Dedicated to the memory of Cabiria Andreian-Cazacu

THE PERMUTATION ENTROPY AND THE ASSESSMENT OF COMPARTMENT FIRE DEVELOPMENT: GROWTH AND DECAY

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We investigate the order/disorder characteristics of the compartment fire based on experimental data. From our analysis, we claim that the permutation entropy is suitable to detect the occurrence of the flashover and eventual unusual data in fire experiments.

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1. INTRODUCTION

Given the sparse literature on the usefulness of the entropy in characterizing fire data, our goal is to develop a new method to perform a local entropic analysis of the time variation of the temperature during a full-scale fire experiment, aiming to judge whether a flashover occurred or not and estimate the time of occurrence.

The experimental data was collected during a full-scale fire experiment conducted at Fire Officers Faculty in Bucharest. We do not include here the description of the experimental setup (materials and methods), which can be found in [2]. During the test, flames were observed to impinge on the ceiling and exit through the front door opening, which is generally known as indicating that the fire reaches the flashover.

In Section 2 we present the theoretical background (as notation, notions, algorithms needed to perform the analysis).

Section 3 is dedicated to the results of the analysis of the collected raw data and their interpretation. For recent research on the fire phenomena performed using entropic tools the interested reader is referred to [4] and [6]. The methodological challenge is that the data under study is likely to be produced and affected by endogenous factors, but also confounded by external factors.

2. THEORETICAL BACKGROUND AND REMARKS

2.1. The permutation entropy and the PYR-algorithm

The natural logarithm is used below, as elsewhere in this paper.

Shannon's entropy [5] is defined as

$$H(P) = -\sum_{i=1}^{n} p_i \log p_i,$$

where $P = (p_1, \ldots, p_n)$ is a probability distribution. It is nonnegative and its maximum value is $H(U) = \log n$, where $U = (1/n, \ldots, 1/n)$. Throughout the paper we use the convention $0 \log 0 = 0$.

For reader's convenience, we describe below the procedure and the theoretical background we have used to determine a probability distribution out of our experimental data.

The permutation entropy PE [1] quantifies uncertainty and disorder of a time series based on the appearance of ordinal patterns, that is on comparisons of neighboring values of time series. For other details on the PE-algorithm applied to the present experimental data, see [2].

Let $T = (t_1, \ldots, t_n)$ be a time series with distinct values.

Step 1. The increasing rearranging of the components of each j-tuple $(t_i, \ldots, t_{i+j-1})$ as $(t_{i+\pi_1-1}, \ldots, t_{i+\pi_j-1})$ yields a unique permutation of order j denoted by $\pi = (\pi_1, \ldots, \pi_j)$, an encoding pattern that describes the up-and-downs in the considered j-tuple.

Step 2. The absolute frequency of this permutation (the number of j-tuples which are associated to this permutation) is

$$k_\pi \equiv \#\{i : i \leq n-(j-1), (t_i, \ldots, t_{i+j-1}) \text{ is of type } \pi\}.$$

These values have the sum equal to the number of all consecutive j-tuples, that is $n-(j-1)$.

Step 3. The permutation entropy of order j is defined as

$$PE(j) \equiv -\sum_{\pi} p_\pi \log p_\pi,$$

where $p_\pi = k_\pi/(n-(j-1))$ is the relative frequency.

In [1] the measured values of the time series are considered distinct. The authors neglect equalities and propose to break them by adding small random perturbations (random noise) to the original series.

In [3] we propose a method to redistribute the ties (equalities), called PYR-algorithm, which fits better in the context of fire phenomena. We briefly describe it here.

Encoding Step. Let $mx(d) = \min\{s : t_s = \max(t_1, \ldots, t_n)\}$, determined for the data collected at the thermocouple Td. The j-tuples with distinct elements are counted on behalf of a permutation as in the PE-algorithm encoding step (Step 1 above). The same holds for each j-tuple $(t_i, \ldots, t_{i+j-1})$ that contains ties, after ordering the ties chronologically if $i \leq mx(d)$, or in reversed chronological order if $i > mx(d)$. This step is an adjustment of the counting procedure inspired by the evolution of the fire: a j-tuple is considered on the ascending trend before the maximum value of the temperature is reached (the growth period), respectively on the descending trend afterwards (the decay period).

