ON THE p-PART OF G-CLASS SIZES OF A NORMAL SUBGROUP OF A FINITE GROUP G

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Let G be a finite group. For an element x of G, x^G denotes the conjugacy class of x in G. $|x^G|$ is called the size of the conjugacy class x^G. Let N be a normal subgroup of G. For $x \in N$, we have $x^G \subseteq N$ and x^G is called a G-class of the normal subgroup N. In this paper, we develop several results on the p-part of G-class sizes of a normal subgroup of a finite group G.

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1. INTRODUCTION AND PRELIMINARIES

We shall always use the term group to refer to a finite group. The letter G always denotes a group, and the letter p always denotes a prime. For an element x of G, $o(x)$ denotes the order of x, and x^G denotes the conjugacy class of x in G. $|x^G|$ is called the size of the conjugacy class x^G, that is the positive integer $|G : C_G(x)|$. If n is a positive integer, then n_p denotes the highest power of the prime p dividing n. We denote by $\pi(n)$ the set of prime divisors of n. For a group G, we set $\pi(G) = \pi(|G|)$. Our remaining notation is standard (see [9]).

Let N be a normal subgroup of a group G. For $x \in N$, we have $x^G \subseteq N$ and we say that x^G is a G-class of the normal subgroup N.

An important and interesting problem in finite group theory is the study of the influence of the conjugacy class sizes of a group G on the structure of G. Naturally, it is also an interesting problem to study the influence of the G-class sizes of a normal subgroup N of a group G on the structure of N. For instance, in [2], A. Beltran and M. J. Felip have established the following result: let N be a normal subgroup of G, such that $\{|x^G| : x \in N\} = \{1, m, mq^a\}$, with q a prime and $(m, q) = 1$. Then N is solvable. However, studying such properties only from partial information, provided by G-class sizes of a normal subgroup N of a group G, can be a more complex problem.

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The purpose of the present paper is to investigate the influence of the p-part of G-class sizes of a normal subgroup N of a group G on the structure of N. The main result is the following:

THEOREM A. *Let N be a normal subgroup of a group G, and let p be a fixed prime factor of $|G|$. Suppose that*

$$\{|x^G|_p : x \in N)\} = \{1, p^e\},$$

where e is a fixed integer and $e > 0$. Then N is solvable and p-nilpotent.

Theorem A of [2], Theorem A of [3] and the main part of Theorem A of [1] are immediate consequences of Theorem A of the present paper. In addition, we are going to improve Theorem A of [2] and Theorem A of [3].

In this section, we list several lemmas which will be used. The following Lemma 1.1 and Lemma 1.2 are well-known.

LEMMA 1.1. *Let $x \in G$. Assume that $o(x) = p_1^{m_1} ... p_n^{m_n}$, where $p_1, ..., p_n$ are distinct primes and $m_1, ..., m_n$ are positive integers. Then, $x = x_1 ... x_n$ with $o(x_i) = p_i^{m_i}$ and $x_r x_s = x_s x_r$ for $s, r = 1, ..., n$. Furthermore, there exist integers k_i such that $x^{k_i} = x_i$ for $i = 1, \cdots, n$.*

LEMMA 1.2. *Let N be a normal subgroup of G. The following two propositions hold:*

(1) *For every $x \in N$, $|x^N| \mid |x^G|$;*

(2) *For every $x \in G$, $|(xN)^{G/N}| \mid |x^G|$.*

The following Lemma 1.3 is Thompson's $P \times Q$-Lemma (see [6, Theorem 3.4, p. 179]).

