Dedicated to Professor Octav Onicescu,
founder of the Bucharest School of Probability

LAPLACE TYPE PROBLEMS FOR A DELONE LATTICE AND NON-UNIFORM DISTRIBUTIONS

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This work considers a Delone lattice with the fundamental cell represented in figure 1. The probability is determined that a constant-length segment with a random exponential distribution direction and $\gamma(2)$ will intersect a side of the lattice.

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1. PRELIMINARIES

Let $\Re(a,\alpha)$ with $\frac{\pi}{4} \leq \alpha \leq \frac{\pi}{3}$ be a Delone lattice with the fundamental cell C_0 represented in Fig. 1.

This relation leads to the following other relations.

$$\widehat{BAD} = \frac{\pi}{2} - \alpha, \qquad \widehat{DGE} = \widehat{DGF} = \pi - \alpha,$$

$$\widehat{BAC} = \pi - 2\alpha, \qquad \widehat{EGF} = 2\alpha. \tag{1}$$

$$|BD| = |CD| = 2a\cos\alpha, \qquad |EG| = |FG| = a\,\mathrm{ctg}\,\alpha,$$

$$|AG| = \frac{a}{\sin\alpha}, \qquad |DG| = 2a\sin\alpha - \frac{a}{\sin\alpha}. \tag{2}$$

$$\mathrm{area}\, C_0 = \frac{a^2}{2}\sin 2\alpha. \tag{3}$$

We want to determine the probability that a segment s with a constant length l with $l < \frac{a\sqrt{3}}{6}$ and with a random non-uniform distribution length, will intersect a side of the lattice $\Re$, *i.e.* the probability P_{int} that the segment s will intersect a side of the fundamental cell C_0.

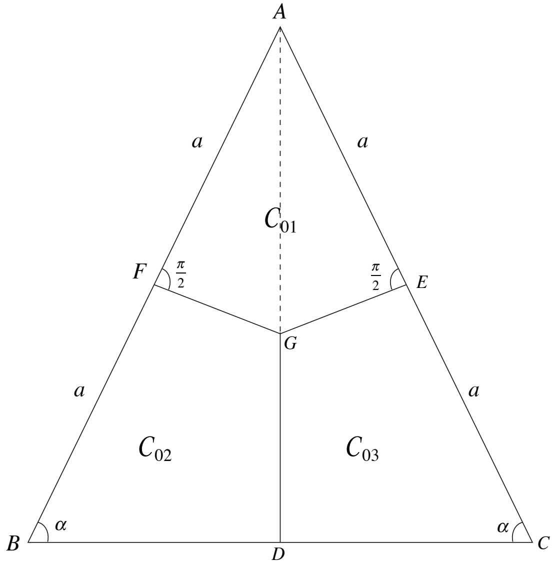

Fig. 1

2. MAIN RESULTS

Let φ be the angle between the segment s and the line BC. By considering the limit positions of the segment s in the cell C_0 for a given φ, we obtain Fig. 2.

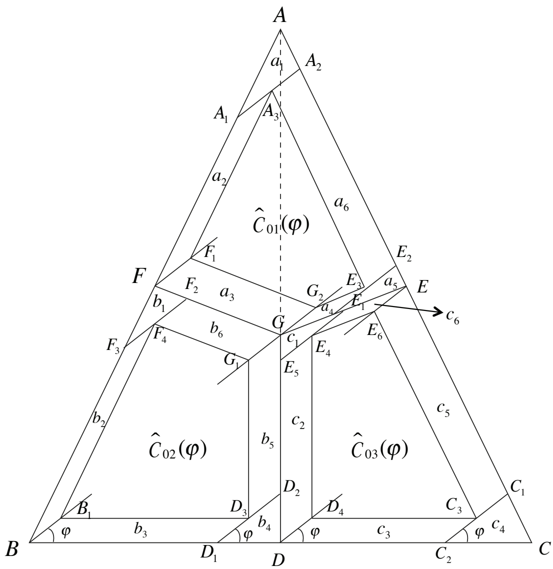

Fig. 2

And the formulas

$$\text{area}\,\widehat{C}_{01}(\varphi) = \text{area}\,C_{01} - \sum_{i=1}^{6} \text{area}\,a_i(\varphi), \tag{4}$$

$$\text{area}\,\widehat{C}_{02}(\varphi) = \text{area}\,C_{02} - \sum_{i=1}^{6} \text{area}\,b_i(\varphi), \tag{5}$$

$$\text{area}\,\widehat{C}_{03}(\varphi) = \text{area}\,C_{03} - \sum_{i=1}^{6} \text{area}\,c_i(\varphi). \tag{6}$$

