FROM STOILOW SEMINAR ON COMPLEX ANALYSIS TO THE SEMINAR ON POTENTIAL THEORY *

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I have the privilege to know Professor Nicu Boboc for a half century and to have been a witness of all his brilliant scientific and didactic achievements.

For a few weeks during the autumn of 1953, I was the teaching assistant for the course of Professor Simion Stoilow, *Theory of functions of one complex variable*, to an exceptional group of students, who later on strongly distinguished themselves by their work in the international mathematics life. The young Nicu Boboc was one of those enthusiastic and passionate students, together with Aurel Cornea, Ciprian Foiaş, George Gussi, Dragoş Lazăr, Paul Mustață, Valentin Poenaru, Nicolae Radu, Marius Stoka, Kostake Teleman, Samuel Zaidman and others. They tumultuously engaged in discussions, bringing up ideas and proposing solutions.

Surely, I have got to know him better in the following years, at the special courses of our Professor Stoilow, and after defending his diploma thesis in 1955. To highlight that, he published his first 3 papers as an undergraduate student: L'unicité du problème de Dirichlet pur des équations de type elliptique with his colleague S. Zaidman (1954), Un exemple de fonction continue de Darboux (1954), and Sur un théorème de type Sturm et applications au problème de la séparation des zéros des fonctions propres de l'opérateur Δ (1955).

Immediately after graduation, he was retained at Professor's Stoilow Chair as teaching assistant (1955), assistant (1957), lecturer (1959), associate professor (1969), professor (1973), so that we worked 48 years in the same Analysis Department of the Faculty of Mathematics, University of Bucharest.

He also attended Stoilow's Seminar at the Institute of Mathematics of the Romanian Academy, where – after his Ph.D. thesis (1961) – he succeeded to spend some very profitable years (1963–69), dedicated only to research work.

The Institute of Mathematics of the Romanian Academy has been founded in 1949, having Dimitrie Pompeiu as a Director. However, it was organized

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and effectively led till 1961 by Simion Stoilow – first as a Deputy Director, and since 1954 (when D. Pompeiu died) as a Director.

Professor Stoilow gathered in the Institute all the active mathematicians in university education and succeeded to set up their cooperation around some great problems, which could be tackled with the existent forces.

Scientific seminars showed up then as the main sources of information about the principal trends and results in the mathematics of the time. These seminars offered the possibility to expose and discuss personal results and were an essential factor in forming the future researchers.

Stoilow's Seminar dedicated to Complex Analysis and Topology had at the beginning few participants (for instance in 1950-51 young people such as Tudor Ganea, Liuba Stecolcic, Marcel Rădulescu and myself), but every year the Seminar strengthened by the participation of Faculty graduates of great value: Martin Jurchescu, Corneliu Constantinescu, Ionel Bucur, Israel Berstein, Felix Albrecht, Aristide Deleanu. Nevertheless, only in 1955, when Nicu Boboc and his colleagues joined the Seminar, it took its definite form.

Due to Professor Stoilow, around whom the other professors constituted a united front, in those difficult years of afflicting transformations, our Institute of Mathematics was a true oasis of creative work, where the Romanian mathematics could intensively develop. In research teams and in seminars reigned friendship and harmonious cooperation.

Initially, Stoilow's Seminar – held every Thursday at 9 o'clock in the Institute - consisted in lectures given by its members in connection with their work, and also for initiation in then actual themes, especially in the topological and geometric function theory, in the R. Nevanlinna and L.V. Ahlfors value distribution, in Riemann surface theory. One used special courses and papers by Professor Stoilow or other papers and books, for instance in '53 appeared R. Nevanlinna's Uniformisierung. Simion Stoilow received many abstracts by mathematicians from all over the world, which be distributed to the Seminar members in order to be presented and discussed. He was acquainted with the mathematical evolution, and in spite of the difficulties of the '50s he preserved connections with France, Germany, Italy, Finland, and evidently with Soviet mathematicians. Professors from other universities - like Professor George Călugăreanu from Cluj, Professor Mendel Haimovici from Iasi – where sometimes guests of the Seminar. The public debate of doctoral theses was preceded by expositions in the Seminar, and generally its members attended also Professor Stoilow's special lectures.