LEMMA 1.3. *Let $P \times Q$ be a direct product of a p-group P and a p'-group Q represented a group of automorphisms of a p-group G. Suppose that $C_G(P) \subseteq C_G(Q)$. Then Q acts trivially on G.*

LEMMA 1.4 ([11]). *Let P be a p-group. Suppose that $\{|x^P| : x \in P\} = \{1, p^e\}$, where e is a fixed integer and $e > 0$. Then P has nilpotency class at most 3 and $P/Z(P)$ has exponent p.*

LEMMA 1.5 ([4, Proposition 3]). *Let G be a non-abelian simple group, and let $p \in \pi(G)$. Then, there exists an element x of G such that $|x^G|_p = |G|_p$.*

2. NORMAL $K(p^e)$-SUBGROUPS

In this section, we discuss the so-called normal $K(p^e)$-subgroups of a group. We first establish two results on non-abelian simple groups as follows.

THEOREM 2.1. *Let G be a non-abelian simple group, and let $p \in \pi(G)$. Then, $|G|_p > |M(G)|_p$, where $M(G)$ denotes the Schur multiplier of G.*

Proof. Clearly, we may assume that $|M(G)| \neq 1, 2$. By the Classification of the Finite Simple Groups, G is either an alternating group $A_n (n \geq 5)$ or a sporadic simple group or a simple group of Lie type. We separately discuss these cases as follows.

(1) Suppose that $G \cong A_n (n \geq 5)$.

We have $|M(A_n)| = 2$ for all $n \geq 5$ except for $n = 6, 7$, where $|M(A_n))| = 6$ (see [10, (11.17), p. 197]). Then, since $|M(G)| \neq 1, 2$, we have that $|G| = 2^3 \cdot 3^2 \cdot 5$ or $2^3 \cdot 3^2 \cdot 5 \cdot 7$ and $|M(G)| = 2 \cdot 3$, and it is obvious that $|G|_p > |M(G)|_p$.

(2) Suppose that G is a sporadic simple group.

Since $|M(G)| \neq 1$ and 2, by checking the Atlas [5], we conclude that G is isomorphic to one of the following groups: M_{22}, Fi_{22}, Suz, J_3 and ON. Again, by checking the Atlas [5], we conclude that $|G|_p > |M(G)|_p$. For example, assume that $G \cong M_{22}$. Then, we have that $|G| = 2^7 \cdot 3^2 \cdot 5 \cdot 7^3 \cdot 11$ and $|M(G)| = 2^2 \cdot 3$ (see the Atlas [5]), and it is obvious that $|G|_p > |M(G)|_p$.

(3) Suppose that G is a simple group of Lie type.

We have $PSL_2(3^2) \cong A_6$ and in (1) we have dealt with the case that $G \cong A_6$. Hence, we may assume that $G \ncong PSL_2(3^2))$. Then, since $|M(G)| \neq 1, 2$, G is isomorphic to one of the following groups: $D_n(q)(n \geq 4)$, ${}^2D_n(q)(n \geq 5)$, $E_6(q)$, ${}^2E_6(q)$, $PSL_3(4)$, $U_4(3)$, $U_6(2)$, $O_7(3)$, $SO_8^+(2)$, ${}^2B_2(8)$, $G_2(3)$, ${}^2E_6(2)$, $A_{n-1}(q)(n \geq 3)$ and ${}^2A_{n-1}(q)(n \geq 3)$ (see [12, p. 211 and p. 214]).

(3.i) Assume that $G \cong D_n(q)(n \geq 4)$.

Since $|M(G)| \neq 1, 2$, we have that $|M(G)| = 4$ and $q \equiv 1 (\text{mod } 4)$ or $q \equiv -1 (\text{mod } 2)$ (see [12, p. 211]). Clearly, if $p \neq 2$, then we have $|G|_p > |M(G)|_p$. So, we assume that $p = 2$. We have

$$|G| = |D_n(q)| = q^{n(n+1)}(q^n - 1)(q^2 - 1) \cdots (q^{2(n-1)} - 1)/(4, q^n - 1).$$

Since $n \geq 4$ and $q \equiv 1 \pmod 4$ or $q \equiv -1 \pmod 2$, from the above equality we know that $|G|_2 \geq 8$. Then, $|G|_2 > |M(G)|_2 = 4$.

