The 8th Bucharest Number Theory Day

TUESDAY, DECEMBER 29, 2020

Organizers: A.C. Cojocaru, Vicentiu Pasol, and Alexandru A. Popa

University of Illinois at Chicago, USA, and Institute of Mathematics of the Romanian Academy, Bucharest, Romania

SCHEDULE

Each talk is 30 minutes, followed by a 10 minute interval for questions & discussion.

(0) 16:45-17:00 (Bucharest & Tel Aviv) / 08:45-09:00 (Chicago)

To join the zoom meeting, please e-mail math.cojocaru at gmail.com. While no registration is necessary, all participants are asked to please display their real name.

(1) 17:00-17:30 (Bucharest & Tel Aviv) / 09:00-09:30 (Chicago)

Nathan Jones, University of Illinois at Chicago, USA

Title: *The Lang-Trotter Conjecture for products of non-CM elliptic curves*

(2) 17:40-18:10 (Bucharest & Tel Aviv) - 09:40-10:10 (Chicago)

Tian Wang, University of Illinois at Chicago, USA

Title: *Bounds for the distribution of the Frobenius traces associated to products of non-CM elliptic curves*

(3) 18:20-18:50 (Bucharest & Tel Aviv) - 10:20-10:50 (Chicago)

Vlad Matei, Tel Aviv University, Israel

Title: *Average size of the automorphism group of smooth projective hypersurfaces*

(4) 19:00-19:30 (Bucharest & Tel Aviv) - 11:00-11:30 (Chicago)

Jacob Mayle, University of Illinois at Chicago, USA

Title: *Galois representations of abelian varieties*

(5) 19:40-20:10 (Bucharest & Tel Aviv) - 11:40-12:10 (Chicago)

George Catalin Turcas, Babes -Bolyai University, Cluj Napoca, Romania

Title: *Irreducibility of mod p Galois representations of elliptic curves with multiplicative reduction over number fields*

(6) 20:20-20:30 (Bucharest & Tel Aviv) - 12:20-12:30 (Chicago)

Concluding remarks

ABSTRACTS

(1) 17:00-17:30 (Bucharest & Tel Aviv) / 09:00-09:30 (Chicago)

Nathan Jones, University of Illinois at Chicago, USA

Title: *The Lang-Trotter Conjecture for products of non-CM elliptic curves*

Abstract: Inspired by the work of Lang-Trotter on the densities of primes with fixed Frobenius traces for elliptic curves defined over $\mathbb{Q}$ and by the subsequent generalization of Cojocaru-Davis-Silverberg-Stange to generic abelian varieties, in this talk I will consider the analogous question for abelian surfaces isogenous to products of non-CM elliptic curves over $\mathbb{Q}$. I will formulate the corresponding conjectural asymptotic, provide upper bounds, and describe explicitly (when the elliptic curves lie outside a thin set) the arithmetically significant constants appearing in the asymptotic. Finally, I will show computational evidence for the conjecture. This is based on joint work with Hao Chen and Vlad Serban.

(2) 17:40-18:10 (Bucharest & Tel Aviv) - 09:40-10:10 (Chicago)

Tian Wang, University of Illinois at Chicago, USA

Title: *Bounds for the distribution of the Frobenius traces associated to products of non-CM elliptic curves*

Abstract: Let $E_1/\mathbb{Q}, \ldots, E_g/\mathbb{Q}$ be elliptic curves over $\mathbb{Q}$, without complex multiplication and pairwise non-isogenous over $\overline{\mathbb{Q}}$. For an integer t and a positive real number x, denote by $\pi_A(x, t)$ the number of primes $p \leq x$, of good reduction for the abelian variety $A := E_1 \times \ldots \times E_g$, for which the Frobenius trace associated to the reduction of A modulo p equals t. Building on prior approaches to the Lang-Trotter Conjecture for the Frobenius traces associated to the reductions of an elliptic curve, we prove non-trivial upper bounds in x for $\pi_A(x, t)$. This is joint work with A.C. Cojocaru.

(3) 18:20-18:50 (Bucharest & Tel Aviv) - 10:20-10:50 (Chicago)

Vlad Matei, Tel Aviv University, Israel

Title: *Average size of the automorphism group of smooth projective hypersurfaces*

Abstract: We show that, as d goes to infinity, the average size of the automorphism group $\mathrm{Aut}_{\mathbb{F}_q}(X)$ of a smooth degree d hypersurface X in $\mathbb{P}^n(\mathbb{F}_q)$ is equal to 1. We also describe work in progress showing that the average size of the automorphism group over the algebraic closure of degree $d \geq 4$ smooth curves in $\mathbb{P}^2(\mathbb{F}_q)$ is equal to 1, as $q \to \infty$ and under a certain assumption on the characteristic.

(4) 19:00-19:30 (Bucharest & Tel Aviv) - 11:00-11:30 (Chicago)

Jacob Mayle, University of Illinois at Chicago, USA

Title: *Galois representations of abelian varieties*

Abstract: Over the past decade, there has been considerable interest and progress in computing the Galois image of an elliptic curve. Notably, Sutherland gave an algorithm that effectively computes the mod ℓ image, and Brau gave an algorithm that (in principle) computes the full adelic image. In this talk, we will discuss recent progress toward these problems in the realm of higher-dimensional abelian varieties.

(5) 19:40-20:10 (Bucharest & Tel Aviv) - 11:40-12:10 (Chicago)

George Catalin Turcas, Babes-Bolyai University, Cluj Napoca, Romania

Title: *Irreducibility of mod p Galois representations of elliptic curves with multiplicative reduction over number fields*

Abstract: For every integer $d \geq 1$, there exists an explicit constant B_d such that the following holds. Let K be a number field of degree d, let $q > \max\{d - 1, 5\}$ be any rational prime that is totally inert in K, and let E be any elliptic curve defined over K such that E has potentially multiplicative reduction at the prime above q. Then, for every rational prime $p > B_d$, E has an irreducible mod p Galois representation. In this talk, we will discuss the ingredients that go into the proof of the aforementioned result and, time permitting, we will present a Diophantine application. Everything is joint work with Filip Najman.