Dedicated to Vasile Brînzănescu on his 75^{th} birthday

INDECOMPOSABLE FILTRABLE VECTOR BUNDLES ON OELJEKLAUS-TOMA MANIFOLDS

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In this short note, we prove that there are finitely many indecomposable filtrable bundles with trivial determinant and vanishing second Chern class on Oeljeklaus-Toma manifolds that satisfy certain conditions. With few precise exceptions, they are simple bundles.

AMS 2010 Subject Classification: 32J18; 32L10; 58H15.

Key words: Oeljeklaus-Toma manifold, flat line bundle, deformation, non-filtrable vector bundle.

1. INTRODUCTION

Let X be a complex manifold and E a holomorphic rank-r vector bundle on X. The bundle E is called *filtrable* if there exists an ascending chain of coherent sheaves

$$0 = \mathcal{F}_0 \subset \mathcal{F}_1 \subset \cdots \subset \mathcal{F}_{r-1} \subset \mathcal{F}_r = E$$

with $\mathrm{rk}(\mathcal{F}_i) = i$ for any $i = 0, \dots, r$. The bundle E is called *reducible* if there exists a subsheaf $0 \neq \mathcal{F} \subset E$ with $\mathrm{rk}(\mathcal{F}) < \mathrm{rk}(E)$, and in rank-two the two notions coincide.

If X is projective (or more generally if X is algebraic) it is well-known that any vector bundle is filtrable. By way of contrast, if X is non-algebraic, there may exists non-filtrable vector bundles on X. One of the first examples was given by Elencwajg-Forster on two-dimensional tori X with $\mathrm{NS}(X) = 0$ [6]. The next example, found by Schuster [12], is the tangent bundle of a K3-surface X with $\mathrm{Pic}(X) = 0$. For 2-dimensional tori, Brînzănescu-Flondor found a necessary numerical criterion for the existence of filtrable vector bundles [2], and this criterion was extended by Bănică-Le Potier for arbitrary non-algebraic surfaces [5]. We refer to [3], [4] for extensive surveys on the subject and a comprehensive literature.

The second-named author is partially supported by the Project PN-III-P4-ID-PCE-2016-0065.

In this note we focus on *Oeljeklaus-Toma manifolds* (short, OT manifolds). Their class provides a fertile ground for testing various statements concerning non-algebraic manifolds, and they are relevant for the theory of vector bundles. In the main result Theorem 1 we prove that, under certain conditions which are satisfied for infinitely many OT manifolds and in arbitrarily high dimension, there are only finitely many isomorphism classes of indecomposable filtrable vector bundles of rank two with trivial determinant and vanishing second Chern class. In the process, we prove a result on the first cohomology of line bundles over such manifolds, see Proposition 2.1. This Proposition also gives a precise classification, in terms of extensions of all the indecomposable filtrable vector bundles of rank two with trivial determinant and vanishing second Chern class. Finally, we obtain a precise description of the simple ones.

2. OELJEKLAUS-TOMA MANIFOLDS AND LINE BUNDLES

2.1. Oeljeklaus-Toma manifolds

We recollect the definition and some basic properties of Oeljeklaus-Toma manifolds (OT, for short). The basic reference is the original paper of K. Oeljeklaus and M. Toma [8].

Let K be an algebraic number field; K admits $n = s + 2t$ embeddings in $\mathbb{C}$, more precisely, s real embeddings $\sigma_1, \ldots, \sigma_s \colon K \to \mathbb{R}$, and $2t$ complex embeddings $\sigma_{s+1}, \ldots, \sigma_{s+t}, \sigma_{s+t+1} = \overline{\sigma}_{s+1}, \ldots, \sigma_{s+2t} = \overline{\sigma}_{s+t} \colon K \to \mathbb{C}$. Note that, for any choice of natural numbers s and t, there is an algebraic number field with s real embeddings and $2t$ complex embeddings [8, Remark 1.1].

Denote by $\mathcal{O}_K$ the ring of algebraic integers of $K$and by $\mathcal{O}_K^*$ the multiplicative group of units of $\mathcal{O}_K$, namely, invertible elements in $\mathcal{O}_K$. Denote by $\mathcal{O}_K^{*,+}$ the subgroup of finite index of $\mathcal{O}_K^*$ whose elements are totally positive units, namely, units being positive in any real embedding: $u \in \mathcal{O}_K^*$ such that $\sigma_j(u) > 0$ for any $j \in \{1, \ldots, s\}$.