(3.ii) Assume that $G \cong^2 D_n(q)(n \geq 5)$.

Since $|M(G)| \neq 1, 2$, we have that $|M(G)| = 4$ and $q \equiv -1 (\text{mod } 4)$ (see [12, p. 211]). Clearly, if $p \neq 2$, then we have $|G|_p > |M(G)|_p$. So, we assume that $p = 2$. We have

$$|G| = |{}^2D_n(q)| = q^{n(n+1)}(q^n + 1)(q^2 - 1) \cdots (q^{2(n-1)} - 1)/(4, q^n + 1).$$

Since $n \geq 5$ and $q \equiv -1 (\mathrm{mod}\ 4)$, from the above equality we know that $|G|_2 \geq 8$. Then, $|G|_2 > |M(G)|_2$.

(3.iii) Assume that $G \cong E_6(q)$.

Since $|M(G)| \neq 1, 2$, we have that $|M(G)| = 3$ and $q \equiv 1 \pmod 3$ (see [**?**]. 211]10). Clearly, if $p \neq 3$, then we have $|G|_p > |M(G)|_p$.

Now, we assume that $p = 3$. Since $q \equiv 1 \pmod 3$, we have $|G|_3 \geq 3^5$ (see [7, p.135]), and so $|G|_3 > |M(G)|_3$.

(3.iv) Assume that $G =^2 E_6(q)$.

Since $|M(G)| \neq 1, 2$, we have that $|M(G)| = 3$ and $q \equiv -1 \pmod 3$ (see [12, p. 211]). By the same arguments as in (3.iii), we conclude that $|G|_p > |M(G)|_p$.

(3.v) Assume that $G = PSL_3(4)$.

We have that $|M(G)| = 2^4 \cdot 3$ (see [12, p. 214]) and

$$|G| = |PSL_3(4)| = 4^3(4^2 - 1)(4^3 - 1)/3 = 2^6 \cdot 3^3 \cdot 5 \cdot 7.$$

Then, it is obvious that $|G|_p > |M(G)|_p$.

(3.vi) Assume that G is isomorphic to one of the following groups: $U_4(3)$, $U_6(2)$, $O_7(3)$, $SO_8^+(2)$, ${}^2B_2(8)$, $G_2(3)$ and ${}^2E_6(2)$.

By checking the Atlas [5], we conclude that $|G|_p > |M(G)|_p$.

For example, assume that $G \cong U_4(3)$. By checking the Atlas [5], we get that $|G| = 2^7 \cdot 3^6 \cdot 5 \cdot 7$ and $|M(G)| = 3^2 \cdot 4$. It is obvious that $|G|_p > |M(G)|_p$.

(3.vii) Assume that $G = A_{n-1}(q)(n \geq 3)$.

We have $|M(G)| = (n, q-1)$ (see [12, p. 211]). Then, if $p \nmid (q-1)$, then $|M(G)|_p = 1$, and hence $|G|_p > |M(G)|_p$. So, we assume that $p|(q-1)$. We have (see [7, p. 135]):

$$\begin{aligned}|G| &= |A_{n-1}(q)| = |PSL_n(q)| \\ &= (q^n - 1) \cdots (q^n - q^{n-1})/(q-1)(n, q-1).\end{aligned}$$

Assume that $n \geq 4$. Then, it is easy to see that $|G|$ is divisible by $(q-1)^2$. In particular, $((q-1)^2)_p$ divides $|G|_p$, and hence $|G|_p > |M(G)|_p$. Now, we assume that $n = 3$. Then, since $|M(G)| \neq 1$, we have that $|M(G)| = 3$ and $q \equiv 1 \pmod 3$. If $p \neq 3$, then $|G|_p > |M(G)|_p = 1$. Hence, we assume that $p = 3$. Then, since $q \equiv 1 \pmod 3$, we have $|G|_3 \geq 3^2 > |M(G)|_3 = 3$.

(3.viii) $G =^2 A_{n-1}(q)(n \geq 3)$.