Figure 2 and formula (1) lead to

$$\widehat{A_1AA_2} = \pi - 2\alpha, \qquad \widehat{AA_1A_2} = \widehat{F_3BB_1} = \alpha - \varphi, \qquad \widehat{AA_2A_1} = \varphi + \alpha. \tag{7}$$

With these angles, the triangle AA_1A_2 leads to

$$|AA_1| = \frac{l\sin(\varphi+\alpha)}{\sin 2\alpha}, \qquad |AA_2| = \frac{l\sin(\alpha-\varphi)}{\sin 2\alpha}, \tag{8}$$

therefore

$$\text{area}\,a_1(\varphi) = \frac{l^2\sin(\alpha-\varphi)\sin(\varphi+\alpha)}{2\sin 2\alpha}. \tag{9}$$

Moreover, from Fig. 2 we obtain

$$\varphi \in [0,\alpha]. \tag{10}$$

Figure 2 and relations (7) and (8) lead to

$$h_2 = \frac{l}{2}\sin(\alpha-\varphi), \qquad |A_1F| = a - |AA_1| = a - \frac{l\sin(\varphi+\alpha)}{\sin 2\alpha},$$

therefore

$$\text{area}\,a_2(\varphi) = \frac{al}{2}\sin(\alpha-\varphi) - \frac{l^2\sin(\alpha-\varphi)\sin(\varphi+\alpha)}{2\sin 2\alpha}. \tag{11}$$

From Fig. 2 and formulas (2) and (7), we obtain

$$\widehat{F_1FG} = \frac{\pi}{2} - \widehat{AA_1A_2} = \frac{\pi}{2} - (\alpha-\varphi), \qquad h_3 = \frac{l}{2}\cos(\alpha-\varphi).$$

Therefore

$$\text{area}\,a_3(\varphi) = |FG|\,h_3 = \frac{al}{2}\text{ctg}\alpha\cos(\alpha-\varphi). \tag{12}$$

Similarly, figure 2 and relation (7) lead to

$$\widehat{EE_2E_1} = \pi - \widehat{AA_2A_1} = \pi - (\varphi+\alpha), \qquad \widehat{EE_1E_2} = \alpha + \varphi - \frac{\pi}{2}, \tag{13}$$

therefore

$$(14)\qquad |EE_1| = l\sin(\varphi+\alpha), \qquad |EE_2| = -l\cos(\varphi+\alpha).$$

Therefore

$$(15)\qquad \text{area}\, a_5(\varphi) = -\frac{l^2}{2}\sin(\varphi+\alpha)\cos(\varphi+\alpha)$$

LEMMA 1. *Taking into account relation (1) and (10) we obtain*

$$\cos(\varphi+\alpha) \le 0.$$

Figure 2 and formulas (2), (13) and (14) lead to

$$\widehat{E_4GG_2} = \widehat{EE_1E_2} = \varphi+\alpha-\frac{\pi}{2}, \qquad h_4 = \frac{l}{2}\sin\left(\widehat{E_4GG_2}\right) = -\frac{l}{2}\cos(\varphi+\alpha),$$

$$|GE_1| = |EG| - |EE_1| = a\text{ctg}\alpha - l\sin(\varphi+\alpha).$$

Therefore

$$(16)\qquad \text{area}\, a_4(\varphi) = -\frac{al}{2}\text{ctg}\alpha\cos(\varphi+\alpha) + \frac{l^2}{4}\sin 2(\varphi+\alpha).$$

Finally, Fig. 2 and formulas (7), (8) and (14) lead to

$$\widehat{A_3A_2E_2} = \pi - \widehat{AA_2A_1} = \pi - (\varphi+\alpha), \qquad h_6 = \frac{l}{2}\sin(\varphi+\alpha),$$

$$|A_2E_2| = a - |AA_2| - |EE_2| = a - \frac{l\sin(\alpha-\varphi)}{2\sin 2\alpha} + l\cos(\varphi+\alpha),$$

therefore

$$(17)\quad \text{area}\, a_6(\varphi) = \frac{al}{2}\sin(\varphi+\alpha) - \frac{l^2}{2}\sin(\varphi+\alpha)\left[\frac{\sin(\alpha-\varphi)}{\sin 2\alpha} - \cos(\varphi+\alpha)\right].$$

