CONJUGATE EFFECTS OF HEAT SOURCE AND CHEMICAL REACTION ON MHD FREE CONVECTION FLOW WITH SHEAR ON THE BOUNDARY AND RAMPED WALL TEMPERATURE

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An exact solution for dimensionless velocity corresponding to the free convection flow of an incompressible, electrically conducting, viscous fluid over an infinite plate that applies arbitrary shear stress to the fluid is developed in the presence of ramped wall temperature, heat source and chemical reaction. It is presented as a sum of its mechanical, thermal and concentration components whose contribution to the fluid motion is graphically underlined. Radiative effects are not taken into consideration but they can be immediately included by a simple rescaling of Prandtl number. The influence of heat absorption/generation and chemical parameters as well as the combined magnetic and porous effects on the fluid motion is brought to light for a time dependent shear on the boundary.

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1. INTRODUCTION

Double diffusion due to the temperature and concentration variations appears in many real situations such as condensation, evaporation and chemical reactions. It has various applications in geothermal and geophysical engineering and the unsteady free convection flow with mass transfer over an infinite flat plate has received continuous attention due to its industrial and technological utilization. On the other, hand the influence of magnetic field on such a flow is significant and Soundalgekar *et al.* [1] seem to be the first authors who took into consideration mass transfer and magnetic effects in their work. It is well known the fact that the properties of some end products are usually improved using electrically conducting fluids and an applied magnetic field [2].

Sometimes, a foreign mass reacts with the fluid producing chemical reactions that influence both the heat and mass transfer. According to Bird *et al.* [3], a chemical reaction between two species can generate heat and its reaction rate depends on the species' concentration. A chemical reaction is of the first order if the rate of reaction is proportional to the species' concentration [4]. It is homogeneous, if it takes place in solution and plays an important role in food processing and polymer production. During time, many researchers studied the influence of chemical reaction on the free convection over a moving plate. For a fast review, we recommend the recent works of Ahmed and Dutta [5] and Narahari *et al.* [6]

Recently, Narahari *et al.* [7] found that the fluid velocity is smaller in the case of ramped wall temperature in comparison to the isothermal plate. On the other hand, the radiation effects on free convection flow with ramped wall temperature are important in some industrial and technological processes such as glass production, plasma physics and power generation systems. Further, the influence of ramped wall temperature on the MHD free convection flow of a viscous fluid over an infinite plate has been extensively studied. The most recent and interesting results seem to be those of Seth *et al.* [8], Rajesh [9], Kalidas [10], Narahari [11], Patra *et al.* [12], Samiulhaq *et al.* [13], Ghara *et al.* [14], Narahari *et al.* [15], Nandkeolyar *et al.* [16], Ismail *et al.* [17], Rajesh and Chamkha [18], Khan *et al.* [19], Narahari *et al.* [20], Seth *et al.* [21] and Samiulhaq *et al.* [22]. However, in all above-mentioned papers the plate is moving in its plane with a given velocity. Recently, Khan *et al.* [23] developed exact solutions for the unsteady MHD conjugate flow in a porous medium with ramped wall temperature and arbitrary shear stress on the boundary.

The main objective of this note is to extent the results of Khan *et al.* [23] to the case when the motion takes place in the presence of heat source and chemical reaction. Secondly, we want to point to one fact that is completely neglected in the existing literature, namely [24] "the velocity of a viscous fluid in such motions does not depend on magnetic and porous effects independently and a two parameter approach is superfluous or even misleading". Radiative effects are not taken into consideration but, according to Magyari and Pantokratoras [25], they can be easily included by a simple rescaling of Prandtl number. The fluid velocity is determined as sum of its mechanical, thermal and concentration components whose contribution to the fluid motion is graphically brought to light for time-dependent shear on the boundary. Finally, the influence of dimensionless heat absorption/generation and chemical reaction parameters as well as the combined magnetic and porous effects on the fluid motion are graphically underlined and discussed for time-dependent shear on the boundary.