We have that $|M(G)| = (n, q+1)$ (see [12, p.121]) and

$$\begin{aligned}|G| &= |{}^2A_{n-1}(q)| \\ &= q^{(n-1)n/2}(q^2 - (-1)^2) \cdots (q^n - (-1)^n).\end{aligned}$$

By the same arguments as in (3.vii), we conclude that $|G|_p > |M(G)|_p$.

So, we conclude that $|G|_p > |M(G)|_p$. This completes the proof of the theorem. □

THEOREM 2.2. *Let G be a non-abelian simple group, and let $p \in \pi(G)$. Then there exists an element x of G such that $|x^G|_p$ does not divide $|M(G)|$, where $M(G)$ denotes the Schur multiplier of G.*

Proof. By Lemma 1.5, there exists an element x of G such that $|x^G|_p = |G|_p$, and so by Theorem 2.1, we conclude that $|x^G|_p$ does not divide $|M(G)|$. This completes the proof. □

Definition 2.3. Let N be a normal subgroup of G, and let $p \in \pi(G)$. We say that N is a $K(p^e)$-subgroup of G if $|x^G|_p = p^e$ for every element x in $N \backslash Z(G)$, where e is a fixed integer and $e > 0$.

We say that G is a $K(p^e)$-group if $|x^G|_p = p^e$ for every noncentral element x of G, where e is a fixed integer and $e > 0$ (see [3, Definition]).

By Lemma 2 of [2], we obtain the following:

THEOREM 2.4. *Let N be a normal subgroup of G, and suppose that N is a $K(p^e)$-subgroup of G. Let M be a norma p'-subgroup of G with $M < N$. If $N/M \nsubseteq Z(G/M)$, then N/M is a $K(p^e)$-subgroup of G/M.*

By Lemma 4 of [2] and its proof, we obtain the following:

THEOREM 2.5. *Let N be a normal subgroup of G, and suppose that N is a $K(p^e)$-subgroup of G. Then $C_N(O_p(N))$ contains all p'-elements of N. Furthermore, if N is p-solvable, then N is p-nilpotent.*

Suppose that N is a normal non-solvable subgroup of G and suppose that an integer m divides $|x^G|$ for every $x \in N \backslash Z(N)$. Then m divides $|Z(N)|$ [2, Theorem 7]. By this result, we obtain the following:

THEOREM 2.6. *Let N be a normal subgroup of G, and suppose that N is a $K(p^e)$-subgroup of G. If N is non-solvable, then $p^e||Z(N)|$.*

THEOREM 2.7. *Let N be a normal subgroup of G, and suppose that N is a $K(p^e)$-subgroup of G. Then, N is solvable.*

Proof. Suppose that the theorem does not hold, and let N be a counterexample of minimal order. We proceed by the following series of steps.

STEP 1. Let K be a normal subgroup of G such that $K < N$. Then K is solvable.

Clearly, either $K \subseteq Z(G)$ or K is a $K(p^e)$-subgroup of G, and so K is solvable by minimality of N.

Step 2. Any Sylow subgroup of N is not a direct factor of N.

Suppose that a Sylow q-subgroup Q of N is a direct factor of N. Then, we have $N = K \times Q$, where K is a q-complement of N. Clearly, $K \trianglelefteq G$ and $K < N$, and so K is solvable by Step 1. As a group of prime-power order, Q is solvable. It follows that $N = K \times Q$ is solvable, a contradiction.

Step 3. $p||N/Z(N)|$.

By Theorem 2.6, we have $p||N|$. It follows by Step 2 that $p|N/Z(N)|$.

Step 4. $O_{p'}(N) = 1$. In particular, $F(N)_{p'} = 1$.

