we study the equation

$$-M\left(\int_{\Omega} |\nabla u|^{p}\,\mathrm{d}x\right)\Delta_{p} u = au^{p-1} - bu^{\gamma} - f(u) - (c/u^{\alpha}) \tag{1.2}$$

with Dirichlet boundary conditions. Let $F(u) := au^{p-1} - bu^{\gamma} - f(u) - (c/u^{\alpha})$, then $\lim_{u\to 0^{+}} F(u) = -\infty$ and hence we refer to (1.1) as an infinite semipositone problem. In recent years, problems involving Kirchhoff type operators have been studied in many papers, we refer to [1–3, 6, 8, 17–19, 21] in which the authors have used variational method and topological method to get the existence of solutions for (1.1). In this paper, motivated by the ideas introduced in [9] and the properties of Kirchhoff type operators in [11–13], we study problem (1.1) in the semipositone case ($F(0) < 0$ but finite) (see [4,5,7]). The main tool used in this study is the method of sub- and super solutions. Our result in this note improves the previous one [9] in which $M(t) \equiv 1$. To our best knowledge, this is a new research topic for nonlocal problems, see [1,2,13].

2. MAIN RESULT

A function ψ is said to be a subsolution of problem (1.1) if it is in $W^{1,p}(\Omega)$ such that $\psi = 0$ on $\partial\Omega$ and satisfies

$$\begin{aligned} M\left(\int_{\Omega} |\nabla \psi|^{p}\,\mathrm{d}x\right)\int_{\Omega} |\nabla \psi|^{p-2}\nabla\psi\cdot\nabla w\,\mathrm{d}x \\ \le \int_{\Omega}\left(a\psi^{p-1} - b\psi^{\gamma} - f(\psi) - \frac{c}{\psi^{\alpha}}\right) w\,\mathrm{d}x, \quad \forall w \in W, \end{aligned} \tag{2.1}$$

where $W := \{w \in C_0^{\infty}(\Omega) :\ w \ge 0 \text{ in } \Omega\}$. A function $z \in W^{1,p}(\Omega)$ is said to be a supersolution if $z = 0$ on $\partial\Omega$ and satisfies

$$M\left(\int_{\Omega} |\nabla z|^{p}\,\mathrm{d}x\right)\int_{\Omega} |\nabla z|^{p-2}\nabla z\cdot\nabla w\,\mathrm{d}x \tag{2.2}$$

$$\geq \int_\Omega \Big(az^{p-1} - bz^\gamma - f(z) - \frac{c}{z^\alpha}\Big) w \, \mathrm{d}x, \quad \forall w \in W.$$

Our main result is given by the following theorem.

THEOREM 2.1. *Let (H1), (H2) and (H3) hold. If* $a > (\frac{p}{p-1+\alpha})^{p-1}\lambda_1$, *then there exists a positive constant* $c^* := c^*(a, A, p, \alpha, \beta, \gamma, \Omega)$ *such that for* $c \leq c^*$, *problem (1.1) has a positive solution, where* λ_1 *is the first eigenvalue of the p-Laplacian operator with Dirichlet boundary conditions.*

Proof. We shall establish our result by constructing positive sub-super-solutions to equation (1.1). From an anti-maximum principle (see [10, pages 155–156]), there exists a $\sigma(\Omega) > 0$ such that the solution z_λ of

$$\begin{cases} -\Delta_p z - \lambda z^{p-1} = -1, & x \in \Omega, \\ z = 0 & x \in \partial\Omega \end{cases}$$

for $\lambda \in (\lambda_1, \lambda_1 + \sigma)$ is positive in Ω and is such that $\frac{\partial z}{\partial v} < 0$ on $\partial\Omega$, where v is outward normal vector on $\partial\Omega$. Fix $\lambda^* \in (\lambda_1, \min\{\lambda_1 + \sigma, (\frac{p-1+\alpha}{p})^{p-1} a\})$ and let

$$K := \min\left\{ \left(\frac{M_\infty (p/p-1+\alpha)^{p-1}}{2b\|z_{\lambda^*}\|_\infty^{\frac{\gamma p-(p-1)(\alpha-1)}{p-1+\alpha}}} \right)^{\frac{1}{\gamma-p+1}}, \left(\frac{a - (\frac{p}{p-1+\alpha})^{p-1}\lambda^*}{3M_\infty b \|z_{\lambda^*}\|_\infty^{\frac{p(\gamma-p+1)}{p-1+\alpha}}} \right)^{\frac{1}{\gamma-p+1}}, \right.$$
$$\left. \left(\frac{M_\infty (p/p-1+\alpha)^{p-1}}{2A\|z_{\lambda^*}\|_\infty^{\frac{\beta p-(p-1)(\alpha-1)}{p-1+\alpha}}} \right)^{\frac{1}{\beta-p+1}}, \left(\frac{a - (\frac{p}{p-1+\alpha})^{p-1}\lambda^*}{3M_\infty A \|z_{\lambda^*}\|_\infty^{\frac{p(\beta-p+1)}{p-1+\alpha}}} \right)^{\frac{1}{\beta-p+1}} \right\}.$$