2. STATEMENT OF THE PROBLEM

Consider the unsteady MHD free convection flow of an incompressible viscous electrically conducting fluid over an infinite vertical plate embedded in a porous medium. Thermal and concentration buoyancy effects are taken into consideration in the presence of a heat source/sink and chemical reaction. A uniform transverse magnetic field of constant strength B acts perpendicular to the plate. Thermal radiation and viscous dissipation heat are neglected but, according to Magyari and Pantokratoras [25], the radiative effects can be easily introduced by a simple rescaling of the Prandtl number if the Rosseland approximation is adopted for the heat flux.

Initially, the whole system is at rest at the constant temperature T_∞ and a uniform concentration level C_∞. The coordinate system is chosen so that the $x-$axis is along the plate in the upward direction while the $y-$axis is normal to the plate. After time $t = 0$ the plate, whose temperature is raised or lowered to $T_\infty + (T_w - T_\infty)t/t_0$ until $t = t_0$ applies an arbitrary shear stress $-f(t)$ to the fluid. It is also maintained at a constant temperature T_w after the time $t = t_0$ and at a constant concentration C_w all the time. Due to the shear the fluid is gradually moved and the magnetic field induced by the fluid motion is assumed to be negligible in comparison to the applied one. This assumption is valid at least for metallic liquids and partially ionized fluids [26] whose magnetic Reynolds number is very small.

Bearing in mind the above considerations as well as the usual Boussinesq's assumption for the radiative heat flux [5], the governing equations corresponding to the laminar free convection flow of such a fluid through a uniform porous medium can be written as [5,23,27]

$$\begin{aligned}\frac{\partial u(y,t)}{\partial t} &= \nu\frac{\partial^2 u(y,t)}{\partial y^2} + g\beta_T[T(y,t) - T_\infty] + g\beta_C[C(y,t) - C_\infty]\\ &- \frac{\sigma B^2}{\rho}u(y,t) - \frac{\nu}{K}u(y,t),\end{aligned} \tag{1}$$

$$\rho c_p\frac{\partial T(y,t)}{\partial t} = k\frac{\partial^2 T(y,t)}{\partial y^2} - Q[T(y,t) - T_\infty]; \quad y, t > 0, \tag{2}$$

$$\frac{\partial C(y,t)}{\partial t} = D_m\frac{\partial^2 C(y,t)}{\partial y^2} - R[C(y,t) - C_\infty]; \quad y, t > 0, \tag{3}$$

where u, T, C, ν, ρ, g, β_T, β_C, σ , K, c_p , k, Q, D_m and R are the velocity, temperature, species concentration, kinematic viscosity, fluid density, acceleration due to gravity, volumetric coefficient of thermal expansion, volumetric coefficient of concentration expansion, electrical conductivity, permeability of the porous medium, specific heat at constant temperature, thermal conductivity, heat absorption/generation coefficient, chemical molecular diffusivity and the chemical reaction parameter.

The corresponding initial and boundary conditions are

$$u(y,0) = 0, \quad T(y,0) = T_\infty, \quad C(y,0) = C_\infty; \quad y \geq 0,$$

$$\frac{\partial u(y,t)}{\partial y}\Big|_{y=0} = -\frac{f(t)}{\mu}, \quad C(0,t) = C_w; \quad t > 0, \tag{4}$$

$$T(0,t) = T_\infty + (T_w - T_\infty)\frac{t}{t_0} \quad if \quad 0 < t \leq t_0$$

$$and \quad T(0,t) = T_w \quad for \quad t > t_0, \tag{5}$$

$$u(y,t) \to 0, \quad T(y,t) \to T_\infty, \quad C(y,t) \to C_\infty \;\; as \;\; y \to \infty. \tag{6}$$