Let P_0 be a Sylow p-subgroup of N, and let P be a Sylow p-subgroup of G such that $P_0 \leq P$. Suppose that $O_{p'}(N) \neq 1$. Then, by Theorem 2.4, we know that either $N/O_{p'}(N) \subseteq Z(G/O_{p'}(N))$ or $N/O_{p'}(N)$ is a $K(p^e)$-subgroup of $G/O_{p'}(N)$. If $N/O_{p'}(N) \subseteq Z(G/O_{p'}(N))$, then $[P_0, P] \leq O_{p'}(N)$, and so $P_0 \leq Z(P)$. This implies $P_0 \leq Z(G)$ as $e > 0$, and thus P_0 is a direct factor of N, contradicting Step 2. It follows that $N/O_{p'}(N)$ is a $K(p^e)$-subgroup of $G/O_{p'}(N)$. Then, $N/O_{p'}(N)$ is solvable by minimality of N.

We have that $O_{p'}(N) \trianglelefteq G$ and $O_{p'}(N) < N$ (see Theorem 2.6), and so $O_{p'}(N)$ is solvable by Step 1. Therefore, both $N/O_{p'}(N)$ and $O_{p'}(N)$ are solvable, and thus N is solvable, a contradiction. So, $O_{p'}(N) = 1$.

Step 5. $F(N) = Z(N)$.

Suppose that $Z(N)_p < O_p(N)$. Then, $C_N(O_p(N)) < N$. Clearly, we have $C_N(O_p(N)) \trianglelefteq G$. Then, $C_N(O_p(N))$ is solvable by Step 1. By Theorem 2.5 $|N : C_N(O_p(N))|$ is a p-number. It follows that $N/C_N(O_p(N))$ is solvable. Then, since $C_N(O_p(N))$ is solvable, N is solvable, a contradiction.

So, by Step 4, we have proved $F(N) = Z(N)$.

Step 6. If K is a normal subgroup of G such that $K \leq N$, then either $K = N$ or $K \leq Z(N)$. In particular, noting that N is non-solvable, $N/Z(N)$ is a non-abelian chief factor of G and N is perfect.

Suppose $K < N$. K is solvable by Step 1. We have $F(K) \leq F(N)$. Then, by Step 5, we have $F(K) = Z(K)$. It follows that $K = C_K(Z(K)) = C_K(F(K)) \leq F(K) = Z(K)$ (see [8, 4.2 Satz, p. 277]). And so $K = Z(K) = F(K) \leq F(N) = Z(N)$.

Step 7. The final contradiction.

By Step 6, $N/Z(N)$ is a non-abelian chief factor of G. Then, we have

$$N/Z(N) = L_1/Z(N) \times \cdots \times L_k/Z(N),$$

where $L_i/Z(N)$ are non-abelian simple groups and isomorphic (see [13, 8.9 Lemma (Remak), p. 169]).

Write $S = L_1/Z(N)$. Then, S is a non-abelian simple group. Clearly, we have $Z(N) = Z(L_1)$, and so $S = L_1/Z(N) = L_1/Z(L_1)$. N is perfect by Step 6, and so L_1 is perfect by the above paragraph. Therefore, $Z(N)(= Z(L_1))$ is isomorphic to a subgroup of the Schur multiplier $M(S)$ of $S(= L_1/Z(N) = L_1/Z(L_1))$ (see [10, (11.20) Corollary, p. 186]). Then, by Theorem 2.6, we have $p^e||M(S)|$.

Notice that since $L_i/Z(N)$ are isomorphic, by Step 3 we have $p||S| = |L_1/Z(N)|$. By hypothesis and Lemma 1.2, we conclude that the p-part of the conjugacy class size of every element of S divides p^e. Then, since $p^e||M(S)|$, the p-part of the conjugacy class size of every element of S divides $|M(S)|$; But this is impossible by Theorem 2.2.

So, we conclude that N is solvable. This completes the proof of the theorem. □

By Theorem 2.7 and Theorem 2.5, we obtain the following Theorem 2.8; it is Theorem A mentioned in Section 1.