Let $\mathbb{H} := \{z \in \mathbb{C} : \mathsf{Im}\, z > 0\}$ denote the upper half-plane. On $\mathbb{H}^s \times \mathbb{C}^t$, consider the following actions:

$$\begin{gathered} T \colon \mathcal{O}_K \circlearrowleft \mathbb{H}^s \times \mathbb{C}^t, \\ T_a(w_1, \ldots, w_s, z_{s+1}, \ldots, z_{s+t}) := (w_1 + \sigma_1(a), \ldots, z_{s+t} + \sigma_{s+t}(a)), \end{gathered} \tag{1}$$

and

$$\begin{gathered} R \colon \mathcal{O}_K^{*,+} \circlearrowleft \mathbb{H}^s \times \mathbb{C}^t, \\ R_u(w_1, \ldots, w_s, z_{s+1}, \ldots, z_{s+t}) := (w_1 \cdot \sigma_1(u), \ldots, z_{s+t} \cdot \sigma_{s+t}(u)). \end{gathered} \tag{2}$$

For any subgroup $U \subset \mathcal{O}_K^{*,+}$, one has the fixed-point-free action $\mathcal{O}_K \rtimes U \circlearrowleft \mathbb{H}^s \times \mathbb{C}^t$. One can always choose an admissible subgroup [8, page 162], namely, a subgroup such that the above action is also properly discontinuous and cocompact. In particular, the rank of admissible subgroups must be s. Conversely, when either $s = 1$ or $t = 1$, every subgroup U of $\mathcal{O}_K^{*,+}$ of rank s is admissible.

One defines the *Oeljeklaus-Toma manifold* associated to the algebraic number field K (with $s > 0$ and $t > 0$) and to the admissible subgroup U of $\mathcal{O}_K^{*,+}$ as

$$X(K,U) := \mathbb{H}^s \times \mathbb{C}^t / \mathcal{O}_K \rtimes U.$$

The couple (s,t) will be referred as *the algebraic type* of X.

In particular, for an algebraic number field K with $s = 1$ real embeddings and $2t = 2$ complex embeddings, choosing $U = \mathcal{O}_K^{*,+}$ we obtain that $X(K,U)$ is an Inoue-Bombieri surface of type S_M.

The Oeljeklaus-Toma manifold $X(K,U)$ is called *of simple type* when there exists no proper intermediate field extension $\mathbb{Q} \subset K' \subset K$ with $U \subseteq \mathcal{O}_{K'}^{*,+}$.

The topology of OT manifolds is rather clear. Letting $\mathbb{T}^s$ for the real s-dimensional torus $\mathbb{T}^s = \left(S^1\right)^s$ one can see that an OT-manifold X of algebraic type (s,t) is a $\mathbb{T}^{s+2t}$-bundle over $\mathbb{T}^s$ (but not a principal bundle). Combined with the Leray spectral sequence, this leads to a clear description of the DeRham cohomology of X, see [7]. On the other hand, the torsion part of $H^i(X,\mathbb{Z})$ is also well understood, at least in the case $i = 2$. Indeed, one can show [1, Proposition 5], that the commutator $[\pi_1(X), \pi_1(X)]$ equals $J(U)$ where $J(U) \subset \mathcal{O}_K$ is the ideal generated by all the elements of the form $1-u$ with $u \in U$. In particular

$$\mathrm{Tors}\,(H_1(X,\mathbb{Z})) \simeq \mathcal{O}_K / J(U). \tag{3}$$

The analytic geometry of these manifolds is rather special. As they have algebraic dimension zero, there are at most finitely many divisors on X. In fact, much more has been shown: any OT manifolds contains no (closed) curves (see [13]), the OT manifolds of algebraic type $(s,1)$ contain no proper closed analytic subspaces [9], and the same holds good more generally, if any element u in the admissible group of units U is a primitive element [10].

2.2. **Line bundles on OT manifolds**

Since we will use some facts about line bundles and their Dolbeault cohomology on OT manifolds, we gather here the information needed. We shall restrict to the case when X is *of simple type* and such that $H_1(X,\mathbb{Z})$ is torsion-free, since this is the case we will further work on.

First, we notice that if X is an OT manifold of simple type of algebraic type (s,t) then, by [11, Corollary 4.6], respectively [11, Corollary 4.10], we have $h^1(X,\mathcal{O}_X)=s$ and respectively $h^2(X,\mathcal{O}_X)=\binom{s}{2}$. On the other hand, by [8, Proposition 2.3] we have that $b_1(X)=s$ and $b_2(X)=\binom{s}{2}$ hence the natural maps $H^1(X,\mathbb{C})\to H^1(X,\mathcal{O}_X)$ and $H^2(X,\mathbb{C})\to H^2(X,\mathcal{O}_X)$ are bijective. Since flat line bundles on X are parametrized by $H^1(X,\mathbb{C}^*)$ it follows that all line bundles on X are flat, i.e. induced by representations $\varrho:\pi_1(X)\to\mathbb{C}^*$.