By introducing the following non-dimensional variables and functions

$$y^* = \frac{y}{\sqrt{\nu t_0}}, \quad t^* = \frac{t}{t_0}, \quad u^* = \frac{u}{U}, \quad T^* = \frac{T-T_\infty}{T_w-T_\infty}, \quad C^* = \frac{C-C_\infty}{C_w-C_\infty},$$

$$M^* = \frac{\sigma B^2}{\rho}t_0, \;\; \frac{1}{K^*} = \frac{\nu t_0}{K}, \;\; Q^* = \frac{Qt_0}{\rho c_p}, \;\; R^* = Rt_0, \;\; f^*(t^*) = \frac{\sqrt{\nu t_0}}{\mu U}f(t_0 t^*). \tag{7}$$

where U is a characteristic velocity and dropping out the star notation, we attain to the next problem with initial and boundary conditions

$$\frac{\partial u(y,t)}{\partial t} = \frac{\partial^2 u(y,t)}{\partial y^2} + T(y,t) + NC(y,t) - K_{eff}u(y,t); \quad y,t > 0 \tag{8}$$

$$\frac{\partial T(y,t)}{\partial t} = \frac{1}{Pr}\frac{\partial^2 T(y,t)}{\partial y^2} - QT(y,t); \quad y,t > 0 \tag{9}$$

$$\frac{\partial C(y,t)}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C(y,t)}{\partial y^2} - RC(y,t); \quad y,t > 0 \tag{10}$$

where

$$u(y,0) = 0, \quad T(y,0) = 0, \quad C(y,0) = 0, \quad y \geq 0 \tag{11}$$

$$\frac{\partial u(y,t)}{\partial y}\Big|_{y=0} = -f(t), \qquad C(0,t) = 1, \qquad t > 0 \tag{12}$$

$$T(0,t) = t \;\; if \;\; 0 < t < 1 \;\; and \;\; T(0,t) = 1 \;\; for \;\; t > 1 \tag{13}$$

$$u(y,t) \to 0, \quad T(y,t) \to 0, \quad C(y,t) \to 0, \quad as \quad y \to \infty. \tag{14}$$

Into above relations N is the ratio of buoyancy forces due to temperature and concentration, K_{eff} is the effective permeability [24], Pr is the Prandtl number, Q is the non-dimensional heat absorption/generation parameter, Sc is the Schmidt number and R is the dimensionless chemical reaction parameter. They are defined by Eq. (7) and

$$N = \frac{\beta_C(C_w - C_\infty)}{\beta_T(T_w - T_\infty)}, \quad K_{eff} = M + \frac{1}{K}, \quad Sc = \frac{\nu}{D_m}, \tag{15}$$

where M is the magnetic parameter, K is the porosity parameter and the characteristic velocity U has been taken to be equal to $g\beta_T t_0(T_w - T_\infty)$.

As we already told before, a similar problem without heat source and chemical reaction has been recently studied by Khan *et al.* [23]. However, their graphical representations are restricted to a constant shear on the boundary while magnetic and porous effects are wrongly brought to light using a two parameter approach. Actually, such a mistake appears in all previous mentioned papers. Furthermore, as the variations of temperature and concentration can be obtained adjusting or using results from the existing literature (see for instance [5] and [14]), closed form solutions will be here established only for velocity and graphical representations prepared for constantly accelerating shear shress on the boundary.

3. VELOCITY CALCULATION

Applying the Laplace transform to Eq. (8) and using the initial and boundary conditions $(11)_1$, $(12)_1$, $(14)_1$ as well as the Laplace transforms (see [14, Eq. (17) with $Ra = Q$] for $\overline{T}(y,q)$ and [27, Eq. (18) with Sc and R instead of Pr_{eff} and Q] for $\overline{C}(y,q)$)

$$\overline{T}(y,q) = \frac{1-e^{-q}}{q^2} e^{-y\sqrt{Pr(q+Q)}}, \quad \overline{C}(y,q) = \frac{1}{q} e^{-y\sqrt{Sc(q+R)}} \tag{16}$$

of $T(y,t)$ and $C(y,t)$ we find that

$$\frac{\partial^2 \overline{u}(y,q)}{\partial y^2} - (q+K_{eff})\overline{u}(y,q) + \frac{1-e^{-q}}{q^2} e^{-y\sqrt{Pr(q+Q)}} + \frac{N}{q} e^{-y\sqrt{Sc(q+R)}} = 0, \tag{17}$$