THEOREM 2.8. *Let N be a normal subgroup of G, and suppose that N is a $K(p^e)$-subgroup of G. Then, N is solvable and p-nilpotent.*

The following Corollary 2.9 contains Theorem A of [3].

COROLLARY 2.9. *Let G be a $K(p^e)$-group, and let P be a Sylow p-subgroup of G. Then, G is solvable and has a norma p-complement [3, Theorem A]. Furthermore, P has nilpotency class at most 3 and $P/Z(P)$ has exponent p.*

Proof. By Theorem 2.8 (taking $N = G$), G is p-nilpotent. Then, G has a normal p-complement H and $G = PH$. It follows that $P \cong G/H$. G/H is non-abelian; otherwise, P is abelian, contradicting $e > 0$. Then, by Theorem 2.4 we conclude that P is a $K(p^e)$-group. This means that $\{|x^P| : x \in P\} = \{1, p^e\}$, and thus by Lemma 1.4, we conclude that P has nilpotency class at most 3 and $P/Z(P)$ has exponent p. This completes the proof. □

COROLLARY 2.10 ([1, Theorem A]). *If N is a normal subgroup of a group G and the size of any G-class of N is 1 or m, for some integer m, then N is nilpotent. More precisely, N is abelian or N is the direct product of a non-abelian p-group P by a central subgroup of G. In this case, $P/(Z(G) \cap P)$ has exponent p.*

Proof. Let $q \in \pi(N)$, and let Q be a Sylow q-subgroup of N. If $q \notin \pi(m)$, then by Lemma 1.2 and [9, 33.4 Theorem, p. 444], we conclude that $Q \leq Z(N)$. So, in order to prove that N is nilpotent, we may assume that $\pi(N) \subseteq \pi(m)$.

Then, by Theorem 2.8 we conclude that N is nilpotent. Thus, N is a direct product of its Sylow subgroups.

In order to complete the proof, without loss we may assume that $\pi(N) = \{p, q\}$ and $N = P \times Q$, where P and Q are a Sylow p-subgroup and a Sylow q-subgroup of N, respectively.

We assume that $P \nsubseteq Z(G)$ and take an element $x \in P \backslash Z(G)$. Let y be any element in $Q \backslash Z(G)$. We have that $C_G(xy) = C_G(x) \cap C_G(y)$. Hence, by hypothesis we have $C_G(xy) = C_G(x) = C_G(y)$. Then, we conclude that $y \in Z(C_G(x))$. This means that $Q \subseteq Z(C_G(x))$ because y is any element in $Q \backslash Z(G)$, and thus Q is abelian.

If $Q \nsubseteq Z(G)$, then by the same arguments, we conclude that P is abelian.

This has proved that either N is abelian or N is the direct product of a non-abelian p-group P by a central subgroup of G. In the second case, P is a non-abelian normal p-subgroup of G and P has only two G-class sizes, and hence by [1, Theorem 11], we know that $P/(Z(G) \cap P)$, and in particular $P/Z(P)$, has exponent p. This completes the proof. $\square$

The following Corollary 2.11 contains Theorem A of [2].

Corollary 2.11. *Let N be a normal subgroup of G, such that $\{|x^G| : x \in N\} = \{1, m, mq^b\}$, with q a prime and $(m, q) = 1$. Then N is solvable [2, Theorem A]. Furthermore, N has a norma Sylow q-subgroup Q (including $Q = 1$, that is, $q \nmid |N|$) and a nilpotent q-complement.*

Proof. Let $r \in \pi(N) \backslash (\pi(m) \cup \{q\})$, and let R be a Sylow r-subgroup of N. Then, by Lemma 1.2 and [9, 33.4 Theorem, p. 444], we conclude that $R \leq Z(N)$. Hence, without loss we may assume that $\pi(N) \subseteq \pi(m) \cup \{q\}$. Then, by Theorem 2.8 we conclude that N is solvable and has a normal Sylow q-subgroup Q and a nilpotent q-complement. This completes the proof. $\square$

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