Define $\psi = K z_{\lambda^*}^{\frac{p}{p-1+\alpha}}$. Then

$$\nabla\psi = K(\frac{p}{p-1+\alpha}) z_{\lambda^*}^{\frac{1-\alpha}{p-1+\alpha}} \nabla z_{\lambda^*}$$

and

$$\begin{aligned} & M\left(\int_\Omega |\nabla\psi|^p \, \mathrm{d}x\right) \int_\Omega |\nabla\psi|^{p-2} \nabla\psi \cdot \nabla w \, \mathrm{d}x \\ = \ & M\left(\int_\Omega |\nabla\psi|^p \, \mathrm{d}x\right) K^{p-1} (\frac{p}{p-1+\alpha})^{p-1} \\ & \int_\Omega \left[\left\{ (\frac{(p-1)(1-\alpha)}{p-1+\alpha}) z_{\lambda^*}^{\frac{-\alpha p}{p-1+\alpha}} |\nabla z_{\lambda^*}|^p + z_{\lambda^*}^{\frac{(p-1)(1-\alpha)}{p-1+\alpha}} \Delta_p z_{\lambda^*} \right\}\right] w \, \mathrm{d}x \\ = \ & M\left(\int_\Omega |\nabla\psi|^p \, \mathrm{d}x\right) K^{p-1} (\frac{p}{p-1+\alpha})^{p-1} \\ & \int_\Omega \left[\left\{ (\frac{(p-1)(1-\alpha)}{p-1+\alpha}) z_{\lambda^*}^{\frac{-\alpha p}{p-1+\alpha}} |\nabla z_{\lambda^*}|^p + z_{\lambda^*}^{\frac{(p-1)(1-\alpha)}{p-1+\alpha}} (1 - \lambda^* z_{\lambda^*}^{p-1}) \right\}\right] w \, \mathrm{d}x \end{aligned}$$

$$\leq M_\infty K^{p-1}(\frac{p}{p-1+\alpha})^{p-1}$$

$$\int_\Omega \left[\left\{\lambda^* z_{\lambda^*}^{\frac{p(p-1)}{p-1+\alpha}} - z_{\lambda^*}^{\frac{(p-1)(1-\alpha)}{p-1+\alpha}} - \left((\frac{(p-1)(1-\alpha)}{p-1+\alpha})\right) \frac{|\nabla z_{\lambda^*}|^p}{z_{\lambda^*}^{\frac{\alpha p}{p-1+\alpha}}}\right\}\right] w\, \mathrm{d}x.$$

Let $\delta > 0$, $\mu > 0$ and $m > 0$ be such that $|\nabla z_{\lambda^*}|^p \geq m$ in $\overline{\Omega}_\delta$ and $z_{\lambda^*} \geq \mu$ in $\Omega \setminus \overline{\Omega}_\delta$, where $\overline{\Omega}_\delta := \{x \in \Omega : d(x, \partial\Omega) \leq \delta\}$. Let

$$c^* := K^{p-1+\alpha} \min \left\{ M_\infty(\frac{p}{p-1+\alpha})^{p-1} \left(\frac{(p-1)(1-\alpha)}{p-1+\alpha}\right) m^p, \right.$$
$$\left. \frac{1}{3}\mu^p \left(a - M_\infty(\frac{p}{p-1+\alpha})^{p-1}\lambda^*\right)\right\}.$$

Let $x \in \overline{\Omega}_\delta$ and $c \leq c^*$. Since $(\frac{p}{p-1+\alpha})^{p-1}\lambda^* < a$, we have

$$M_\infty K^{p-1}(\frac{p}{p-1+\alpha})^{p-1}\lambda^* z_{\lambda^*}^{\frac{p(p-1)}{p-1+\alpha}} < a\left(K z_{\lambda^*}^{\frac{p}{p-1+\alpha}}\right)^{p-1}. \tag{2.3}$$

From the choice of K, we have

$$\frac{M_\infty}{2}(\frac{p}{p-1+\alpha})^{p-1} \geq bK^{\gamma-p+1}\|z_{\lambda^*}\|_\infty^{\frac{\gamma p(p-1)(\alpha-1)}{p-1+\alpha}}, \tag{2.4}$$

$$\frac{M_\infty}{2}(\frac{p}{p-1+\alpha})^{p-1} \geq AK^{\beta-p+1}\|z_{\lambda^*}\|_\infty^{\frac{\beta p(p-1)(\alpha-1)}{p-1+\alpha}}. \tag{2.5}$$