$$\frac{\partial \overline{u}(y,q)}{\partial y}|_{y=0} = -F(q), \quad \overline{u}(y,q) \to 0 \quad as \quad y \to \infty, \tag{18}$$

where $\overline{u}(y,q)$ and $F(q)$ are the Laplace transforms of $u(y,t)$, respectively $f(t)$. The solution of Eq. (17) with the boundary conditions (18) is

$$\begin{aligned} \overline{u}(y,q) = {} & F(q)\frac{\exp(-y\sqrt{q+K_{eff}})}{\sqrt{q+K_{eff}}} + \frac{\sqrt{Pr}}{(Pr-1)}\frac{1-e^{-q}}{q^2}\frac{\exp(-y\sqrt{q+K_{eff}})}{\sqrt{q+K_{eff}}} \\ & \frac{\sqrt{q+Q}}{q+a} - \frac{\sqrt{Pr}}{(Pr-1)}\frac{1-e^{-q}}{q^2}\frac{\exp(-y\sqrt{Pr(q+Q)})}{q+a))} + \frac{N\sqrt{Sc}}{Sc-1} \\ & \frac{\exp(-y\sqrt{q+K_{eff}})}{\sqrt{q+K_{eff}}}\frac{\sqrt{q+R}}{q(q+b)} - \frac{N}{Sc-1}\frac{\exp(-y\sqrt{Sc(q+R)})}{q(q+b)} \end{aligned} \tag{19}$$

where $a = \frac{QPr-K_{eff}}{Pr-1}$ and $b = \frac{RSc-K_{eff}}{Sc-1}$.

Now, applying the inverse Laplace transform to Eq. (19), using Eqs. $(A1)-(A3)$ from Appendix and the convolution theorem, the velocity field $u(y,t)$ can be written in the form

$$u(y,t)=u_m(y,t)+u_T(y,t)+u_C(y,t), \tag{20}$$

where

$$u_m(y,t)=\frac{1}{\sqrt{\pi}}\int_0^t \frac{f(t-s)}{\sqrt{s}}\exp(-\frac{y^2}{4s}-K_{eff}s)ds, \tag{21}$$

$$\begin{aligned} u_T(y,t) &= \frac{H(t-1)}{Pr-1}\int_0^t (t-s-1)\Phi(y\sqrt{Pr},s;c)e^{-as}ds \\ &- \frac{H(t)}{Pr-1}\int_0^t (t-s)\Phi(y\sqrt{Pr},s;c)e^{-as}ds \\ &+ \frac{H(t)\sqrt{Pr}}{(Pr-1)\sqrt{\pi}}\int_0^t (t-s)\int_0^s \frac{g(y,s,\tau,Q)}{\sqrt{s-\tau}}h(\tau,c)d\tau ds \\ &- \frac{H(t-1)\sqrt{Pr}}{(Pr-1)\sqrt{\pi}}\int_0^{t-1} (t-s-1)\int_0^s \frac{g(y,s,\tau,Q)}{\sqrt{s-\tau}}h(\tau,c)d\tau ds \end{aligned} \tag{22}$$

$$\begin{aligned} u_C(y,t) &= \frac{N\sqrt{Sc}}{(Sc-1)\sqrt{\pi}}\int_0^t\int_0^s \frac{g(y,s,\tau,R)}{\sqrt{s-\tau}}h(\tau,d)d\tau ds \\ &- \frac{N}{Sc-1}\int_0^t \Phi(y\sqrt{Sc},s;d)e^{-bs}ds \end{aligned} \tag{23}$$

and $c=\frac{K_{eff}-Q}{Pr-1}$, $d=\frac{K_{eff}-R}{Sc-1}$, $h(t,a)=\frac{1}{\sqrt{t\pi}}+\sqrt{a}e^{at}erf(\sqrt{at})$, $g(y,s,\tau,a)=\exp(-\frac{y^2}{4(s-\tau)}-K_{eff}(s-\tau)-a\tau)$ and the function $\Phi(y,t;a)$ is defined in Appendix.