Example. A 6-tuple which satisfies $t_{i+2} < t_{i+3} = t_{i+6} < t_{i+1} < t_{i+4} = t_{i+5}$, should be counted on behalf of the permutation $(2, 3, 6, 1, 4, 5)$ if it occurs on a growth period, or on behalf of the permutation $(2, 6, 3, 1, 5, 4)$ if it appears during the decay period.

The resulting entropy, computed following Step 2 and Step 3 in the PE-algorithm above, is denoted by PYR$PE(j)$. The name comes from the Greek word $\pi\tilde{\upsilon}\rho$ (pyr), meaning *fire*.

3. **RESULTS AND DISCUSSION**

The raw data set under consideration consists of measured temperatures during a compartment fire: six thermocouples T1, ..., T6 measure the temperatures every second during the experiment. Hence, we get six time-series consisting of 3046 entries (data points) each.

3.1. **Growth and decay period**

Reaching the maximum temperature is a significant event in the evolution of the fire. We aim to assess the performance of the PYR-algorithm and the permutation entropy during the growth and the decay period of the fire event.

We plot the values of the permutation entropy for 3 and 4-tuples: the line connecting the values of the permutation entropy for the growth period is denoted by $G(j)$, for the decay period by $D(j)$, where $j = 3$ or 4 is the embedding dimension. The probability distribution is determined on the time intervals $[1, mx(i)]$ and $[mx(i), 3046 - j + 1]$, where $i = 1, 2, ..., 6$ and $mx(i)$ is the time when the maximum temperature value is reached at the thermocouple

Ti. The entropy on the decay period is usually bigger, that is, the temperature has more fluctuations after the maximum temperature has been reached. When we increase the distance to the fire, the entropy looks similar on the two periods under consideration. See Figure 1 and Figure 2.

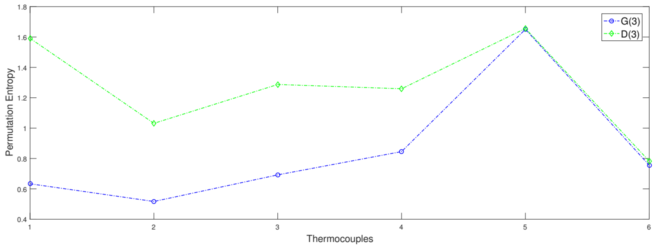

Fig. 1 – PYR$PE(3)$ on the growth and decay intervals.

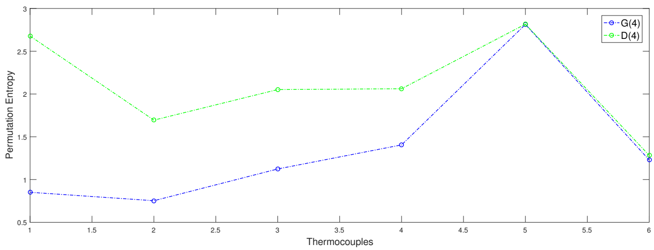

Fig. 2 – PYR$PE(4)$ on the growth and decay intervals.

3.2. The permutation entropy plotted against time. An important local minimum

It is of interest to observe the evolution of the permutation entropy in time, so we plot it cumulatively (denoted by CPYRPE): the probability distribution corresponding to each interval $[1, t]$, where $t \in [490, 3046 - j + 1]$, is determined by counting the patterns as prescribed by the PYR-algorithm for the j-tuples $(t_n, \ldots, t_{n+j-1})$, where $1 \leq n \leq t - j + 1$. The plots against the time t, for 3 and 4-tuples, appear in Figure 3, respectively Figure 4.

At the moment $t = mx(i) - j + 1$, the maximum temperature value at the thermocouple Ti becomes the last element of the last j-tuple counted for the plot of PYRPE. Then the graph of the entropies changes its appearance, we observe there a local minimum (see red star plots of PYRPE at each thermocouple). It is due to the structure of the compartment fire data, so one can infer that this minimum would appear regardless of the occurrence of a flashover or the position of the thermocouple. The descent of the plot while approaching the moment $mx(i)$ is due to the monotonous increase in temperature.

We remark that the plot of the values corresponding to the thermocouple T5 looks different, which leads us to the conclusion that it might also indicate a turbulence, a white noise perturbation of undetermined origin or a malfunction of this thermocouple (an improperly calibrated scale), however this is beyond the scope of the present paper to discuss it in detail.