From formulas (9), (11), (12), (15), (16) and (17) we obtain

$$(18)\qquad A_1(\varphi) = \sum_{i=1}^{6}\text{area}\, a_i(\varphi) = al\sin(\varphi+\alpha) + \frac{l^2}{4}\text{ctg}2\alpha\left[1-\cos 2(\varphi+\alpha)\right].$$

By substituting this expression into (4), it follows that

$$(19)\qquad \text{area}\,\widehat{C}_0(\varphi) = \text{area}\, C_{01} - A_1(\varphi).$$

Figure 2 leads to

$$(20)\qquad \widehat{EF_3F_2} = \alpha - \varphi, \qquad \widehat{FF_2F_3} = \frac{\pi}{2} - (\alpha-\varphi)$$

therefore

$$(21)\qquad |FF_2| = l\sin(\alpha-\varphi), \qquad |FF_3| = l\cos(\alpha-\varphi).$$

Therefore

(22) $$\operatorname{area} b_1(\varphi) = \frac{l^2}{4}\sin 2(\alpha-\varphi).$$

Figure 2 and formulas (20) e (21) lead to

$$h_2' = \frac{l}{2}\sin(\alpha-\varphi), \qquad |BF_3| = a - |FF_3| = a - l\cos(\alpha-\varphi),$$

therefore

(23) $$\operatorname{area} b_2(\varphi) = \frac{al}{2}\sin(\alpha-\varphi) - \frac{l^2}{4}\sin 2(\alpha-\varphi).$$

Similarly, we obtain

(24) $$|DD_1| = l\cos\varphi, \qquad |DD_2| = l\sin\varphi,$$

therefore

(25) $$\operatorname{area} b_4(\varphi) = \frac{l^2}{4}\sin 2\varphi.$$

From Fig. 2 and relations (2) and (24), it follows that

$$h_3' = \frac{l}{2}\sin(\varphi), \quad |BD_1| = |BD| - |DD_1| = 2a\cos\alpha - l\cos\varphi.$$

Therefore

(26) $$\operatorname{area} b_3(\varphi) = al\cos\alpha\sin\varphi - \frac{l^2}{4}\sin 2\varphi.$$

Figure 2 and formulas (2) e (24) lead to

$$h_5' = \frac{l}{2}\cos\varphi, \quad |D_2G| = |DG| - |DD_2| = 2a\sin\alpha - \frac{a}{\sin\alpha} - l\sin\varphi,$$

that is

(27) $$\operatorname{area} b_5(\varphi) = al\sin\alpha\cos\varphi - \frac{al}{2\sin\alpha}\cos\varphi - \frac{l^2}{4}\sin 2\varphi.$$

Finally, from Fig. 2 and relations (2) and (21), it follows that

$$h_6' = \frac{l}{2}\cos(\alpha-\varphi), \ |F_2G| = |FG| - |FF_2| = a\mathrm{ctg}\alpha - l\sin(\alpha-\varphi),$$

therefore

(28) $$\operatorname{area} b_6(\varphi) = \frac{al}{2}\mathrm{ctg}\alpha\cos(\alpha-\varphi) - \frac{l^2}{4}\sin 2(\alpha-\varphi).$$

Taking into account formulas (22), (23), (25)–(27) and (28), we obtain

(29)
$$A_2(\varphi) = \sum_{i=1}^{6}\operatorname{area} b_i(\varphi) = al\sin(\varphi+\alpha) - \frac{l^2}{4}\left[\sin 2\alpha\cos 2\varphi + (1-\cos 2\alpha)\sin 2\varphi\right].$$

By substituting this expression into (5), it follows that

$$\operatorname{area} \widehat{C}_{02}(\varphi) = \operatorname{area} C_{01} - A_2(\varphi). \tag{30}$$

From Fig. 2 we obtain

$$\widehat{E_1GE_5} = \pi - \alpha, \qquad \widehat{E_1E_5G} = \frac{\pi}{2} - \varphi, \qquad \widehat{E_5E_1G} = \varphi + \alpha - \frac{\pi}{2}. \tag{31}$$

With these angles, from triangle E_1E_5G it follows that

$$|E_1G| = \frac{l\cos\varphi}{\sin\alpha}, \qquad |E_5G| = \frac{l\cos(\varphi+\alpha)}{\sin\alpha}. \tag{32}$$

Therefore

$$\operatorname{area} c_1(\varphi) = -\frac{l^2\cos\varphi\cos(\varphi+\alpha)}{2\sin\alpha}. \tag{33}$$