The solutions (22) and (23) are not valid for $Pr=Sc=1$. In order to determine the thermal and concentration components corresponding to this case, we take $Pr=Sc=1$ into Eq. (17) and follow the same way as before. Direct computations show that (see also Eqs. $(A4)$ from Appendix)

$$\begin{aligned} u_T(y,t) &= \frac{1}{K_{eff}-Q}\{tH(t)\Phi(y,t;Q)-(t-1)H(t-1)\Phi(y,t-1;Q) \\ &+ \frac{y}{2\sqrt{Q}}[H(t)\Psi(y,t;Q)-H(t-1)\Psi(y,t-1;Q)] \\ &+ \frac{1}{2\pi}\left[H(t)\int_0^t (t-s)\int_0^s \frac{g(y,s,\tau,Q)}{\tau\sqrt{\tau(s-\tau)}}d\tau ds\right. \end{aligned}$$

$$- \quad H(t-1)\int_0^{t-1}(t-s-1)\int_0^s \frac{g(y,s,\tau,Q)}{\tau\sqrt{\tau(s-\tau)}}d\tau ds\Bigg]\Bigg\} \tag{24}$$

$$u_C(y,t) = \frac{NH(t)}{K_{eff}-R}\{\Phi(y,t;R) + \frac{1}{2\pi}\int_0^t\int_0^s \frac{g(y,s,\tau,R)}{\tau\sqrt{\tau(s-\tau)}}d\tau ds\} \tag{25}$$

where the function $\Psi(y,t;a)$ is defined in Appendix.

Direct computations show that $u(y,t)$ given by Eq. (20) satisfies the associated initial and boundary conditions. In order to verify the condition $(12)_1$, for instance, we must use the next identity

$$\int_0^t \frac{f(t-s)}{s\sqrt{s}}\exp(-\frac{y^2}{4s} - K_{eff}s)ds = \frac{4}{y}\int_{\frac{y}{2\sqrt{t}}}^{\infty} f(t-\frac{y^2}{4s^2})\exp(-s^2 - \frac{K_{eff}y^2}{4s^2})ds. \tag{26}$$

Finally, it is worth pointing out that the solutions corresponding to the same motion in the absence of heat source and chemical reaction are immediately obtained from the above solutions using the limits

$$\lim_{a\to 0}\Phi(y,t;a) = erfc(\frac{t}{2\sqrt{t}}),\ \lim_{a\to 0}\frac{\Psi(y,t;a)}{\sqrt{a}} = yerfc(\frac{t}{2\sqrt{t}}) - 2\sqrt{\frac{t}{\pi}}\exp(-\frac{y^2}{4t})$$

The mechanical component $u_m(y,t)$ given by Eq. (21), as expected, is identical to that obtained in [23, Eq. (31)]. Unfortunately, the complementary part of our solution in the absence of heat source and chemical reaction cannot be compared with the similar solution from [23, Eq. (30)] because the dimensionless governing equations for velocity are different.

4. NUMERICAL RESULTS AND DISCUSSION

In order to avoid repetition, we do not present here the solutions corresponding to different forms of the function $f(\cdot)$ and graphical representations which are similar to those of Khan *et al.* [23]. Our interest is to bring to light the following new contributions: 1) The influence of the dimensionless parameters N, Q and R on the fluid velocity. 2) Combined magneto and porous effects on the fluid velocity and 3) A comparison between the contributions of mechanical, thermal and concentration components of the velocity on the fluid motion in the case of a time dependent shear stress on the boundary when $f(t) = tH(t)$. In this case, the thermal and concentration components of velocity are given by Eqs. (22) and (23), while $u_m(y,t)$ takes the simple forms (see [28, Eq. (B2)] and Eq. (A6) from Appendix)

$$(27)\quad u_m(y,t) = \frac{1}{\sqrt{K_{eff}}}\left(\frac{1}{2K_{eff}} - t\right)\Psi(y,t;K_{eff}) - \frac{y}{2K_{eff}}\Psi(y,t;K_{eff}); \quad K_{eff} \neq 0,$$

and

$$(28)\quad u_m(y,t) = \frac{y^2+4t}{3}\sqrt{\frac{t}{\pi}}\exp\left(-\frac{y^2}{4t}\right) - y\frac{y^2+6t}{6}erfc\left(\frac{y}{2\sqrt{t}}\right), \quad if \quad K_{eff} = 0$$

