CORRIGENDUM TO "*INTRANSITIVE PERMUTATION GROUPS WITH BOUNDED MOVEMENT HAVING MAXIMUM DEGREE*"

MEHDI REZAEI, MEHDI ALAEIYAN, and ZEINAB FORUZANFAR

Communicated by Sorin Dăscălescu

AMS 2020 Subject Classification: 20B05.

Key words: intransitive permutation groups, orbits, movement.

In this note, we point out a mistake in part (II) of Theorem 1.1 of [1] and correct it here. In [1], the authors investigated the structure of intransitive permutation groups with bounded movement having maximum degree. In part (II) of Theorem 1.1, they classified all permutation groups with bounded movement m having maximum degree $n = |\Omega| = 3m + 1$. Unfortunately, they missed a group satisfying in this condition. In fact, at the bottom of page 6, they omitted the cases (d) and (g) of the cases $(a) - (o)$, i.e., the cases $r_3 = s = 1, r_2 = r'_3 = r_4 = 0$ and $r'_3 = s = 1, r_2 = r_3 = r_4 = 0$.

In this note, we consider these two cases and provide an example satisfying in the first case which was not mentioned in part (II) of Theorem 1.1. Also, we note that the cases (h), (i), (j), (m), (n) (o), were not necessary to check. Because 3 is the least odd prime dividing the order of G and it is not possible to have $r_3 = r'_3 = 0$. As it was checked in [1], the cases (a), (b), (e), (k), (l) do not arise by the equality $n = |\Omega| = 3r_3 + 3r'_3 + 2r_2 + 4r_4 + |\Sigma_5| = 3m + 1$. So, only the following four cases remain:

(I) $r_3 = r_4 = 1, r'_3 = r_2 = s = 0$,

(II) $r_3 = s = 1, r_2 = r'_3 = r_4 = 0$,

(III) $r'_3 = r_4 = 1, r_3 = r_2 = s = 0$,

(IV) $r'_3 = s = 1, r_2 = r_3 = r_4 = 0$,

The cases (I) and (III) were verified in [1]. Below, we give an example which satisfies the case (II).

MATH. REPORTS **25(75)** (2023), *3*, 451–453
doi: 10.59277/mrar.2023.25.75.3.451

Example 1. Let $G_1 := \mathbb{Z}_7$ and $G_2 := \mathbb{Z}_3$ be permutation groups on the sets $\Omega_1 = \{1,2,3,4,5,6,7\}$ and $\Omega_2 = \{1',2',3'\}$, respectively, where $\mathbb{Z}_7 \cong \langle(1\cdots 7)\rangle$ and $\mathbb{Z}_3 \cong \langle(1'2'3')(2\,3\,5)(4\,7\,6)\rangle$. Set $\Omega := \Omega_1 \cup \Omega_2$.

Then $G := G_1 \rtimes G_2$ is a permutation group on Ω which has $t = 2$ orbits, and since each non-identity element of G has three cycles of length 3 or one cycle of length 7, so $m = \text{move(G)} = 3$. It follows that $n = 3m + 1 = 10$.

Now we consider the cases (II) and (IV). For the sake of simplicity, we set $\sum_5 := \Delta$. So, we have $n = 3m + 1 = 3 + |\Delta|$.

Suppose that the transitive constituent G^Δ of G has movement m'. Let $p \geq 5$ be the least odd prime dividing $|G^\Delta|$. Since we are trying to classify permutation groups with bounded movement having maximum degree, by Lemma 1.1 of [2], we can consider the maximum degree $\lfloor \frac{2m'p}{p-1} \rfloor$ for G^Δ. Then by checking Theorem 1.2 of [2], one can see that the cases (1) and (2) do not happen. So, G^Δ is a p-group of order p^a on the set Δ of size p^a with movement $m' = \frac{p^{a-1}(p-1)}{2}$, for some $a \geq 1$. Therefore, case (IV) does not arise.

Now inequality $m \geq m'$ implies that $\frac{p^a+2}{3} \geq \frac{p^{a-1}(p-1)}{2}$. But this only happens when $p = 7$, $a = 1$, $m = m' = 3$ and $n = 10$, which implies that the only group satisfying in this case is the group mentioned in Example 1.

Now Theorem 1.1 of [1] can be corrected as follows:

Theorem 2. *Let G be a permutation group on a set Ω with t, $t \geq 2$, orbits which have no fixed points in Ω. Suppose further that m is a positive integer such that move$(G) = m$ and $n = 3m + t - 1$. Then*

I) $n \leq \lfloor \frac{9m-3}{2} \rfloor$, *and the equality holds if and only if G is one of the following:*

(a) *G is an elementary abelian 3-group and all its orbits have length 3.*

(b) *G is the semidirect product of $\mathbb{Z}_2^2$ and $\mathbb{Z}_3$ with normal subgroup $\mathbb{Z}_2^2$.*

II) *Let $n = 3m + 1$, which is the maximum bound for $t = 2$. Then either G is the group mentioned in the case (b) or G is the semidirect product of $\mathbb{Z}_7$ and $\mathbb{Z}_3$ with normal subgroup $\mathbb{Z}_7$.*

Acknowledgments. The authors would like to thank the referees for carefully reading the manuscript and for giving constructive comments.

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Received April 10, 2020

Buein Zahra Technical University
Department of Mathematics
Buein Zahra, Qazvin, Iran
mehdrezaei@gmail.com, m_rezaei@bzte.ac.ir

Iran University of Science and Technology
Department of Mathematics
Narmak, Tehran, 16844 Iran
alaeiyan@iust.ac.ir

Buein Zahra Technical University
Department of Mathematics
Buein Zahra, Qazvin, Iran
zforouzanfar@gmail.com, z_foruzanfar@bzte.ac.ir