Mihai Prunescu

Vita

Synopsis: I was born at November 24, 1967 in Bucharest, Romania. Between 1987 and 1992 I studied Mathematics at the University of Bucharest. My final year project was called *Ordered fields satisfying Rolle's Theorem* and was done under the direction of Victor Alexandru. Since 1992 I am member of IMAR in the Algebra group. My main motivational interest is Mathematical Logic, but Number Theory furnished to me the most subjects of reflection for different problems coming from the logic framework. At December 21, 1998 I have defended my doctoral dissertation *A structural approach to diophantine definability*, supervised by Alexander Prestel at the University of Konstanz, Germany (magna cum laude). Between 1999 and 2003 I have been assistant at the University of Greifswald, Germany. Between 2003 and 2005 I received a post doctoral grant by the Deutsche Forschungsgemeinschaft (DFG) at the University of Freiburg, Germany. Since 2005 I collaborate with the company Brain Products GmbH on numeric software for the analysis and evaluation of electroencephalograms.

Results: The first article in the publication list was done together with Paul Horja, Gheorghe Craciun and Tudor Zamfirescu under his direction at the University Dortmund in 1992 and studies the Baire metric space of all homeomorphisms of the circle S^1. My contribution to this article was to find and prove the Theorem 1, that tells that up to a rare set, all homeomorphisms of S^1 are semiperiodic, following a classification by Fritz John.

My doctoral dissertation done in Konstanz had as a goal to replace the artisanal construction used to prove the Theorem of Matiyasevich and other definability results in Number Theory by more structural algebraic facts. The most important result of the dissertation says that if one considers the non-trivial ultrapowers $\mathbb{Z}^*$ and $\mathbb{Z}[T]^*$ of the ring of rational integers and of its polynomial ring, the application of natural restriction $\text{res} : \text{Emb}(\mathbb{Z}[T]^*) \to \text{Emb}(\mathbb{Z}^*)$ between the monoids of embeddings of this rings in themselves, is an isomorphism of monoids. Moreover, the surjectivity is equivalent with the Theorem of Matiyasevich, telling that all recursively enumerable subsets of $\mathbb{Z}$ are diophantine, and the injectivity is equivalent with a result of Jan Denef telling the same about the recursively enumerable subsets of the polynomial ring $\mathbb{Z}[T]$. Diophantine problems and definability in arithmetic remained for me a topic of interest. Four other articles in my list contain different results in this field.

In Greifswald I started to work in a direction which is defined in Bruno Poizat's book *Les petits cailloux* and currently called *unit cost complexity.* This direction of research belongs to the algebraic complexity theory and is based on an ideal model of computation where Turing machines are able to write in their cells names for the elements of some infinite algebraic structure of given signature, and the heads of the machine are able to perform operation in the structure or to test signature relations - for example the order in some ordered field. Although an ideal model, the analogon of the P versus NP problem for this model of computation is the question of fast deciding the satisfiability of existential formula with free variables and parameters in the given structure, which is a very natural problem to ask and to study. Differently from the classical P versus NP problem, the unit cost variant can be solved in many situations. Structures not allowing elimination of quantifiers trivially have P $\neq$ NP. I was able to prove that all infinite abelian groups and all infinite boolean algebras have P $\neq$ NP, although some of them admit elimination of quantifiers. Quite interesting is the case of ordered abelian semigroups because the unit cost knapsack problem proves to be equivalent with the classical P versus NP problem. My best result in this area was the construction of an ad hoc structure allowing fast elimination of quantifiers and consequently satisfying P = NP from the point of view of unit cost complexity.

In Freiburg, as I worked more in programming, I started to experiment with recurrent double sequences over finite sets. Given a finite set A, a constant $1 \in A$ and a fixed function $f : A^3 \to A$, one has a recurrent double sequence with initial conditions (say) $a(i, 0) = a(0, j) = 1$ and the recurrence $a(i, j) = f(a(i, j-1), a(i-1, j-1), a(i-1, j))$. My first result with those objects was to prove a conjecture of Passoja and Lakhtakia concerning the self-similarity of the recurrent double sequences in a narrow class defined by some linear functions of a given special form over finte fields. Encouraged by this result I have experimented with general functions f and I have supposed that recurrent double sequences are building a powerful model of computation, related with Wang's tilings, that is at least Turing complete. I succeded in proving this, even for the particular case of functions f depending of only two variables and being commutative. Finally I developped an automatic proof method that is able to find out if a recurrent double sequence can be alternatively constructed by a system of substitutions of a given type. Although the method is purely computational and works only in concrete cases, it made possible a lot of results. Finally, I conjectured that if $A = G$ an arbitrary finite abelian group and $f : G^3 \to G$ ia a homomorphism of abelian groups, the resulting recurrent double sequence is always a product of a system of substitutions, like for Penrose's Tiling and similar combinatorial objects. The conjecture seems to be difficult to prove, but its verification on particular cases reveals us objects of unbelievable beauty and complexity.