NICU BOBOC AND POTENTIAL THEORY, INSEPARABLE POINTS IN THE ROMANIAN MATHEMATICAL SPACE *

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Anyone who tries to talk or write about a personality undertakes a vague impiety, especially when the subject has recently passed away, and the time rolled by has not yet fulfilled its role in decanting the essential aspects of his life. Regarding the scientific work, the assessments cannot be far from reality, due to the "written evidence", whereas in opinions concerning other aspects of the life and activity of this personality, subjectivity would certainly play a part.

Bearing that in mind, within this exposition we will give more space to the cultural environment in which Nicu Boboc established himself as a mathematician, even as one of the most representative Romanian mathematicians from the second half of the 20th century and the first decade from this millennium. A much narrower extent will bring to light other sides of life and the unmistakable features of this great professor. We didn't put aside this human portrait, which might give rise to controversy, because in the place of a neutral canvas quietly hanging on the wall under the motto "De mortuis nil nisi bonum", one would prefer a lively presence in all its dimensions, in the minds of mathematicians from today and tomorrow.

The passing of time is beneficial for the sedimentation of some essential doings, but the same passing is the realm of silence of other doings, of no less importance.

After the end of the war, in those states placed under the influence of the Soviet Union, among which our country was too, there were significant changes both in the educational and academical life, following the eastern model. One cannot say that they were not partly favorable for many cultural areas. The foundation of research institutes directly subordinated to the Academy was a new step, making clear the status of those passionate for doing research. The Institute of Mathematics and the Faculty of Mathematics would host a young generation, with a great potential for development. Students would make their

* This article was translated from Romanian by Anca Iuliana Bonciocat

debut in research, being supervised by their mentors, experienced professors who had a very good knowledge of the stage of development of the respective fields, as well as of the problems that were posed. The small group of Nicu Boboc, Corneliu Constantinescu, Aurel Cornea, Martin Jurchescu and Cabiria Andreian Cazacu, young people who had closely followed their professor Simion Stoilow during their undergraduate and graduate years, and who had grown up under his careful eye, was organized according to the Parisian model, in a scientific seminar. The first fruit of this group was the monograph *Teoria Potenţialului pe Suprafeţe Riemanniene* [Potential Theory on Riemannian Surfaces], elaborated by N. Boboc, C. Constantinescu and A. Cornea, published by Editura Academiei [Academy Publishing House].

The 4th Congress of the Romanian Mathematicians, organized in Bucharest in 1956, was a significant event in the activity of the seminar, offering a direct contact with renown mathematicians like J. Hadamard, A. Denjoy, G. de Rham, W. Blaschke, P. Erdös, B. Segre, K. Kuratowski, I. N. Vekua, S. Eilemberg and others. In the fall of 1958, the members of the seminar, together with Professor Stoilow, decided to organize the seminar in cycles of more extensive lectures, lasting a semester or even a whole year. In this way, the academic year 1959 – 1960 was devoted to Potential Theory on Riemannian surfaces, with lectures given by N. Boboc, C. Constantinescu and A. Cornea, together with M. Jurchescu for the chapter on capacity and capacitability. These lectures built up a volum from a series of the Academy Publishing House in 1962, followed by other volumes on various topics.

The extension of the range of topics approached by the Stoilow Seminar, and the enlargement of the group of participants naturally led to splitting into more research directions, with their own seminars. In this way, the Potential Theory Seminar emerged. The initiators of this seminar, Nicu Boboc, Corneliu Constantinescu and Aurel Cornea, being at the same time its base pillars for a long time, had a double lineage: they had been the prominent students of professors Simion Stoilow and Miron Nicolescu as well. From each of them, they had gained everything those great mentors could have offered: solid knowledge on complex functions, and respectively real analysis. The first assessment test of this young school in Potential Theory from Romania, in the challenging competition of founding an axiomatic theory, had been already successfully passed by the publishing of the aforementioned volume treating Potential Theory on Riemannian surfaces. This theory on Riemannian surfaces would be the test bench for all the axiomatization attempts in the subsequent future.