To obtain some physical insight of these results, the velocity profiles against y are presented for different values of physical parameters N, Q, R and K_{eff}. From Fig. 1 it clearly results that the fluid velocity is an increasing function with respect to N. If $N > 0$, the mass buoyancy force acts in the same direction with the thermal buoyancy force and the fluid velocity increases for increasing values of the concentration. In the case $N < 0$, the negative buoyancy force causes the occurrence of a reverse flow. If $N = 0$, there is no contribution from the species diffusion and the free convection is due to the thermal buoyancy force only. In all cases, the dimensionless velocity of the fluid smoothly decreases from a maximum value near the plate to an asymptotic value for large values of y.

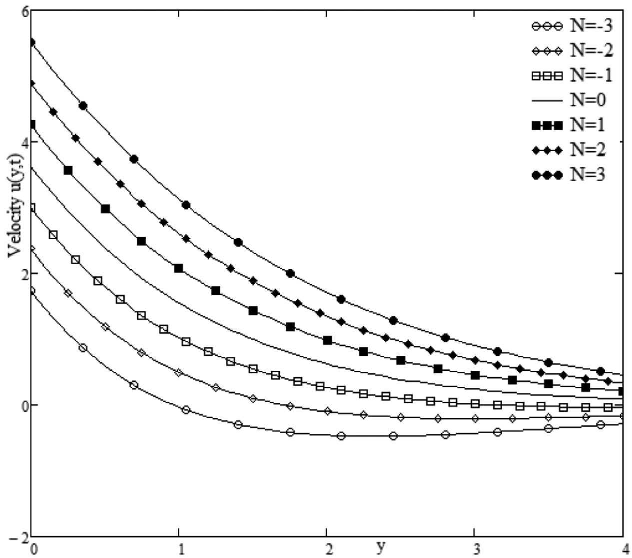

Fig. 1 – Velocity profiles against y for $Pr = 0.71$, $Sc = 0.5$, $Q = 1$, $R = 0.4$, $t = 3$, $K_{eff} = 1$, and different values of N.

The influence of heat generation ($Q < 0$) or heat absorption ($Q > 0$) on the fluid velocity is shown by Fig. 2. It is found that the fluid velocity decreases

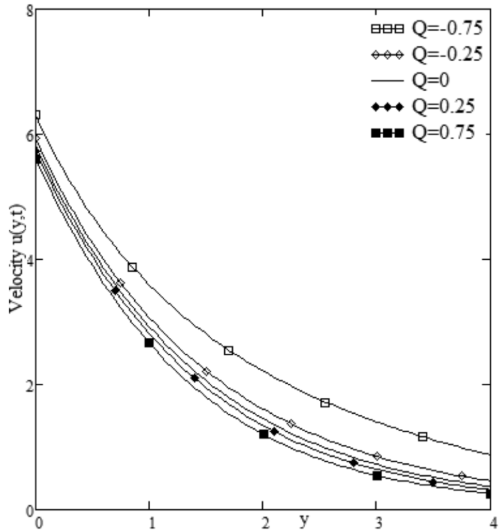

Fig. 2 – Velocity profiles against y for $Pr = 0.71$, $N = 1$, $Sc = 0.5$, $R = 0.4$, $K_{eff} = 1$, $t = 4$, and different values of Q.