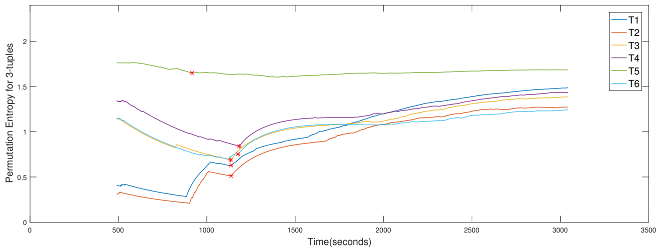

Fig. 3 – CPYR$PE(3)$ plotted against time.

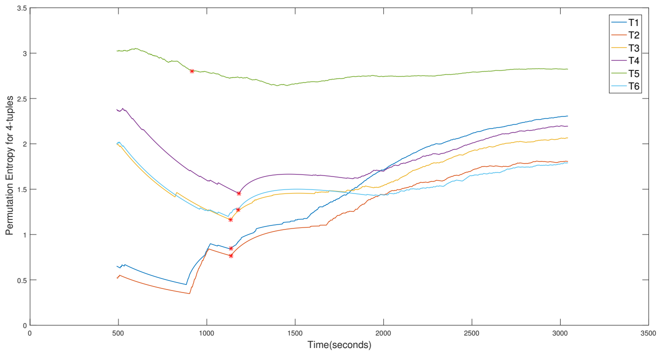

Fig. 4 – CPYR$PE(4)$ plotted against time.

Of major importance is the adequacy of the (PYR-) algorithm used to determine the probability distribution. The similarities of the plots, for $j = 3$ and 4, are not consequences of the mathematical properties of the entropy, but a result of the PYR-algorithm, that is of the fire phenomena evolution.

3.3. **Flashover**

Another meaningful event during a compartment fire development is the eventual occurrence of the flashover. We further describe a new procedure to detect the *prior* occurrence and estimate the time of the flashover by analyzing the temperature time series collected during the fire experiment.

First, we plot the evolution of the complementary cumulative (with respect to the maximum temperature) permutation entropy, denoted CCPYRPE: the probability distribution is determined by the PYR-algorithm for the j-tuples $(t_n, \ldots, t_{n+j-1})$, where $t \leq n \leq mx(i) - j + 1$, where $t \in [1, mx(i) - j]$ and $i = 1, 2, .., 6$. By plotting these values against time, we determine the time when the entropies take the value zero (the zero entropy time denoted further by $ZE(j)$). See Figure 5 and Figure 6. We note that the conclusions remain unchanged when running the analysis for different embedding dimensions. The fact that the CCPYRPE takes the value zero while approaching the maximum temperature shows that the temperature has no other fluctuations before reaching its maximum, its increase is monotonous, hence the fire development can be further interpreted as the flashover occurrence.

Unexpectedly, $ZE(j)$ is the same for the embedding dimensions $j = 3$ and $j = 4$ at every thermocouple. This fact cannot be mathematically explained (the counting of the j-tuples yields probability distributions with 6, respectively 24 components) and we consider that the results characterize compartment fire events.

As expected, the way the temperature fluctuates depends on the position of the thermocouple, that is on the distance to the upper layer and on the distance to the fire source. The thermocouples T2 and T3, respectively T4 and T6, have only little delays in between their $ZE(j)$ (see Table 1). The thermocouples are situated outside the area of the fire, so the values do not coincide.

The fact that at T1 the temperature has fluctuations in the 4-tuple which ends with $mx(1)$ (and therefore the entropy is not zero) could be intuitively explained by its position at the highest layer, where the resulting faster-moving smoke and fire gases are encountered. We stress that some isolated fluctuations at T1 seem to occur when other thermocouples recordings reach the zero entropy time, that is when they record the flashover. Therefore, we do not ignore such singletons of fluctuating data, but consider them significant for further analysis of the dynamics of the fire.

