IMAR 75
Bucharest, September 27 and 28, 2024

ABSTRACTS

Nonlinear Fokker–Planck flows and McKean–Vlasov stochastic differential equations

Viorel Barbu (Iași)

This talk is concerned with the existence theory for the Fokker–Planck equations (NFPEs)

$$u_t - \sum_{i,j=1}^{d} D_{ij}^2(a_{ij}(x,u)u) + \operatorname{div}(b(x,u)u) = 0 \text{ in } (0,\infty)\times\mathbb{R}^d,$$

$$u(0,x) = u_0(x),\ x\in\mathbb{R}^d.$$

This equation is relevant in statistical mechanics, mean field theory and also in stochastic analysis. In fact, a distributional solution u of this equation describes the microscopic dynamics of the stochastic differential differential equation (McKean–Vlasov SDE)

$$dX = b(x,u)dt + \sigma(x,u)dW.$$

The results surveyed here were obtained in collaboration with Michael Röckner (Bielefeld).

Optimal polarization pairs of codes in the Leech lattice

Peter Boyvalenkov (Sofia)

It was previously shown by the authors that the discrete potentials of almost all known sharp codes attain universal lower bounds for polarization (PULB) for spherical τ-designs, where "universal" is meant in the sense of applying to a large class of potentials that includes absolutely monotone functions of inner products and in the sense that the computational parameters of the bound are invariant with respect to the potential. In this talk we characterize the sets of universal minima D for some of these sharp codes C found in the Leech lattice and establish a duality relationship, namely that the normalized discrete potentials

of C and D have the same minimum value and the sets C and D are each others minima sets (up to antipodal symmetrization in some cases). The extremal duality is obtained by utilizing the natural embedding of the PULB pair codes in the Leech lattice and its properties, which simplifies the analysis significantly. In the process we discover a new universally optimal code in the projective space $\mathbb{RP}^{21}$ with cardinality 1408.

Joint work with S. Borodachov (Towson University), P. Dragnev (Purdue University Fort Wayne, USA), D. Hardin and E. Saff (Vanderbilt University, Nashville, USA), M. Stoyanova (Sofia University, Bulgaria).

Reaction-diffusion systems: Carleman estimates, unique continuation, controllability and inverse source estimates

Cătălin Lefter (Iaşi)

We consider systems of reaction-diffusion equations and we discuss various problems and corresponding results involving, as a central tool, Carleman estimates with either internal or boundary observations: Controllability to stationary solutions with a low number of localized, distributed controls entering only a part of the equations; Inverse source stability estimates with boundary observations; Unique continuation at initial time for parabolic systems.

Reaching consensus from different perspectives: self-supervised hypergraphs with multiple world interpretations

Marius Leordeanu (Bucharest)

The world can be interpreted from many perspectives and levels of semantic understanding, and all these interpretations are highly interdependent. We can often tell what happens in one part of a scene, at a specific time and location, by knowing what takes place in other parts. One way of interpreting the world can tell us a lot about other ways: geometry, semantics, motion and measurements from different sensors are strongly interrelated, each providing contextual information for the others.

In my talk, I will present our work during the last five years towards developing a complex multi-layer system that can exploit the connectivity between the many layers of sensing and understanding the world. Our multi-layer hypergraph is able to learn by itself, deal with missing data, estimate its own uncertainty and provide robust estimations. It achieves all these by reaching consensus among its many different layers through an iterative estimation and learning process.

I will also present applications of our model to one of the most relevant domains of our times, that of Earth Observations (EO) from satellites and drones — a field in which the search for an efficient multi-interpretation model is becoming increasingly important and active.

Proof mining in optimization and nonlinear analysis

Laurenţiu Leuştean (Bucharest)

The research program of proof mining is concerned with the extraction of hidden finitary content from mathematical proofs. The new information is obtained after a logical analysis, using proof-theoretic tools, and can be both of quantitative nature, such as algorithms and effective bounds, as well as of qualitative nature, such as uniformities in the bounds or weakening the premises. Thus, even if one is not particularly interested in the numerical details of the bounds themselves, in many cases such explicit bounds immediately show the independence of the quantity in question from certain input data. This line of research, developed by Kohlenbach in the 1990's, has its roots in Kreisel's program on unwinding of proofs, put forward in the 1950's. In this talk we give an introduction to proof mining and present some recent applications in optimization and nonlinear analysis.

Exponential stabilization of parabolic equations by H-infinity control technique

Gabriela Marinoschi (Bucharest)

The H-infinity control is a technique used in control theory to design robust stabilizing feedback controllers that force a system corrupted by perturbations to achieve stability with a prescribed performance. This method involves a transfer function which incorporates the effects of the input perturbations towards the output observation. The aim is to determine the optimal feedback controller which minimizes the effect of these perturbations on the output, by ensuring that the L^2-norm of the transfer function is smaller than the L^2-norm of the perturbation with a certain prescribed bound. We present the H-infinity control problem with state feedback for infinite dimensional boundary control systems of parabolic type with distributed disturbances and discuss some applications.