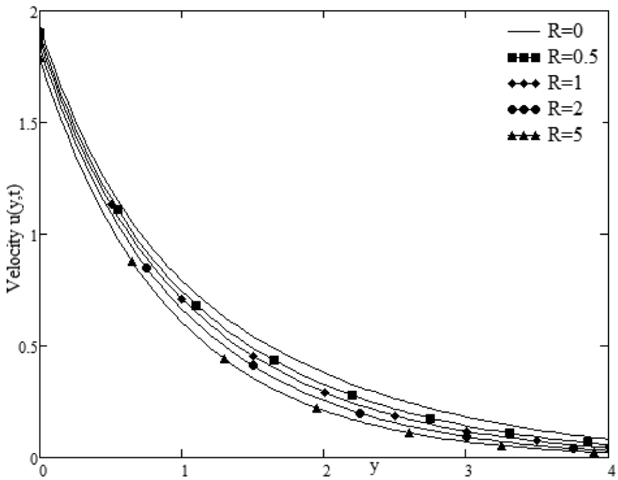

Fig. 3 – Velocity profiles against y for $Pr = 0.71$, $N = 1$, $Sc = 0.5$, $Q = 0.25$, $K_{eff} = 1$, $t = 2$, and different values of R.

with increasing values of the heat absorption and an opposite trend appears in the case of heat generation. Further, the values of velocity at any distance y from the plate are lower for $Q = 0.75$ than those for $Q = 0.25$. Fig. 3 reveals the effects of chemical reaction parameter R. The fluid velocity is a decreasing function with respect to this parameter. In order to be realistic, the numerical value of Prandtl number has been taken to be 0.71 which corresponds to water. The values of other parameters are arbitrary chosen.

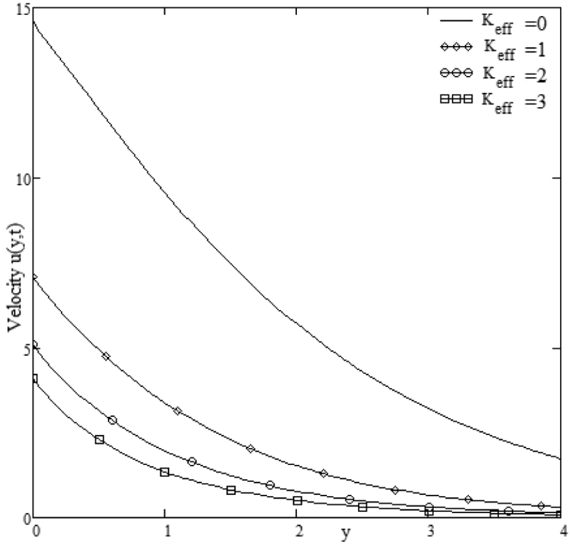

Fig. 4 – Velocity profiles against y for $Pr = 0.71$, $N = 1$, $Sc = 0.5$, $Q = 0.25$, $R = 1$, $t = 2$, and different values of K_{eff}.

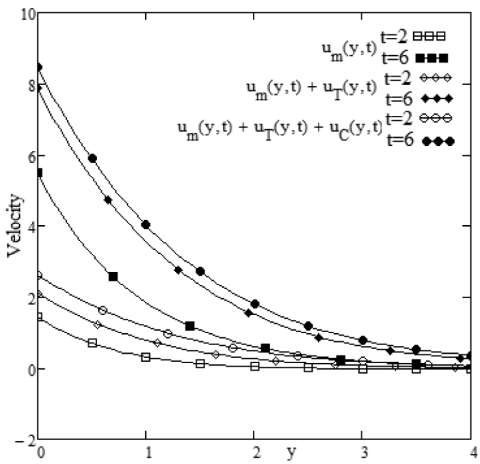

Fig. 5 – Profiles of the Velocities $u_m(y,t)$, $u_m(y,t) + u_T(y,t)$ and $u_m(y,t) + u_T(y,t) + u_C(y,t)$ against y for $Pr = 0.71$, $N = 1$, $Sc = 0.5$, $Q = 0.25$, $K_{eff} = 1$, $R = 1$, and different values of t.