By (2.4), (2.5) and (H2), we know that

$$-\frac{M_\infty}{2}K^{p-1}(\frac{p}{p-1+\alpha})^{p-1} z_{\lambda^*}^{\frac{(p-1)(1-\alpha)}{p-1+\alpha}} \leq -b\left(K z_{\lambda^*}^{\frac{(p}{p-1+\alpha}}\right)^\gamma, \tag{2.6}$$

$$-\frac{M_\infty}{2}K^{p-1}(\frac{p}{p-1+\alpha})^{p-1} z_{\lambda^*}^{\frac{(p-1)(1-\alpha)}{p-1+\alpha}} \leq -A\left(K z_{\lambda^*}^{\frac{(p}{p-1+\alpha}}\right)^\beta \leq -f\left(K z_{\lambda^*}^{\frac{(p}{p-1+\alpha}}\right). \tag{2.7}$$

Since $|\nabla z_{\lambda^*}|^p \geq m$ in $\overline{\Omega}_\delta$, from the choice of c^* we have

$$\begin{aligned} & -M_\infty K^{p-1}(\tfrac{p}{p-1+\alpha})^{p-1}\left(\tfrac{(p-1)(1-\alpha)}{p-1+\alpha}\right)\frac{|\nabla z_{\lambda^*}|^p}{z_{\lambda^*}^{\frac{\alpha p}{p-1+\alpha}}} \\ & \leq M_\infty K^{p-1}(\tfrac{p}{p-1+\alpha})^{p-1}\left(\tfrac{(p-1)(1-\alpha)}{p-1+\alpha}\right)\frac{m^p}{z_{\lambda^*}^{\frac{\alpha p}{p-1+\alpha}}} \\ & \leq -\frac{c}{\left(K z_{\lambda^*}^{\frac{p}{p-1+\alpha}}\right)^\alpha}. \end{aligned} \tag{2.8}$$

Hence for c^*, combining (2.3), (2.6), (2.7), (2.8) and (H3) we have

$$
\begin{aligned}
&M\left(\int_\Omega |\nabla\psi|^p \,\mathrm{d}x\right)\int_\Omega |\nabla\psi|^{p-2}\nabla\psi\cdot\nabla w \,\mathrm{d}x\\
&\qquad= M\left(\int_\Omega |\nabla\psi|^p \,\mathrm{d}x\right) K^{p-1}(\frac{p}{p-1+\alpha})^{p-1}\\
&\quad\times\int_\Omega\left(\lambda^* z_{\lambda^*}^{\frac{p(p-1)}{p-1+\alpha}} - z_{\lambda^*}^{\frac{(p-1)(1-\alpha)}{p-1+\alpha}} - \left(\frac{(p-1)(1-\alpha)}{p-1+\alpha}\right)\frac{|\nabla z_{\lambda^*}|^p}{z_{\lambda^*}^{\frac{\alpha p}{p-1+\alpha}}}\right) w\,\mathrm{d}x,\\
&\le M_\infty K^{p-1}(\frac{p}{p-1+\alpha})^{p-1}\int_\Omega\Bigg\{\lambda^* z_{\lambda^*}^{\frac{p(p-1)}{p-1+\alpha}} - \frac{1}{2}K^{p-1}(\frac{p}{p-1+\alpha})^{p-1} z_{\lambda^*}^{\frac{(p-1)(1-\alpha)}{p-1+\alpha}}\\
&\qquad - \frac{1}{2}K^{p-1}(\frac{p}{p-1+\alpha})^{p-1} z_{\lambda^*}^{\frac{(p-1)(1-\alpha)}{p-1+\alpha}}\\
&\qquad K^{p-1}(\frac{p}{p-1+\alpha})^{p-1}\left(\frac{(p-1)(1-\alpha)}{p-1+\alpha}\right)\frac{|\nabla z_{\lambda^*}|^p}{z_{\lambda^*}^{\frac{\alpha p}{p-1+\alpha}}}\Bigg\} w\,\mathrm{d}x,\\
&\le \int_\Omega\left(a\left(K z_{\lambda^*}^{\frac{p}{p-1+\alpha}}\right)^{p-1} - b\left(K z_{\lambda^*}^{\frac{p}{p-1+\alpha}}\right)^{\gamma} - f\left(K z_{\lambda^*}^{\frac{p}{p-1+\alpha}}\right) - \frac{c}{\left(K z_{\lambda^*}^{\frac{p}{p-1+\alpha}}\right)^{\alpha}}\right) w\,\mathrm{d}x,\\
&\qquad\le \int_\Omega\left(a\psi^{p-1} - b\psi^{\gamma} - f(\psi) - \frac{c}{\psi^\alpha}\right) w\,\mathrm{d}x, \quad \forall w\in W, \quad x\in\overline{\Omega}_\delta.
\end{aligned}
$$