PROPOSITION 2.1. *Let X be an OT manifold of simple type and such that $H_1(X,\mathbb{Z})$ is torsion free, let $\varrho:\pi_1(X)\to\mathbb{C}^*$ be a representation and L_ϱ the associated holomorphic line bundle on X. Then*

$$h^1(X,L_\varrho)=\begin{cases} s, & \text{if } \varrho\equiv 1;\\ 1, & \text{if } \varrho=(\overline{\sigma_i})^{-1} \text{ for some } i=s+1,\dots,s+t;\\ 0, & \text{otherwise.}\end{cases}$$

Proof. We will use the following well-known result by Mumford: if $\pi:\tilde{X}\to X$ is a cover of complex manifolds and $\mathcal{F}$ is a coherent sheaf of X such that $\pi^*(\mathcal{F})$ is acyclic on $\tilde{X}$ then one has a canonical isomorphism

$$H^i(X,\mathcal{F})\simeq H^i(\pi_1(X),H^0(\tilde{X},\pi^*(\mathcal{F}))$$

where the right-hand side stands for group cohomology.

Take X to be our OT manifold, $\tilde{X}=\mathbb{H}^s\times\mathbb{C}^t$ its universal cover and $\mathcal{F}=L_\varrho$. Then the above isomorphism works well, since $\tilde{X}$ is Stein. Notice that $\pi^*(L_\varrho)$ is in fact trivial, hence $H^0(\tilde{X},\pi^*(L_\varrho))$ identifies with the ring $\mathcal{R}$ of global holomorphic functions on $\mathbb{H}^s\times\mathbb{C}^t$. Notice that under the isomorphism $\pi*(L_\varrho)\simeq\mathcal{O}_{\tilde{X}}$ the structure of $\pi_1(X)-$module on $\mathcal{R}$ is given by

$$\gamma\cdot f:=\varrho(\gamma)\,(f\circ\gamma)\,,\forall\gamma\in\pi_1(X),f\in\mathcal{R}.$$

Let $X^{ab}:=\tilde{X}/\mathcal{O}_K$; then X^{ab} is a Galois cover of X with deck group U. Notice that X^{ab} is an open subset of the Cousin group $\mathbb{C}^{s+t}/\mathcal{O}_K$ whose preimage in $\mathbb{C}^{s+t}$ is convex, [11]. From the exact sequence of groups

$$1\to\mathcal{O}_K\to\pi_1(X)\to U\to 1$$

we get the Lyndon-Hochschild-Serre spectral sequence for group cohomology

$$E_2^{pq}=H^q(U,H^p(B,\mathcal{R})).$$

This induces the exact sequence

$$0\to H^1(U,\mathcal{R}^{\mathcal{O}_K})\to H^1(X,\mathcal{L}_\varrho)\to H^1(\mathcal{O}_K,\mathcal{R})^U\to H^2(U,\mathcal{R}^{\mathcal{O}_K}) \tag{4}$$

Next, notice that by the above result of Mumford one has $\mathcal{R}^{\mathcal{O}_K}\simeq H^0(X^{ab},p^*(\mathcal{L}_\varrho)))$ and $H^1(\mathcal{O}_K,\mathcal{R})\simeq H^1(X^{ab},p^*(\mathcal{L}_\varrho))$ (where $p:X^{ab}\to X$ is the canonical projection). Now $p^*(\mathcal{L}_\varrho)$ is the flat line bundle on X^{ab} corresponding to the representation $\varrho_{|\mathcal{O}_K}$; as $H_1(X,\mathbb{Z})$ is torsion-free, we see $\mathcal{O}_K$ is actually the commutator $[\pi_1(X),\pi_1(X)]$ hence $\varrho_{|\mathcal{O}_K}$ is trivial. We get that $p^*(\mathcal{L}_\varrho) \simeq \mathcal{O}_{X^{ab}}$. But $H^0(X^{ab},\mathcal{O}_{X^{ab}})$ consists of constants only, by [11, Proposition 2.3], so the above sequence (4) becomes

(5) $$0 \to H^1(U,\mathbb{C}) \to H^1(X,L_\varrho) \to H^1(X^{ab},\mathcal{O}_{X^{ab}})^U \to H^2(U,\mathbb{C}).$$

Note that by [11, Theorem 3.1], the set

$$\{d\overline{z}_i, i = s+1,\dots,s+t\}$$

is a basis for $H^1(X^{ab},\mathcal{O}_{X^{ab}})$.