The IVth Congress of Romanian Mathematicians in Bucharest 1956 was a significant event in the life of the Seminar, offering a direct contact with many outstanding mathematicians like J. Hadamard, A. Denjoy, G. de Rham, W. Blaschke, P. Erdös, B. Segre, K. Kuratowski, I.N. Vekua, S. Eilenberg and others.

Professor Boboc was one of the most active members of Stoilow's Seminar. A specific feature of his character is to be always open for collaboration. Even from undergraduate years, his first paper in 1954 was written together with his colleague S. Zaidman, and in 1958 he published (in Annales Scientifiques de l'École Normale supérieure, Paris) a paper *Sur la détermination d'une fonction par les valeurs prises sur certain ensemble* with Solomon Marcus, and two papers with Nicolae Radu. In the frame of Stoilow's Seminar, he developed the collaboration with Corneliu Constantinescu and Aurel Cornea, with such important results.

In the '60s, Prof. Boboc worked and published between 1965 and 1975 together with Paul Mustață. The cooperation with Aurel Cornea led to many papers till 1981 and those with Gheorghe Bucur (first joint paper in 1969), respectively with Lucian Beznea (first joint paper in 1992), continue fruitfully till now. We also mention his care for involving in research the young members of the department, like for instance Gheorghe Sireţchi, with whom be carried out in 1961 the paper Sur la compactification d'un espace topologique, a vast extension of Stoilow's theory concerning the ideal boundary of Riemann surfaces, and in the same year two papers with Gheorghe Mocanu: Sur la notion de métrique harmonique sur une surface riemanniene hyperbolique and Sur la propriété (N) de Menchov.

In the autumn of 1958, the Seminar members together with Professor Stoilow decided to organize the Seminar with longer lecture cycles, during a semester or even a whole year. Thus the school year 1959-60 was dedicated to Potential theory on Riemann surfaces, lectures held by N. Boboc, C. Constantinescu and A. Cornea together with M. Jurchescu for the chapter: Capacity and Capacitability. These lectures formed a volume published in the series: Seminar S. Stoilow by the Romanian Academy Publishing House in 1962 and the series continued by the volume Modern Problems in Complex Analysis, published in 1965 and containing Local Theory of Functions of several complex variables by M. Jurchescu, Current Theory on orientable manifolds by C. Constantinescu, and Quasiconformal mappings by Cabiria Andreian Cazacu, as well as by the volume Topology, Categories, Riemann surfaces containing Dimension theory by A. Deleanu, Theory of Categories by M. Jurchescu and Riemann Surfaces (Teichmüller Spaces) by C. Andreian Cazacu. Other cycles of lectures were held by I. Bucur, C. Foiaş, V. Poenaru, N. Radu, Alex. Lascu, P. Caraman.

But time ran out. In the autumn of 1960 Professor Stoilow retired from the Faculty's department, remaining only Director of the Institute and President of the Mathematics Section of the Romanian Academy. It was then that Professor Nicu Boboc spoke at the festivity dedicated to Professor Stoilow on this occasion at the University House at the beginning of 1961, in the name of Stoilow's disciples.

Even after the hard unexpected blow from 4th of April 1961, the loss of our great and so much admired and loved professor, the Seminar went on, following the directions he had initiated: several complex variables, quasiconformality, potential theory.

The volume mentioned before, *Potential theory on Riemann surfaces* by N. Boboc, C. Constantinescu and A. Cornea, contained not only a very valuable systematization, but also important results due to the authors (as underlined by Professor Gh. Vrănceanu in the Preface) and was the starting point for more general research on Potential Theory.

At that time, in 1961, Professor Boboc had finished his remarkable Ph.D. thesis Potential Theory on Riemannian manifolds infinitely differentiable with a metric structure defined by a metric tensor.

Professor Boboc always aims in his work at the complete understanding of the concepts and the necessity of imposed conditions. He always endeavors to find the most adequate frame. These tendencies are already presented in his thesis, where he constructs a tool in order to study the Riemannian manifolds such that results from Newtonian potential theory remain valid, and introduced now notions like the thinness functions, *c*-absolutely continuous and *c*-singular measures, the manifold quasi-equivalence.

In the same period, N. Boboc, C. Constantinescu and A. Cornea directed their research to the axiomatic theory of harmonic functions and harmonic spaces. The result was a more general axiomatization than that of Brelot and Bauer, published in a series of important papers *On the Dirichlet problem*, in Nagoya Math. J., 1963, *On the Balayage* respectively on *Non-negative super-harmonic functions* in Annales de l'Institut Fourier (Grenoble) 1965. These papers formed the base for the book *Potential theory on harmonic spaces*, Springer-Verlag 1972, by C. Constantinescu and A. Cornea.