Next in $\Omega\setminus\overline{\Omega}_\delta$, for $c\le c^*$ from the choice of c^* and K, we know that

$$\frac{c}{K^\alpha} \le \frac{1}{3}K^{p-1} z_{\lambda^*}^{p}\left(a - (\frac{p}{p-1+\alpha})^{p-1}\lambda^*\right), \tag{2.9}$$

$$bK^{\gamma-p+1} z_{\lambda^*}^{\frac{p(\gamma-p+1)}{p-1+\alpha}} \le \frac{1}{3}\left(a - (\frac{p}{p-1+\alpha})^{p-1}\lambda^*\right), \tag{2.10}$$

and

$$AK^{\beta-p+1} z_{\lambda^*}^{\frac{p(\beta-p+1)}{p-1+\alpha}} \le \frac{1}{3}\left(a - (\frac{p}{p-1+\alpha})^{p-1}\lambda^*\right) \tag{2.11}$$

By combining (2.9), (2.10) and (2.11), we have

$$
\begin{aligned}
&M\left(\int_\Omega |\nabla\psi|^p \,\mathrm{d}x\right)\int_\Omega |\nabla\psi|^{p-2}\nabla\psi\cdot\nabla w \,\mathrm{d}x\\
&\qquad= M\left(\int_\Omega |\nabla\psi|^p \,\mathrm{d}x\right) K^{p-1}(\frac{p}{p-1+\alpha})^{p-1}\\
&\quad\times\int_\Omega\left(\lambda^* z_{\lambda^*}^{\frac{p(p-1)}{p-1+\alpha}} - z_{\lambda^*}^{\frac{(p-1)(1-\alpha)}{p-1+\alpha}} - \left(\frac{(p-1)(1-\alpha)}{p-1+\alpha}\right)\frac{|\nabla z_{\lambda^*}|^p}{z_{\lambda^*}^{\frac{\alpha p}{p-1+\alpha}}}\right) w\,\mathrm{d}x,
\end{aligned}
$$

$$\leq \int_\Omega \left(M_\infty k^{p-1} (\frac{p}{p-1+\alpha})^{p-1} \lambda^* z_{\lambda^*}^{\frac{p(p-1)}{p-1+\alpha}} \right) w \, \mathrm{d}x,$$

$$= \int_\Omega \frac{M_\infty}{z_{\lambda^*}^{\frac{\alpha p}{p-1+\alpha}}} \sum_{i=1}^{3} \left(\frac{1}{3} k^{p-1} (\frac{p}{p-1+\alpha})^{p-1} \lambda^* z_{\lambda^*}^{p} \right) w \, \mathrm{d}x,$$

$$\leq \int_\Omega \Bigg\{ \frac{1}{z_{\lambda^*}^{\frac{\alpha p}{p-1+\alpha}}} \left(\frac{1}{3} k^{p-1} z_{\lambda^*}^{p} a - \frac{c}{k^\alpha} \right) + k^{p-1} z_{\lambda^*}^{p} \left(\frac{1}{3} a - b k^{\gamma-p+1} z_{\lambda^*}^{\frac{p(\gamma-p+1)}{p-1+\alpha}} \right)$$

$$+ k^{p-1} z_{\lambda^*}^{p} \left(\frac{1}{3} a - A k^{\beta-p+1} z_{\lambda^*}^{\frac{p(\beta-p+1)}{p-1+\alpha}} \right) \Bigg\} w \, \mathrm{d}x,$$

$$\leq \int_\Omega \left(a \left(K z_{\lambda^*}^{\frac{p}{p-1+\alpha}} \right)^{p-1} - b \left(K z_{\lambda^*}^{\frac{p}{p-1+\alpha}} \right)^{\gamma} - f \left(K z_{\lambda^*}^{\frac{p}{p-1+\alpha}} \right) - \frac{c}{\left(K z_{\lambda^*}^{\frac{p}{p-1+\alpha}} \right)^{\alpha}} \right) w \, \mathrm{d}x,$$

$$\leq \int_\Omega \left(a\psi^{p-1} - b\psi^\gamma - f(\psi) - \frac{c}{\psi^\alpha} \right) w \, \mathrm{d}x, \quad \forall w \in W, \quad x \in \Omega \setminus \overline{\Omega}_\delta.$$

Thus ψ is a positive subsolution of (1.1). From (H_1) and $\gamma > p-1$, it is obvious that $z = M$, where M is sufficiently large constant is a supersolution of (1.1) with $z \geq \psi$. Thus, the proof is complete. □

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