Combined effects of magnetic and porosity parameters on the dimensionless velocity of the fluid are underlined in Fig. 4. As expected, the fluid velocity is a decreasing function with regard to K_{eff}. Its profiles smoothly descend from maximum values at the boundary to lowest values for large values of y. Finally, in order to bring to light the effective contributions of the three components u_m, u_T and u_C of velocity on the fluid motion, the diagrams of $u_m(y,t)$, $u_m(y,t)+u_T(y,t)$ and $u(y,t) = u_m(y,t)+u_T(y,t)+u_C(y,t)$ against y are depicted for two values of the time t. The contribution of each component on the fluid velocity, as it is clearly seen from this figure, is significant and has to be taken into consideration. Further, as expected, the fluid velocity is an increasing function with respect to the time t.

5. CONCLUSIONS

Unsteady MHD free convection flow of an incompressible, electrically conducting, viscous fluid over an infinite plate that applies an arbitrary time-dependent shear stress to the fluid has been studied in the presence of ramped wall temperature, heat source and chemical reaction. Thermal radiation and viscous dissipation are neglected but the radiative effects can immediately be included by a simple resealing of the Prandtl number. Closed form solutions are established for the dimensionless velocity as a sum of mechanical, thermal and concentration components.

Porous effects are taken into consideration and we again emphasized the fact that the fluid velocity does not depend on the porosity parameter independently, but by a combination with the magnetic parameter that is called the effective permeability. A two parameter approach is superfluous or even misleading although the recent literature ignores this very important remark. Finally, in order to get some physical insight of results that have been obtained, the influence of pertinent parameters and of the three components of velocity on the fluid motion is graphically underlined and discussed in the case of a ramped shear stress on the boundary. The main findings are:

• Fluid velocity is an increasing function with respect to $N > 0$ (aiding flows). A reverse trend appears for opposing flows when $N < 0$.

• In the presence of heat absorption ($Q > 0$) the fluid velocity decreases for increasing values of Q. An opposite effect appears when $Q < 0$ (heat generation).

• The fluid velocity is a decreasing function both with respect to the chemical reaction parameter R and the effective permeability K_{eff}.

• Contributions of mechanical, thermal or concentration components of velocity on the fluid motion are significant and cannot be neglected.

6. APPENDIX

(A1) $$L^{-1}[\frac{e^{-y\sqrt{q}}}{\sqrt{q}}] = \frac{1}{\sqrt{\pi t}}\exp(-\frac{y^2}{4t}),\ L^{-1}[\frac{\sqrt{q}}{q-a}] = \frac{1}{\sqrt{\pi t}} + \sqrt{a}e^{at}erf(\sqrt{at}),$$

(A2) $$L^{-1}[\frac{e^{-y\sqrt{q+a}}}{q-b}] = e^{bt}\Phi(y,t;a+b)$$

where $\Phi(y,t;a) = \frac{1}{2}\{e^{y\sqrt{a}}erfc[\frac{y}{2\sqrt{t}} + \sqrt{(a)t}] + e^{-y\sqrt{a}}erfc[\frac{y}{2\sqrt{t}} - \sqrt{(a)t}]\}$,

(A3) $$L^{-1}[e^{-aq}F(q)] = f(t-a)H(t-a) \quad if \quad L^{-1}[F(q)] = f(t),$$

(A4) $$L^{-1}\{\sqrt{q}\} = -\frac{1}{2t\sqrt{\pi t}},\ L^{-1}[e^{-y\sqrt{q}}] = \frac{y}{2t\sqrt{\pi t}}\exp(-\frac{y^2}{4t}),$$

(A5) $$\Psi(y,t;a) = \frac{1}{2}\{e^{y\sqrt{a}}erfc[\frac{y}{2\sqrt{t}} + \sqrt{(a)t} - e^{-y\sqrt{a}}erfc[\frac{y}{2\sqrt{t}} - \sqrt{(a)t}]\},$$

(A6) $$\int_0^t \sqrt{s}\exp(-\frac{y^2}{4s})ds = \frac{2t-y^2}{3}\sqrt{t}\exp(-\frac{y^2}{at}) + \frac{y^3}{6}\sqrt{\pi}erfc(\frac{y}{2\sqrt{t}}).$$

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