Another fruitful collaboration with P. Mustață produced between 1965– 75 a series of papers and the book *Espaces harmoniques associés aux opérateurs differentiels linéaires du second ordre de type elliptique*, Lect. Notes in Math., Springer-Verlag, 1968.

Between 1963 and 1966, a Seminar on Potential Theory and Probability was organized by N. Boboc, C. Constantinescu, A. Cornea together with I. Cuculescu, in order to thoroughly study the connections between these domains, which led to N. Boboc - C. Constantinescu - A. Cornea's papers on Markov processes associated to harmonic spaces (e.g. *Semigroups of transitions on* harmonic spaces in Rev. Roumaine Math. Pures Appl., 1997).

In parallel with the Potential Theory Seminar, which, due to Professor Boboc, still continues every Tuesday at the Faculty of Mathematics, another Seminar on Convexity was organized by N. Boboc and A. Cornea from 1967 till early '80s. From this Seminar resulted a vast cycle of papers not only in collaboration with A. Cornea, but beginning from 1969 also with Gh. Bucur, together with whom Prof. Boboc published also 3 important monographs: *Convex cones of continuous functions on compact spaces*, Romanian Academy Publishing House (1976), *Order and convexity in Potential Theory H-cones*, Lect. Notes in Math., Springer-Verlag (1981) (this one with A. Cornea, and in collaboration with H. Hollein), *Measure and Capacity*, Romanian Scientific and Encyclopedic Publishing House (1985).

Gradually, the scope of Potential Theory Seminar enlarged from potential theory associated to differential operators to integro-differential operators, to the Malliavin calculus and infinite dimensional methods, and to the study of Markov processes by analytic methods. Excessive functions and measures make another research theme in the last decade, and on this topic, besides the cooperation with Gh. Bucur, Professor N. Boboc published many valuable papers together with Lucian Beznea.

Another important contribution of Professor Boboc to the development of Romanian mathematics concerns the international contacts, and we remind here only the intense participation to the Romanian-Finnish Seminars. If the first edition of this Seminar, Braşov 1969, was dedicated to Quasiconformality and Teichmüller Spaces, all of the following 7 Seminars organized in Romania from 1976 till 2001 (there have been also two Seminars in Finland) reflected the structure of Stoilow's Seminar adding to quasiconformality other sections: Functions of One respectively Several Complex Variables, Connections with Functional Analysis, and Potential Theory.

Due to Professors Boboc and Cornea, to the whole Potential Theory Seminar, this Potential Section essentially contributed to the success of the Romanian-Finnish Seminars by the brilliant participation from abroad, as well as by the high level of the Romanian lectures. Let's remind too that in 2001 at Braşov the Romanian-Finnish Seminar was continued by the Summer School "Connections between Potential Theory, Geometry and Probability", having as lecturers: Professors N. Boboc, O. Martio, K.T. Sturm, Gh. Bucur, L. Beznea.

It is one of the great merits of Professor N. Boboc that he continued (at the beginning with Profesor A. Cornea, and after '78 alone) this Potential Theory Seminar, where the majority of the members obtained the doctoral degree under his supervision.

If in 1969 at the 20th anniversary of the Institute, Miron Nicolescu – then

the Director of the Institute – began the enumeration of the most remarkable new results obtained in the Institute with those obtained by the Potential group (namely the nuclearity on harmonic spaces), today the Potential Seminar team represents one of the strongest Romanian mathematical schools.

This school is in permanent expansion, essentially because of the Professor Boboc's captivating lectures at the Faculty of Mathematics, remarkable by their clarity and systematization, and also because of the enthusiasm that Professor Boboc transmits to the students.

Endowed with an exceptional labor power and rigorous organization gift, Professor N. Boboc carried on also a great activity in the Department of Analysis, in the Faculty of Mathematics and in the University. Let us mention only the years 1984–89, when he was the Dean of the Faculty and fought in extremely difficult conditions to preserve and develop the traditions of our great professors concerning rectitude and the high level of the scientific and didactic work.

We express our gratitude for all his achievements and wish him good health and new successes!