The program of the Potential Theory Seminar was mainly focused on the fruition of the accomplishments of its members in the field of complex functions and, equally, on the keeping up to date with the evolutions in the axiomatization of Potential Theory at European level, aiming to elaborate its own viewpoint in this field. The first target of the program was accomplished by publishing a considerable number of articles by A. Cornea, alone or in collaboration with C. Constantinescu, and culminating in the publication of the monograph *Ideale Ränder Riemannscher Flächen*, Springer 1963, elaborated by Constantinescu-Cornea. This work presents in a unitary way a series of compactifications of (non-compact) Riemannian surfaces, closely related to the the ideal boundary behavior of analytic mappings between two Riemannian surfaces. Also related to the studies of boundary behavior, we would like to mention the articles *On the invariance of the set of unfolding points under quasi-conformal transformations* (*Comunicările Acad. R.P.R.* **12** (1962)), and *On the notion of angular convergence at the boundary points of a Riemann surface. A generalization of Lindelöf's theorem*, *Certaines principes dans la théorie du potentiel sur de variétés Riemanniennes*, *Rev. Roumaine Math. Pures Appl.* **6** (1961), elaborated by N. Boboc. Other works of his, in collaboration with Gheorghe Mocanu and Nicolae Radu, deal with the harmonic metric on hyperbolic Riemannian surfaces or the first axiomatic attempts of Potential Theory on differentiable or triangulable manifolds.

The second direction of research, the one concerning the axiomatic Potential Theory, had been embraced by an enlarged group of researchers; two other mathematicians from the golden generation joined the founding team Boboc-Constantinescu-Cornea: Nicolae Radu and Paul Mustaţă. The process of establishing an axiomatic system that would allow to reformulate in a more generous context the huge legacy of results obtained in the classical Potential Theory had been sparked by G. Tautz and M. Brelot, by way of the harmonic functions with respect to the Laplacian, and by J.L. Doob and H. Bauer, using the solutions of the heat equation; the first two even before 1950, and the other two a bit later. The axiomatization theory regarding harmonic functions, respectively caloric functions, is due to M. Brelot, respectively H. Bauer. It's a remarkable fact that in a paper from *C.R. Acad. Sci. Paris* **246** (1958), before Bauer's axiomatic works, Boboc-Radu proposed an axiomatic theory – though only on n-dimensional manifolds – in which the axiom that would later bear Bauer's name was formulated for the first time.

In 1963, Boboc-Constantinescu-Cornea proposed a more general axiomatic theory, a symbiosis of the axiomatic theories of Doob and Bauer. More precisely, one assumes the convergence axiom from Doob, combined with the existence of a covering of the local compact space with a class of open sets satisfying a certain minimum principle. Two years later (1965), in *Ann. Inst. Fourier Grenoble* **15** (1965) they enriched the axiomatic system, imposing the condition that the sheaf be non-degenerate in every point. More comprehensive than Bauer's and Doob's axiomatics, therefore easier to fit to various examples of operators, this new axiomatic theory of Boboc-Constantinescu-Cornea is in fact – as they would prove it later – a local version of Bauer's axiomatic.

An impressive collection of results, which didn't miss any issue of Potential Theory to that date, have been elaborated by Romanian mathematicians. We can mention here the fundamental works by Boboc-Constantinescu-Cornea from *C.R. Acad. Sci. Paris* **256** (1963), *Nagoya Math. J.* **23** (1963) and *Ann. Inst. Fourier Grenoble* **15** (1965), which proposed a new axiomatic approach in Potential Theory. These authors had substantial contributions about the axiom of polarity, phrasing statements of it in the balayage language (Boboc-Cornea) or consequences of this axiom concerning the distributivity of the infimum operation with respect to the natural supremum on the set of positive superharmonic functions. An example given by C. Constantinescu shows that, against expectations, the polarity property is not fulfilled in all Brelot-type spaces. In the same spirit, there is a counterexample offered by Boboc-Cornea, regarding the axiom of polarity and the axiom of domination on an harmonic space.