Taking into account Fig. 2 and formulas (2) and (32), we obtain

$$h_2'' = \frac{l}{2}\cos\varphi, \quad |DE_5| = |DG| - |E_5G| = 2a\sin\alpha - \frac{a}{\sin\alpha} + \frac{l\cos(\varphi+\alpha)}{\sin\alpha},$$

therefore

$$\operatorname{area} c_2(\varphi) = al\sin\alpha\cos\varphi - \frac{al}{2\sin\alpha}\cos\varphi + \frac{l^2\cos\varphi\cos(\varphi+\alpha)}{2\sin\alpha}. \tag{34}$$

Figure 2 leads to

$$|CC_1| = \frac{l\sin\varphi}{\sin\alpha}, \qquad |CC_2| = \frac{l\sin(\varphi+\alpha)}{\sin\alpha}. \tag{35}$$

Therefore

$$\operatorname{area} c_4(\varphi) = \frac{l^2\sin\varphi\sin(\varphi+\alpha)}{2\sin\alpha}. \tag{36}$$

From Fig. 2 and relations (2) and (35), it follows that

$$h_3'' = \frac{l}{2}\sin\varphi, \qquad |C_2D| = |CD| - |CC_2| = 2a\cos\alpha - \frac{l\sin(\varphi+\alpha)}{\sin\alpha},$$

therefore

$$\operatorname{area} c_3(\varphi) = al\cos\alpha\sin\varphi - \frac{l^2\sin\varphi\sin(\varphi+\alpha)}{2\sin\alpha}. \tag{37}$$

Figure 2 and formula (35) lead to

$$\widehat{E_6C_3C_1} = \pi - (\varphi+\alpha), \qquad h_5'' = \frac{l}{2}\sin(\varphi+\alpha), \qquad |C_1E| = a - |CC_1| = a - \frac{l\sin\varphi}{\sin\alpha},$$

that is

$$\operatorname{area} c_5 = \frac{al}{2}\sin(\varphi+\alpha) - \frac{l^2\sin\varphi\sin(\varphi+\alpha)}{2\sin\alpha}. \tag{38}$$

Finally, taking into account Fig. 2 and relations (2) and (32), we obtain

$$\text{area}\, c_6 = -\frac{al\text{ctg}\alpha}{2}\cos(\varphi+\alpha) + \frac{l^2\cos\varphi\cos(\varphi+\alpha)}{2\sin\alpha}. \tag{39}$$

From formulas (33), (34), (36)–(38) and (39), it follows that

$$A_3(\varphi) = \sum_{i=1}^{6} \text{area}\, c_i(\varphi) = 2al(\cos\alpha\sin\varphi + \sin\alpha\cos\varphi) - \frac{al}{\sin\alpha}\cos\varphi - \frac{l^2}{2}(\sin 2\varphi - \text{ctg}\alpha\cos 2\varphi). \tag{40}$$

By substituting this expression into (6), we obtain

$$\text{area}\, \widehat{C}_{03}(\varphi) = \text{area}\, C_{03} - A_3(\varphi). \tag{41}$$

Let M_i $(i = 1, 2, 3)$ be the set of segments s the midpoint of which is in cell C_{0i} and let N_i be the set of segments s that are entirely contained in cell C_{0i}. We obtain [3]:

$$P_{int} = 1 - \frac{\sum_{i=1}^{3}\mu(N_i)}{\sum_{i=1}^{3}\mu(M_i)}, \tag{42}$$

where μ is the Lebesgue measure of the Euclidean plane.

Measures $\mu(M_i)$ and $\mu(N_i)$ are determined by using the Poincaré kinematic measure [2]:

$$\mathrm{d}k = \mathrm{d}x \wedge \mathrm{d}y \wedge \mathrm{d}\varphi,$$

where x, y are the coordinates of the midpoint of s and φ is the angle defined above.