We next investigate the possible situations for ϱ. The case when ϱ is trivial is already treated in [11, Corollary 4.6], hence we assume ϱ is nontrivial. By the description of the structure of $\pi_1(X)$-module of $\mathcal{R}$ we get that $H^1(U,\mathbb{C}) = 0$ since $u\cdot z = \varrho(u)z$ for any $u \in U$ and $z \in \mathbb{C}$ so if $\varrho(u) \neq 1$ for some $u \in U$ forces $z = 0$. One also has $H^2(U,\mathbb{C}) = 0$; this is immediate for U of rank one, and for higher ranks doing induction on rank using the Hochschild-Lyndon-Serre spectral sequence and using again the nontriviality of ϱ. So we are left with an isomorphism

$$H^1(X,L_\varrho) \to H^1(X^{ab},\mathcal{O}_{X^{ab}})^U.$$

Taking into account the description of the basis of $H^1(X^{ab},\mathcal{O}_{X^{ab}})$ we see that for any $u \in U$ one has

$$R_u^*(d\overline{z}_i) = \varrho(u)\overline{\sigma_i(u)}d\overline{z}_i$$

hence $d\overline{z}_i \in H^1(X^{ab},\mathcal{O}_{X^{ab}})^U$ if and only f $\varrho(u) = (\overline{\sigma_i})^{-1}(u)$ for any $u \in U$. Since X of simple type, it cannot happen that for two different i,j on has $(\overline{\sigma_i})^{-1}(u) = (\overline{\sigma_j})^{-1}(u)$ (for otherwise all $u \in U$ would live in a proper subfield L of K, namely in $L := \{x \in K | \sigma_i(x) = \sigma_j(x)\}$, and this would contradict the assumption that X is of simple type) so the subspace of invariants is at most one-dimensional, and this can happen only when $\varrho(u) = (\overline{\sigma_i})^{-1}(u)$ for any $u \in U$. Since $\mathcal{O}_K = [\pi_1(X),\pi_1(X)]$, this implies $\varrho = (\overline{\sigma_i})^{-1}$, as stated. □

Remark 2.2. Notice that the above result generalizes the case $(s,t) = (1,1)$, that is, the case of Inoue-Bomberi surfaces. Indeed, if X is such a surface and $L \in \mathrm{Pic}(X)$ then $H^1(X,L) \neq 0$ forces (by Riemann-Roch) that $H^0(X,L) \neq 0$ or $H^2(X,L) \neq 0$. Since X has no curves, in the first case we get $L = \mathcal{O}_X$ while in the second $L = K_X$. It is not hard to see that the canonical bundle K_X of X is the flat line bundle associated to $\sigma_1\sigma_2 = (\overline{\sigma_2})^{-1}$.

3. THE MAIN RESULTS

THEOREM 1. *Let $X = X(K,U)$ be an OT manifold of algebraic type (s,t) with $s = 1$. Assume that U is generated by a primitive element u_0 of K (in particular, X is of simple type) and that $H_1(X,\mathbb{Z})$ is torsion-free. Then X carries* *finitely many indecomposable filtrable rank-2 holomorphic vector bundles with trivial determinant and vanishing second Chern class.*

Proof. We start with the particular case $t = 1$, which is especially enlightening for the proof of the general case. In this case, X is an Inoue-Bombieri surface. If E is filtrable of rank 2 with vanishing Chern classes, then E sits in an extension of the form

$$0 \to L \to E \to L^\vee \otimes \mathcal{I}_Z \to 0 \tag{6}$$

But $c_2(E) = 0$ and $c_1(L) = 0$ as $b_2(X) = 0$. Taking the second Chern class in the above extension we get $\deg(Z) = h^0(Z, \mathcal{O}_Z) = 0$ and hence Z is empty. If E is is indecomposable, we must have $H^1(X, L^{\otimes 2}) \neq 0$; but this leads to $L^{\otimes 2} = \mathcal{O}_X$ or $L^{\otimes 2} = K_X$. Since $\mathrm{Pic}(X) \simeq \mathbb{C}^*$ we see there are exactly 4 possible choices for L. Noticing that in any of these cases one has in fact $\mathrm{Ext}^1(L^\vee, L) \simeq H^1(X, L^{\otimes 2}) \simeq \mathbb{C}$, we see that there are precisely 4 filtrable indecomposable rank-2 vector bundles E with trivial determinant.

