INSTITUTUL DE MATEMATICA “SIMION STOILOW” AL ACADEMIEI ROMANE

Seminarul de Teoria Potentialului

Two lectures on coagulation: 1. Smoluchowski’s coagulation equation. 2. Coagulation models.

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1. Marti, 13 decembrie 2011, ora 14:00, IMAR, etajul 3, sala C.Banica – 306

2. Miercuri, 14 decembrie 2011, ora 10:00, IMAR, etajul 3, sala C.Banica – 306

1. Smoluchowski’s coagulation equation: probabilistic interpretation of solutions for constant, additive and multiplicative kernels: The phenomenon of coagulation applies to and is studied in several domains as for example : in chemistry (polymerization), in astrophysics (formation of stars and planets), in physics (the behavior of fuels in combustion engines) etc. It was first introduced by the Polish physicist Marian von Smoluchowski in 1916, in a paper on the precipitation in colloidal suspension. In order to understand the model, one has to imagine a system of clusters characterized by their masses. The distribution of masses changes over time. When two clusters are close to each other they can merge together and form a cluster of larger mass, this is the coagulation phenomenon. The problem is to characterize the evolution of this system over time, by considering the equation satisfied by the distribution of masses. The Smoluchowski equation is purely deterministic and produces a rather complex model which takes the form of an infinite system of nonlinear equations. In this presentation we will describe basically the coagulation model for the case of constant, multiplicative and additive kernels and their interpretation via branching processes. We will also give a connexion between the additive and the multiplicative case and some renormalisation theorems.

2. Coagulation models - interpretation via a nonlinear stochastic process: The solution of the Smoluchowski’s coagulation equation is associated with a nonlinear pure jump Markov process. This probabilistic approach allows to understand the dynamic of the system and to construct a new stochastic algorithm and a numerical scheme for the solution of the coagulation equation. Jumps in the Markov process are equivalent to instants where coalescence phenomena occur. Under quite stringent hypothesis, we obtain a central limit theorem for our Monte Carlo method. A more natural model, including also the position of the particles, the so-called coagulation equation with diffusion, can also be studied via a couple of stochastic processes (X_t,Z_t), where X denotes the position of the particle and Z its mass. The component X is mainly a Brownian diffusion while Z is as in the homogeneous case, a pure jump Markov process.