(Table 1)

Zero Entropy time	T1	T2	T3	T4	T5	T6
$mx(i)$	1140	1140	1138	1186	918	1180
$ZE(3)$	–	1012	1010	1066	907	1162
$ZE(4)$	–	1012	1010	1066	907	1162

The occurrence of the flashover and its exact time have not been inves-

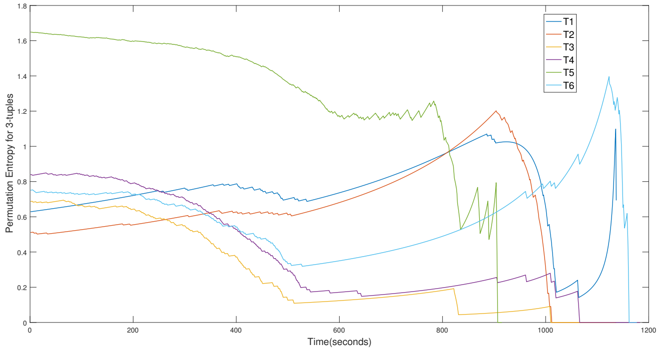

Fig. 5 – CCPYR$PE(3)$ plotted from the ignition time to $mx(i)$

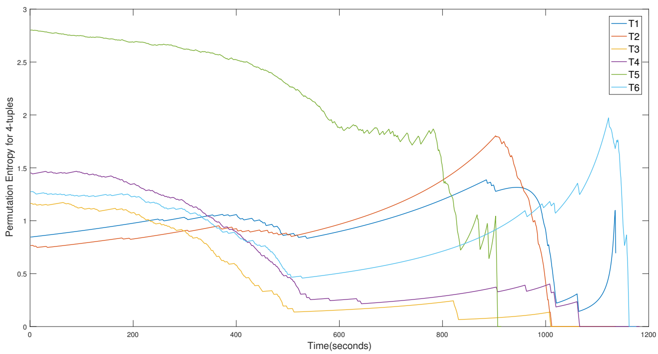

Fig. 6 – CCPYR$PE(4)$ plotted from the ignition time to $mx(i)$.

tigated yet by means of entropy. Our results clearly demonstrate that such analysis is possible and more experimental data gathered from the fire zone might enable a better estimate of this time.

The fluctuation of the temperature (see also Figure 7) is affecting the evolution of the entropy. Figure 5 and Figure 6 show that there is an increase of the local permutation entropy followed by a sudden decrease, for each thermocouple except T5 (since before the flashover, the temperature is highly oscillating before it starts to increase fast).

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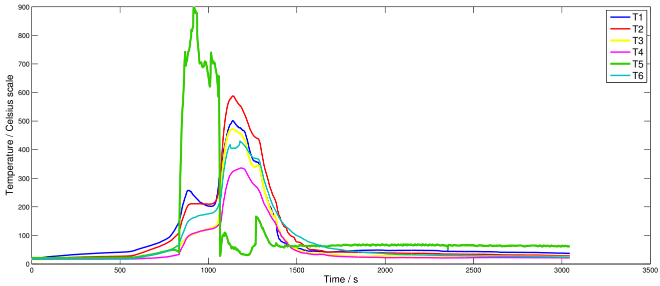

Fig. 7 – Time-temperature plot.

REFERENCES

[1] C. Bandt and B. Pompe, *Permutation entropy: a natural complexity measure for time series.* Phys. Rev. Lett. **88** (2002), *17*, 174102.

[2] F.-C, Mitroi-Symeonidis, I. Anghel, O. Lalu, and C. Popa, *The permutation entropy and its applications on fire tests data.* J. Appl. Comput. Mech. **6** (2020), *SI*, 1380 – 1393.

[3] F.-C. Mitroi-Symeonidis and I. Anghel, *The PYR-algorithm for time series modeling of temperature values and its applications on full-scale compartment fire data.* Acta Technica Napocensis, Series: Applied Mathematics, Mechanics, and Engineering **63** (2020), *IV*, 403 – 410.

[4] S. Murayama, K. Kaku, M. Funatsu, H. Gotoda, *Characterization of dynamic behavior of combustion noise and detection of blowout in a laboratory-scale gas-turbine model combustor.* Proceedings of the Combustion Institute **37** (2019), *4*, 5271 – 5278.

[5] C. E. Shannon, *A mathematical theory of communication.* Bell System Technical Journal **27** (1948), *3*, 379 – 423.

[6] K. Takagi, H. Gotoda, I. T. Tokuda, and T. Miyano, *Dynamic behavior of temperature field in a buoyancy-driven turbulent fire.* Phys. Lett. A **382** (2018), *44*, 3181 – 3186.

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