Among the many accomplishments of the triplet Boboc-Constantinescu-Cornea, one, concerning the reduite and the balayage in cones of superharmonic functions, occupies a special position by the elegance and depth of the statements, but also by their subsequent consequences for the development of Potential Theory and for establishing connections to Markov processes. The notions of "reduite" and "balayage" have appeared chronologically in the reverse order, and have been introduced by M. Brelot in the years 1956 and respectively 1945, in the context of Bauer's axiomatic. There are many other results obtained by N. Boboc, C. Constantinescu, A. Cornea, which placed Bucharest on the map of the most prominent centers of the Potential Theory field, alongside Paris and Erlangen.

The prestige of Romanian mathematicians in this area was confirmed by the numerous invitations they received to participate in some of the most important conferences, in France (Paris) and Germany (Göttingen), or in conferences and workshops dedicated to Potential Theory, like the periodic conferences from Oberwolfach (Germany), Stresa (Italy), Prague, Berlin. On the other hand, the Institute of Mathematics had great personalities amongst its visitors: H. Bauer, M. Brelot, G. Choquet, M. Hervé, G. Anger, J. Kral, G. Guber, Gabriel Mokobodzki, Daniel Sibony, Denis Feyel, Arnaud de La Pradelle.

The new evolutions in Potential Theory in Europe and in the world, during the second half of the 7th decade, follow the general trend in which the exchange of ideas between apparently different domains flourished; Potential Theory and the Theory of Stochastic Processes transfered insights to each other, thanks also to two mathematical oeuvres of the century. *Convexity Theory* and *Capacity Theory*, the great works of Gustave Choquet, influenced the trajectory of Romanian research as well. The first essential paper on convexity theory published in our country was due to N. Boboc and A. Cornea in *Rev. Roumaine Math. Pures Appl.* **12** (1967). One can find there a theorem on the existence of Choquet boundary of a compact space X with respect to a convex cone S of lower semicontinuous real functions. This property will bear the name Bauer's minimum principle. One may observe that Choquet boundary is finer, as an instrument of investigation, than Shilov boundary for algebras of functions. Moreover, the authors generalized Mokobodzki's theorem of characterization of S-minimal measures and performed a deep study on min-stable cones S of continuous functions for which under any point $x \in X$ one can find a unique S-minimal measure. When X is a convex compact set in a locally convex space, and S is the cone of geometrically convex functions, the previous situation occurs only if the convex compact set is a geometric simplex: starting from here, the cone S of continuous functions on X that fulfills the uniqueness property of minimal measures would simply be called simplicial cone. The minimal measures, in most of the cases (X separable metrizable), are those supported by the Choquet boundary. The paper presents examples of such situations and, among them, most importantly the cone of continuous functions on the closure of a relatively compact open set D, with restrictions to D being superharmonic with respect to an harmonic sheaf of functions on a locally compact space. The novelty of the study here consists in the fact that it is totally detached from the underlying convex compact set X, on which the Choquet convexity theory had been previously developed, and that it offered a wider perspective towards possible applications. It is the debut of the third stage from the life of the Potential Theory seminar, which would have, over the subsequent 15 years, the following directions of research:

a) Convexity Theory with its many aspects;

b) The connexion of the previous research achievements in Potential Theory with the theory of stochastic processes, inaugurated by the works of J.L. Doob during 1954 – 1956, and G.A. Hunt in 1957 – 1958;

c) The founding of an axiomatic Potential Theory, in which the hyperharmonic elements to be no longer circumscribed to a locally compact topological structure, since the correspondent of hyperharmonic functions in the theory of processes – the excessive functions – were defined on numerical functions given on a space with Borel structure.

In the beginning of this period, after a suitable "practice" in university teaching, within the department of differential and integral calculus of professor

Miron Nicolescu, the Potential Theory team has "strengthened" by the debut of Gh. Bucur, the last PhD student of the professor. In the area of convexity theory, besides the paper of Boboc-Cornea mentioned above, a large number of publications were elaborated by Boboc-Bucur, Boboc-Bucur-Cornea and Gh. Bucur.