We next move to the general case, $t \geq 2$. Recall from [11, Corollary 4.6] that in this case we have again $h^1(X, \mathcal{O}_X) = 1$. Note also that the second Betti number $b_2(X)$ vanishes. Indeed, by [7, Theorem 3.1], it suffices to show that for any multiindex I of length $|I| \leq 2$ it is impossible to find σ_I such that $\sigma_{I|U} = 1$. This is obvious if $|I| = 1$; now if $I = \{i, j\}$ is such that $\sigma_i(u_0)\sigma_j(u_0) = 1$ then it follows that the unit u_0 is palindromic (reciprocal), in particular its degree is odd. But this is a contradiction, since the degree of u_0 divides $[K : \mathbb{Q}] = 1 + 2t$. Hence $h^2(X, \mathbb{C}) = 0$. But on OT manifolds, the Hodge decomposition holds well by [11, Theorem 4.5.], and hence we obtain $h^2(X, \mathcal{O}_X) = 0$.

If E is a filtrable rank-two vector bundle with trivial determinant on X, it sits in a an extension of the form

$$0 \to L \to E \to L^\vee \otimes \mathcal{I}_Z \to 0 \tag{7}$$

with $Z \subset X$ a locally complete intersection of codimension two. Since $t > 1$ we get $\dim(X) \geq 3$, hence $\dim(Z) \geq 1$. Applying [10], we deduce that Z is empty, hence (7) becomes

$$0 \to L \to E \to L^\vee \to 0. \tag{8}$$

Since X has no divisors, we see that for any E there exists at most two possible line bundles L as above which are also dual one to another. Proposition 2.1 implies $h^1(X, L^{\otimes 2}) \leq 1$ for any $L \in \mathrm{Pic}(X)$ and $h^1(X, L^{\otimes 2}) = 0$ for all but finitely many L. In particular, except for finitely many L, the bundle given by the extension (7) is decomposable. Therefore, there are only finitely many indecomposable filtrable bundles E. □

Remark 3.1. The indecomposable filtrable bundles obtained in the proof Theorem 1 are simple with the exception of the unique non-split extension

$$0 \to \mathcal{O}_X \to E \to \mathcal{O}_X \to 0,$$

and its twists with a line bundle L with $L^{\otimes 2} \cong \mathcal{O}_X$.

Indeed, assume that the line bundle L defining the extension (8) satisfies $L^{\otimes 2} \ncong \mathcal{O}_X$. Recall that non-trivial line bundles on X have no global sections. By twisting (8) by L, we obtain $h^0(X, E \otimes L) = 0$, and twisting it with $L^\vee$ we get $h^0(X, E \otimes L^\vee) = 1$. Since $E \cong E^\vee$, the claim follows from (8) again twisted by E.

If L is trivial, it is clear that the unique non-trivial extension (8) admits endomorphisms that are not homotheties, e.g. the composition $E \to \mathcal{O}_X \hookrightarrow E$. In fact, one can prove that $h^0(X, E \otimes E^\vee) = 2$ in this case. Similarly if L is a square root of $\mathcal{O}_X$.

Even though the hypotheses of Theorem 1 are non–trivial, they are satisfied by a reasonably large class of OT manifolds.

PROPOSITION 3.2. *There exist OT manifolds of arbitrarily high dimension satisfying the assumptions of Theorem 1.*

Proof. Recall that an algebraic unit u is called *exceptional* $1 - u$ is an algebraic unit too. To prove the assertion stated in the Remark, using 3 we need to construct exceptional units of arbitrary high degree which are also totally positive. Consider the polynomial $P(X) = X^3 + X - 1$; it is immediate that P is irreducible and has a single real root $\alpha \in (0, 1)$. Because P is monic and as $P(0) = -1$ it follows that α is an algebraic unit. Since $Q(X) := -P(1 - X)$ is also monic and its free term is -1 we get that $1 - \alpha$ is also a unit, so α is a totally positive exceptional unit. Now for any $n \in \mathbb{N}, n \geq 1$, consider $\varepsilon_n := \sqrt[2n+1]{\alpha}$; it follows plainly that ε_n is also a totally positive exceptional unit, of degree $3(2n + 1)$. $\square$

Acknowledgments. We acknowledge with gratitude the deep influence Professor Vasile Brînzănescu has had upon our development as researchers. We have made the first steps in the theory of vector bundles under his guidance, in particular, we have learnt about the many open problems in the topic of vector bundles on non-algebraic complex manifolds from him directly. His advice, passed with patience and pedagogical tact, was of great value for us.

We are indebted to the anonymous referees for their extremely careful reading and for numerous excellent and constructive comments on the initial submission.

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Received December 22, 2019

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