Invariants and obstructions in low regularity Sobolev spaces to manifolds

Dedicated to the memory of Haim Brézis

Petru Mironescu (Lyon)

We present a number of objects that 'hear' the homotopy properties or the singularities of Sobolev maps to manifolds. A simple starting point is the following: when M is a metric measure space and N is a smooth compact manifold, we may naturally associate with a vanishing mean oscillation (VMO) map $f : M \to N$ a homotopy class, which is a 'robust' object. Another example is $\int_M f^*\omega$, if $f \in \mathrm{VMO}(M; N)$, M is k-dimensional, Lipschitz, and compact, and ω is a closed k-form on N. This is again a robust (homotopy invariant) object. In higher dimensions, say $M = \mathbb{R}^d$, with $d > k$, one may define the current $d(f^*\omega)$, when f is in one of the 'critical' Sobolev spaces $W^{s,p}(\mathbb{R}^d; N)$ with $sp = k$. This is straightforward when $s \geq 1$, but more delicate when $0 < s < 1$. We explain the existence of this current, and its robust character. When N has a simple topology (e.g., when it is $(k-1)$-connected), the above currents detect all the topological the obstructions to the approximation with smooth maps with values into N. These results generalize previous works of Bethuel, Bourgain, Bousquet, Brezis, Coron, Demengel, Hélein, Giaquinta, Mucci, and the author. Joint work with Antoine Detaille and Kai Xiao (Université Claude Bernard Lyon 1).

Multiplier ideals of normal surface singularities

András Némethi (Budapest)

I present several new results regarding the structure of multiplier ideals, jumping number and multiplicities in the context of normal surface singularities. (Joint work with Laszlo T. and Koltai L., arXiv:2407.13413 [math.AG])

Special metrics on non-Kaehler manifolds

Liviu Ornea (Bucharest)

Different kinds of Hermitian metrics can be defined on complex manifolds which cannot bear Kaehler metrics. In this talk I shall discuss balanced, pluriclosed and locally conformal ones. I shall focus on the possibility of their coexistence with respect to the same complex structure.

Permutation and Hilbert-Schmidt stability

Liviu Păunescu (Bucharest)

A system of equations is stable if objects almost satisfying the system are close to objects that are an exact solution. We define permutation and Hilbert-Schmidt stability. We notice that they are group properties and discuss the similarities and the differences between the two notions.

Moments of quadratic Dirichlet L-functions over function fields

Alexandru Popa (Bucharest)

After reviewing recent progress in the moment problem, I will present an asymptotic formula for a "smoothed" fourth moment of quadratic Dirichlet L-functions over function fields, exhibiting infinitely many terms in the asymptotic expansion. The proof involves studying the multiple Dirichlet series associated with the fourth moment, a series in five complex variables satisfying an infinite group of functional equations. We show that this series can be analytically continued to an optimal domain, using a new type of functional equation. This is joint work with Adrian Diaconu and Vicenţiu Paşol.

From stochastic of boundary problems to numerical and neural networks representation

Ionel Popescu (Bucharest)

We will show how to use the stochastic representation of boundary problems to get in the first place a numerical scheme which overcomes the curse of dimensionality. This will be used in turn to get a neural network representation of the solution.

A conjecture on the space of GL_n-cuspidal functions in the function field case

Olivier Schiffmann (Paris)

Let X be a smooth projective curve of genus g defined over a finite field $\mathbb{F}_q$. For $n \geq 1$ and $d \in \mathbb{Z}$, let $C_{n,d}^{cusp} \subset Fun(Bun_{n,d}(X), \mathbb{C})$ denote the space of (everywhere unramified) cuspidal function of rank n and degree d. Its dimension was explicitly computed a few years

ago by Hongjie Yu, in terms of the so-called Kac polynomials of curves. Using the theory of cohomological Hall algebras, we propose a Lie-theoretic interpretation of this space (at least, of its dimension), and associate to it a module (of that dimension) over an algebra slightly larger than the usual Hecke algebra.

Characterising residually finite dimensional C^*-algebras in dynamical contexts

Adam Skalski (Warsaw)

A C^*-algebra is said to be residually finite-dimensional (RFD) when it has 'sufficiently many' finite-dimensional representations. The RFD property is an important, and still somewhat mysterious notion appearing in the theory of operator algebras, admitting several equivalent descriptions and having subtle connections to residual finiteness properties of groups. In this talk I will present certain characterisations of the RFD property for C^*-algebras arising as crossed products by amenable actions of discrete groups, extending (and inspired by) earlier results of Bekka, Exel and Loring. I will also explain the role of the amenability assumption and describe several consequences of our main theorems. Finally I will discuss some examples, notably these related to semidirect products of groups. Based on joint work with Tatiana Shulman (University of Gothenburg)

On the dynamics of polynomial automorphisms of $\mathbb{C}^2$

Raluca Tănase (Bucharest)

We discuss the dynamics of the complex Hénon map, a prototype of a two-dimensional dynamical system exhibiting stretching, folding, chaos and various coexisting phenomena, emphasizing important advances in the field, particularly in the parabolic conservative case. We discuss the Écalle-Hakim theory for germs tangent to the identity in $\mathbb{C}^n$ and its use in the characterization of the Julia set of the Hénon map. Based on joint work with T. Firsova, R. Radu, J. Raissy, and L. Vivas.