In *Rev. Roumaine Math. Pures Appl.* **17** (1972), Boboc-Bucur-Cornea developed the theory of cones of potentials in the framework of topological spaces and of a harmonic support. The notions of support on an ordered cone and cone of potentials have come from the study of superharmonic functions. Remarkable properties of harmonic support for superharmonic functions were pointed out in the well-known PhD thesis of R.M. Hervé. Further developments regarding the notion of abstract support on ordered convex cones are due to Boboc-Constantinescu-Cornea, Mokobodzki-Sibony and Boboc-Bucur. The cone of potentials was introduced by G. Mokobodzki and further developed by Boboc-Bucur-Cornea.

Starting from the concept of abstract cone of potentials proposed by Mokobodzki, Boboc-Bucur-Cornea enriched the structure till the one of normed cone of potentials, concept that transforms the abstract cone of Mokobodzki into a deeper tool of research, offering at the same time the possibility to represent the cone of potentials as a cone of real functions, continuous on a compact space X, represented by the extremal elements of a cap of the cone, in the dual Banach space, formed by the functionals from the polar of the cone C in this dual space.

The papers of Boboc-Bucur from *Math. Scandinavica* **26** (1970) and *Rev. Roumaine Math. Pures Appl.* **17** (1972) are related to the Dirichlet problem associated to the Choquet boundary of the space of affine functions, problem initially approached by E. Alfsen. A more thorough study of this problem, and expressing the extensibility of a given continuous function on the closure of Choquet boundary to a continuous affine function, was worked out by Boboc-Bucur, and later, using their results, by E. Effros in his paper from *Israel. Journal of Math*, 1971. In order to solve the problem, Boboc-Bucur introduced the concept of extremal S-affine function f given on the Shilov boundary of a space X with respect to the cone S of continuous functions on the space X. Another approach of the Dirichlet problem was the one of the topological characterization of functions defined on the Choquet boundary, which can be extended by continuity and affinity over the entire space X. This problem involved also members of Choquet's team like Mark Rogalski, Hi Facouri, and other mathematicians, like for instance Alain Gleit, Erick Alfsen, E. Effros, Bay Andersen. In *Rev. Roumaine Math. Pures Appl.* **17** (1972) Boboc-Bucur introduced two topologies on the boundary of X with respect to S: the Choquet topology and the facial topology, which turns the Choquet boundary into a compact space. Choquet topology proved its utility in the works of S. Teleman on operator algebras. In connection with the facial topology, one constructed a functional calculus, which shows that continuous (semicontinuous) functions in the facial topology can be extended to continuous (semicontinuous) affine functions on the space X.

Many of the results obtained by Boboc-Bucur-Cornea have been the object of a monograph published by Boboc-Bucur in 1976, *Conuri convexe de funcţii continue pe spaţii compacte* [Convex cones of continuous functions on compact spaces], Editura Academiei [Academy Publishing House]. This work is an up-to-date exhibition of convexity theory, comprising several results previously published by their authors, but also original statements that had never been published. Although written in Romanian, this monograph is read by foreign mathematicians as well, maybe due to the review written by G.J.O. Jameson in *Math. Revue*, from which we quote: "The objects studied in this book are cones of continuous (sometimes only lower semicontinuous) functions on compact spaces. Many results concerning affine and concave functions on convex sets have generalizations applying to this situation: Assumptions about the cone replace the affine structure of the set. The theory also takes on 'cones of potentials', which have arisen, by several stage of generalizations, from the cones normally considered in potential theory. Much of the materials is the work of the authors and their associates, most has not previously appeared in book form, and some has not even appeared in journals. [...] A translation into a more widely known language will be eagerly awaited. In the meantime, workers on this field world do well to make use of the AMS instant Romanian course; it really does not take long to equip oneself to read mathematics in Romanian". A tribute to our venerable language! At least mathematics can be read in Romanian language.