Let us assume that the direction of the support line of s is a random variable with a probability density of $f(\varphi)$. Taking into account formula (10), we obtain

$$\mu(M_i) = \int_0^{\alpha} f(\varphi)\,\mathrm{d}\varphi \iint_{\{(\varkappa,y)\in C_{0i}\}} \mathrm{d}x\mathrm{d}y = \int_0^{\alpha} (\text{area}\, C_{0i})\, f(\varphi)\,\mathrm{d}\varphi = (\text{area}\, C_{0i}) \int_0^{\alpha} f(\varphi)\,\mathrm{d}\varphi \tag{43}$$

and taking into account formulas (19), (30) and (49), we obtain

$$\mu(N_i) = \int_0^{\pi} f(\varphi)\,\mathrm{d}\varphi \iint_{\{(\varkappa,y)\in \widehat{C}_{0i}(\varphi)\}} \mathrm{d}x\mathrm{d}y = \int_0^{\alpha} \left[\text{area}\, \widehat{C}_{0i}(\varphi)\right] f(\varphi)$$

$$= \int_0^\alpha \left[\text{area}\, C_{0i} - A_i(\varphi)\right] f(\varphi)\,\mathrm{d}\varphi = (\text{area}\, C_{0i}) \int_0^\alpha f(\varphi)\,\mathrm{d}\varphi - \int_0^\alpha A_i(\varphi) f(\varphi)\,\mathrm{d}\varphi.$$

These formulas lead to

$$\sum_{i=1}^{3} \mu(M_i) = (\text{area}\, C_0) \int_0^\alpha f(\varphi)\,\mathrm{d}\varphi,$$

$$\sum_{i=1}^{3} \mu(N_i) = (\text{area}\, C_0) \int_0^\alpha f(\varphi)\,\mathrm{d}\varphi - \int_0^\alpha \left[\sum_{i=1}^{3} A_i(\varphi)\right] f(\varphi)\,\mathrm{d}\varphi.$$

By substituting these expressions into (42), we obtain

$$P_{int} = \frac{1}{(\text{area}\, C_0) \int_0^\alpha f(\varphi)\,\mathrm{d}\varphi} \int_0^\alpha \left[\sum_{i=1}^{3} A_i(\varphi)\right] f(\varphi)\,\mathrm{d}\varphi. \tag{44}$$

From formulas (18), (29) and (40), it follows that

$$\sum_{i=1}^{3} A_i(\varphi) = 4al(\sin\alpha\cos\varphi + \cos\alpha\sin\varphi) - \frac{al}{\sin\alpha}\cos\varphi - \frac{l^2}{4}$$
$$\left[(\sin 2\alpha + \text{ctg}2\alpha - 2\text{ctg}\alpha)\cos 2\varphi + (3 - 2\cos 2\alpha)\sin 2\varphi - \text{ctg}2\alpha(1 - \cos 2\alpha)\right].$$

With this value (3), relation (44) can be written as

$$P_{int} = \frac{1}{(a^2 \sin 2\alpha) \int_0^\alpha f(\varphi)\,\mathrm{d}\varphi} \int_0^\alpha \{4a(\sin\alpha\cos\varphi + \cos\alpha\sin\varphi) - \frac{a}{\sin\alpha}\cos\varphi - \frac{l}{4}\left[(\sin 2\alpha + \text{ctg}2\alpha - 2\text{ctg}\alpha)\cos 2\varphi + (3 - 2\cos 2\alpha)\sin 2\varphi - \text{ctg}2\alpha(1 - \cos 2\alpha)\right]\} f(\varphi)\,\mathrm{d}\varphi. \tag{45}$$

2.1. **Exponential distribution**

We have

$$f(\varphi) = \mathrm{e}^{-\varphi}.$$

In a previous paper [1], we demonstrated the following formulas:

$$\int_0^\alpha f(\varphi)\,\mathrm{d}\varphi = 1 - \mathrm{e}^{-\varphi},$$

$$\int_0^\alpha f(\varphi)\sin\varphi d\varphi = \frac{1}{2} - \frac{1}{2}(\sin\alpha + \cos\alpha)\,e^{-\alpha},$$

$$\int_0^\alpha f(\varphi)\cos\varphi d\varphi = \frac{1}{2} + \frac{1}{2}(\sin\alpha - \cos\alpha)\,e^{-\alpha},$$

$$\int_0^\alpha f(\varphi)\sin 2\varphi d\varphi = \frac{1}{5}\left[2 - (\sin 2\alpha + 2\cos 2\alpha)\,e^{-\alpha}\right],$$

$$\int_0^\alpha f(\varphi)\cos 2\varphi d\varphi = \frac{1}{5}\left[1 + (2\sin 2\alpha - \cos 2\alpha)\,e^{-\alpha}\right].$$