The Romanian debut in establishing the axiomatic Potential Theory was done by Boboc-Cornea in two papers from *C.R. Acad. Sci. Paris*, 1970, in which the authors define the abstract concept of H-cone (the letter H is chosen to suggest the origin of this notion: harmonic). In an extensive article from *Ann. Inst. Fourier Grenoble*, dedicated to Marcel Brelot on his 70th anniversary, Boboc-Bucur-Cornea introduced the concept of standard H-cone and, besides examples given by harmonic spaces on locally compact spaces with countable basis, they discuss the cone of excessive functions. In this way comes up the notion of standard H-cone of functions, the theory of reduite and balayage on the parts of the representation space. One studies polar and semipolar sets and one proves the Doob-Bauer-type theorem on semipolarity, the axiom of fine sheaf, the localization in H-cones of functions, the relativity to the localized of axiom D of fine sheaf.

In *Rev. Roumaine Math. Pures Appl.* **24** (1979) and **25** (1980), papers which can be considered of top-level importance in abstract Potential Theory research, Boboc-Bucur-Cornea introduced the natural topology in H-cones of functions, as being the topology that makes continuous the universal continuous elements from the dual, and proved that this topology is metrizable and complete.

Boboc-Bucur-Cornea develop a theory of H-cones in ordered Hilbert spaces, the so called theory of Dirichlet spaces. It follows the way opened by Gauss, Dirichlet, Cartan, and resumed firstly by A. Beurling and J. Deny, G. Stampacchia, and later by Japanese mathematicians M. Fukushima, S. Watanabe, K. Ito, and the French squad: Alano Ancona, Yves Le Jan. These works gave a positive answer to the problem posed in 1956 by Beurling and Deny, concerning the continuity of latticial operations in a Dirichlet space. Simultaneously, in the concrete frame of a Dirichlet space, the same answer was found by A. Ancona. Likewise, these papers created a natural setting for the development of Hilbertian methods in Potential Theory, an older challenge proposed also by Beurling and Deny. It was solved the problem of characterization of autodual H-cones that can be seen as the potentials of some symmetric Dirichlet spaces, leaving open the problem of a similar characterization for H-cones that are not autodual. The balayage operators have been characterized in the geometric language of Hilbert spaces, and it was stated the theorem of representation of a Dirichlet space as a functional space in the sense of Aronsjagn-Smith.

The research performed by Boboc-Bucur-Cornea in the abstract Potential Theory aroused a special interest in the world of researchers. That would explain the invitations to talk about those results in the Brelot-Choquet-Deny Seminar in Paris, also in Germany (Eichstaett, Bielefeld, Oberwolfach), Prague, Copenhagen, Nagoya. The publication of Boboc-Bucur-Cornea (in collaboration with H. Hölein) of the monograph *Order and Convexity in Potential Theory: H-cones*, Springer Lecture Notes in Math., 1982, is a prestigious accomplishment of the Romanian mathematicians. Along with the fact that the book synthesizes, as completely as possible, the mathematical acquirements in the field, including Mokobodzki, Sibony, Hansen, Bliedtner, Ancona, Sieveking, Fuglede, Berg, it offers to the mindful reader a balanced construction with deep results and research tools for the future. The research entailed to the publication of this monograph, but subsumed under the same mathematical object, the standard H-cone, was carried by Boboc-Bucur (in papers from *Lecture Notes in Math.* 1983, *Rev. Roumaine Math. Pures Appl.* **30-33** (1985-

88), **35-37** (1990-92),*C.R. Acad. Sci. Paris* **302** (1986), **311** (1990)) and was addressed to various problems, like Hunt's hypothesis, Stonian structures, classifications of potentials in standard H-cones, fine potentials, Green potentials, Frostman property, subordinations and supraordination in H-cones, dilations and adjoints of dilations in H-cones, characterizations of Dirichlet forms, given by excessive and coexcessive elements, the processes associated to standard H-cones, the alternate method of Schwarz, perturbations in resolvents and kernel semigroups. L. Beznea and N. Boboc have been studied in *C.R. Acad. Sci. Paris* **315** (1992) and *Potential Analysis* **4** (1995) the absorbent balayages on H-cones, with applications to the decomposition of H-cones into parabolic and elliptic parts.

As time went on, the challenges concerning connections with the Theory of Markov Processes become preponderant. They were the consequence of the mathematical "production" of the team Nicu Boboc-Lucian Beznea, which completed a cycle of over 20 papers with the monograph *Potential Theory and Right Processes*, Kluwer Academic Publishers/Springer 2004, a work of great importance, both by the results it includes, but also by constructing useful tools in subsequent studies in the field. This book continues the line of the monographs published by Romanian mathematicians, and thus Romanian Mathematics has ensured an important place at the interface between Potential Theory and Markov Processes. Other articles by L. Beznea, alone or in collaboration with N. Boboc, L. Stoica, A. Cornea, M. Röckner, would draw towards Potential Theory a number of young mathematicians, who in turn have become the collaborators of L. Beznea.

The results have been mostly obtained by L. Beznea and N. Boboc in *Potential Analysis*, *C.R. Acad. Sci. Paris*, *Probab. Th. and Related Field*, *Annals of Probab.*, *Acta Math. Sinica, English Series*, *Infinite Dim. Analysis Quant. Probab.* Their aforementioned monograph offers a unitary version of the theory. We present below some of the topics addressed therein.

• The characterization of semipolar sets as exactly those sets that are negligible for all potential kernels of continuous additive functionals with total fine support (of a fixed right Markov process) has been proved by J. Azéma by probabilistic methods (in 1972) and by W. Hansen in an analytic context (in 1981), in the special case with a reference measure. The characterization had been an open problem for the general case (without a reference measure), formulated by C. Dellacherie in 1988. A positive answer has been given by Beznea-Boboc in 1996, and a probabilistic approach has been further carried out by P.J. Fitzsimmons and R.K. Getoor.

• In the same year they have proved the quasi-Lindelöf property of the fine topology (with polar exceptional sets), improving a result of P.J. Fitzsimmons and R.K. Getoor from 1995, where the exceptional set was only semipolar.

• They developed an analytic approach for the Revuz formula and for the Revuz correspondence between classes of strongly supermedian kernels and their associated measures. They answered a question raised by Fitzsimmons-Getoor in 2003, concerning the characterization of strongly supermedian functions by the property of being “universally” supermedian. The authors have come with significant new material along the probabilistic works by Revuz, Azéma, Getoor-Sharpe, Fitzsimmons, Fitzsimmons-Getoor and Dellacherie-Maisonneuve-Meyer.

• Furthermore, they tackled the Feynman-Kac formula and the Kato classes of measures, studying the perturbation of the generator of a right Markov process with a signed measure, by analytic and probabilistic methods of Potential Theory, extending results obtained by Fitzsimmons-Getoor (2003) and Stollmann-Voigt (1996).

• They have proved in collaboration with Michael Röckner that any strongly continuous resolvent of sub-Markovian contractions on an L^p space is in fact associated with a Markov process, thus giving an answer to an open problem raised by G. Mokobodzki in 1991. An extension of the starting space proves to be required. In particular, any Dirichlet form becomes quasi-regular after such an extension. The authors have shown that the right Markov process associated with a quasi-regular semi-Dirichlet form has a dual process, proving that the weak duality hypotheses in the sense of Fitzsimmons-Getoor-Sharpe are satisfied.

Applications to stochastic analysis in infinite dimension have been developed in collaboration with L. Beznea and M. Röckner in papers from *J. Evol. Eq.*, *C.R. Acad. Sci. Paris*, and *Potential Analysis.* They have proved a general procedure for constructing càdlàg Markov processes, starting with a resolvent of kernels, to which they imposed some conditions justified by applications, and controlling the starting points of the process. The main application is to find out the martingale solutions to stochastic differential equations on Hilbert spaces. The regularity of the trajectories was proved by using a technique initiated in *Bull. London Math. Soc.*, about the property of the capacity to be tight and the existence of a superharmonic function having compact level sets (the so-called Lyapunov function)

The scientific work of Nicu Boboc, dedicated mostly to Potential Theory under various aspects, as exposed above, is not just about it. An extensive presentation on this subject was excellently done by Jiří Veselý (2003), in the International Conference on Potential Theory, organized on the occasion of the 70th birthday of Nicu Boboc.