By substituting these values into relation (45), we obtain

$$P_{int} = \frac{2l}{(a^2\sin 2\alpha)(1-e^{-\alpha})}$$

$$\Big\{2a\left[\sin\alpha + \cos\alpha - (\sin 2\alpha + \cos 2\alpha)\,e^{-\alpha}\right] - \frac{a}{2\sin\alpha}\left[1 + (\sin\alpha - \cos\alpha)\,e^{-\alpha}\right]$$
$$-\frac{l}{20}\left[6 + \sin 2\alpha - 4\cos 2\alpha - 4\mathrm{ctg}2\alpha - 2\mathrm{ctg}\alpha + 5\mathrm{ctg}2\alpha\cos 2\alpha\right.$$
$$+ e^{-\alpha}\left(2 + \sin 2\alpha\cos 2\alpha + 2\sin^2 2\alpha - 3\sin 2\alpha - 4\cos 2\alpha\right.$$
$$\left.\left. +5\mathrm{ctg}2\alpha + 2\mathrm{ctg}\alpha\cos 2\alpha - 6\mathrm{ctg}2\alpha\cos 2\alpha\right)\right]\Big\}.$$

2.2. **Demonstration** γ

We have

$$f(\varphi) = \varphi e^{-\varphi}.$$

In a previous paper [1], we demonstrated the following formulas:

$$\int_0^\alpha f(\varphi)\,d\varphi = 1 - (1+\alpha)\,e^{-\alpha},$$

$$\int_0^\alpha f(\varphi)\sin\varphi d\varphi = \frac{1}{2} - \frac{1}{2}e^{-\alpha}\cos\alpha - \frac{\alpha}{2}e^{-\alpha}(\sin\alpha + \cos\alpha),$$

$$\int_0^\alpha f(\varphi)\cos\varphi d\varphi = \frac{1}{2}e^{-\alpha}(\sin\alpha + \cos\alpha) + \frac{\alpha}{2}e^{-\alpha}(\sin\alpha - \cos\alpha),$$

$$\int_0^\alpha f(\varphi)\sin 2\varphi d\varphi = \frac{4}{5} + \frac{1}{25}e^{-\alpha}(3\sin 2\alpha + 8\cos 2\alpha) + \alpha e^{-\alpha}(\sin 2\alpha - 2\cos 2\alpha),$$

$$\int_0^\alpha f(\varphi)\cos 2\varphi \mathrm{d}\varphi = \frac{1}{5} + \frac{1}{25}\mathrm{e}^{-\alpha}(4\sin 2\alpha + 9\cos 2\alpha) + \alpha\mathrm{e}^{-\alpha}(\sin 2\alpha - \cos 2\alpha).$$

By substituting these values into relation (45), we obtain

$$P_{int} = \frac{2l}{a^2 \sin 2\alpha \left[1 - (1+\alpha)\mathrm{e}^{-\alpha}\right]}$$
$$\Bigg(a\left[\mathrm{e}^{-\alpha}(\sin 2\alpha - 2\cos 2\alpha - \frac{1}{2}\mathrm{ctg}\alpha - \frac{1}{2}) - \alpha\mathrm{e}^{-\alpha}(2\sin 2\alpha + 2\cos 2\alpha\right.$$
$$\left.-\frac{1}{2}\mathrm{ctg}\alpha + \frac{1}{2}) + 2\cos\alpha\right] - \frac{l}{4}\left\{\frac{12}{5} + \frac{1}{5}\sin 2\alpha - \frac{8}{5}\cos 2\alpha - \frac{4}{5}\mathrm{ctg}2\alpha\right.$$
$$-\frac{2}{5}\mathrm{ctg}\alpha + \cos 2\alpha\mathrm{ctg}2\alpha + \mathrm{e}^{-\alpha}\left[\frac{9}{25}\sin 2\alpha + \frac{28}{25}\cos 2\alpha + \frac{3}{25}\sin 2\alpha\cos 2\alpha\right.$$
$$-\frac{16}{25}\cos^2\alpha - \frac{16}{25}\cos^2 2\alpha + \frac{4}{25}\sin^2 2\alpha + \frac{9}{25}\cos 2\alpha\mathrm{ctg}2\alpha$$
$$\left.-\frac{18}{25}\cos 2\alpha\mathrm{ctg}\alpha + (1+\alpha)(1-\cos 2\alpha)\mathrm{ctg}2\alpha\right] + \alpha\mathrm{e}^{-\alpha}\left(\sin^2 2\alpha\right.$$
$$\left.\left.-3\sin 2\alpha\cos 2\alpha + 3\sin 2\alpha - 5\cos 2\alpha - \cos 2\alpha\mathrm{ctg}2\alpha + 2\cos 2\alpha\mathrm{ctg}\alpha)\right\}\right).$$

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