Nicu Boboc was very attached to the Faculty of Mathematics of the University of Bucharest starting with his student years, continuing with the long period of discipleship, within the same faculty, firstly as an assistant to Professor Simion Stoilow and ending up as a professor at the same department of Analysis, until his retirement. Even after that, he would continue his research activity, participating in research grants of the Institute of Mathematics of the Romanian Academy, remaining until the end the flag bearer of the Potential Theory seminar, in which generations of mathematicians were trained and substantial research was carried out, thus placing Bucharest, for over a half century, at the forefront of international research in this field.

In his long teaching career, he was the mentor of over 12 000 students, who learned under his magic wand the skill of a mathematical reasoning carried with rigor up to the end, as well as the satisfaction of the first discoveries.

In the most famous high schools in the country, in faculties of mathematics or technical institutes, the former students of professor Nicu Boboc have taught and still teach, offering themselves to their students, as they learned from their mentor. They learned from him the plenary commitment in all the range of activities that characterize the educational act: scientific circles for students, school or university Olympiad contests, elaboration of school textbooks or university courses, patriotic work, diluted today in "social activity".

The measure of his teaching, accompanied by the writing of courses of high scientific standing, is comparable only with the measure of his achievements in the field of research. His stature as a scientist and eminent professor, amplified by his total availability and plenary commitment in everything he did, are the foundation of his will to undertake positions of responsibility, like Vice-rector in charge of research problems at the University of Bucharest, Dean of the Faculty of Mathematics during two complete terms, Inspector in the Ministry of Education and even member of the Romanian Parliament.

Because, as our poet phrased it,

> "The heavy car of state must rumble on, and each
> Of you must pour his blood into the battle's breach,
> That of your endless pain they may grow prosperous."

from *Emperor and proletarian*, by M. Eminescu*

"He was doing maths as easily as he breathed", according to S. Marcus (assistant to the student group including Boboc, Cornea, C. Foiaş, N. Radu, P. Mustaţă etc), and Nicu Boboc was the one who often gave quick solutions

*Translation by Corneliu M. Popescu. In Romanian in original: "A statelor greoaie care trebuie-mpinse/Şi trebuiesc luptate războaiele aprinse,/Căci voi murind în sânge, ei pot să fie mari.", *Împarat şi proletar*, Mihai Eminescu to the problems posed in the tutorial, even before the assistant would finish to give their statements.

A special colleague and a desired collaborator, Nicu Boboc, as well as A. Cornea, transformed their houses into true research institutes, from which the collaborators left late at night, or rather in the morning.

His bold creativity and rare stamina imposed him as an undisputed and demanding leader, perhaps too demanding for some of his colleagues. Maybe each one would hold his own truth: he, with his natural qualities, devoting himself entirely to everything he did, demanded from his colleagues a similar commitment, sometimes overlooking that "Not the burden placed on one's shoulders is so hard, rather one's weak shoulders perceive it as such. "

If we were to characterize the passing of this great Man over here, we would confess without hesitation:

He did everything. He gave everything, forgetting himself!

List of PhD students of Nicu Boboc †

Tran Dinh Truc, *H-operators, kernels and H-cones associated with a degenerate elliptic differential operator*

Eugen Popa, *Cones of superharmonic functions and products of harmonic spaces*

Martha Bănulescu, *Potentials on topological groups*

Liliana Popa, *The Dirichlet integral on autodual standard H-cones*

Lucian Beznea, *Potential theory - Classifications in cones of potentials*

† This is the reprint of the list published in the volume [L. Beznea and Gh. Bucur (editors), *Fifty Years of Modern Potential Theory in Bucharest – To the Anniversary of Nicu Boboc*, Editura Universităţii din Bucureşti, Bucharest 2004, p. 22].

Dumitru Popa, *Classes of operators on spaces of continuous functions with operatorial values*

Cornel Udrea, *Contributions to a nonlinear potential theory*

Emil Popescu, *Potential theory structures associated with pseudo-differential operators*

Emil Moldoveanu, *Potential theory associated with the spaces of trajectories*

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