THE INSTITUTE OF MATHEMATICS "SIMION STOILOW" OF THE ROMANIAN ACADEMY

DOCTORAL THESIS
Summary

Analytical Pseudoconvexity

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Bucharest, 2014

Introduction

General notions and fundamental results in the theory of Stein, q-convex (with corners) and q-complete (with corners) spaces are included in the first half of the thesis. Also, the specific concepts are presented: convexity with respect to a linear set introduced by M. Peternell [28], the Andreotti function of an analytic subset and the definition of a (strictly) q-plurisubharmonic function (see [15]).

The second part contains the original results (Theorem 2, Theorem 5 and Theorem 6). This theorems are included in the papers [18], [19] and [20], which have been accepted for publication.

Theorem 2 generalizes results obtained by Stein [31], Ballico [2], [3], Le Barz [21] and Vâjâitu [34] and says the following: if $\pi : Z \to X$ is a locally semi-proper morphism of complex spaces such that X is q-complete, then Z is $(q+r)$-complete, where r is the dimension of the fiber.

In the next part there are presented generalizations of the solution to the Levi problem on complex spaces with isolated singularities given by Colţoiu and Diederich [8]. They proved that if $p : Y \to X$ is a Riemann domain, where X and Y are complex spaces with isolated singularities such that X is Stein and p is a Stein morphism, then Y is Stein. This result is improved into two ways:

- we suppose that X is q-complete and we get that Y is q-complete (see Theorem 5);
- we suppose that the morphism p is locally q-complete with corners and we get that Y is q-complete with corners (see Theorem 6).

The structure of the thesis. This thesis consists of an introduction, four chapters and a bibliography with 63 titles.

Acknowledgments

First of all I would like to thank Professor Mihnea Colţoiu, my thesis supervisor, for suggesting me the problems included in the thesis and also for his patience and his involvement during the last years.

I am very grateful to Professor Cezar Joiţa who coordinated my Bachelor and Master thesis at SNS-B. I thank him for the kindness with which he answered all the questions and for being the one who gave me the opportunity to start working in Several Complex Variables.

I would like to thank my colleague, Ovidiu Preda, with whom I had very interesting discussions about Stein spaces, q-convexity and locally semi-finite morphisms.

I am very grateful to the referees: Prof. Dr. George Marinescu (Köln University, Germany), C.S. II Dr. Cezar Joiţa (”Simion Stoilow” Institute of Mathematics of the Romanian Academy) and Conf. Dr. Constantin Costara (”Ovidius” University, Constanţa, Romania) for their time and effort in reading my thesis manuscript and for their constructive remarks.

Also, I would like to give special thanks to the ”Simion Stoilow” Institute of Mathematics of the Romanian Academy and to SNS-B. Without their help I would have not arrived at this point.

Last but not least, I would like to thank my friends and family for their moral support and their permanent encouragements.

Contents

The central notions in the first chapter are those of a Stein space, (strictly) plurisubharmonic and q-convex function on an open subset of $\mathbb{C}^n$ and those of a q-convex and a q-complete complex space. An equivalent characterization for a (stricly) plurisubharmonic function is given using the Levi form of that function. Using local embeddings the concepts of a (strictly) plurisubharmonic and a q-convex function are generalized in the context of complex spaces.

The following result represents the most important characterization of Stein spaces.

Theorem 1 ([16] and [23], [24])**.** *A complex space X is Stein if and only if there exists $\varphi : X \to \mathbb{R}$ a continuous strongly plurisubharmonic exhaustion function on X.*

The notions of a strictly plurisubharmonic function and of a Stein space are generalized in the following way.

Definition 1. *Let X be a complex space and $\varphi \in \mathcal{C}^\infty(X, \mathbb{R})$ a smooth function on X. The function φ is called q-convexă if for every $x \in X$ there exists a local chart $\iota : U \hookrightarrow \widetilde{U} \subset \mathbb{C}^n$ and a function $\widetilde{\varphi} \in \mathcal{C}^\infty(\widetilde{U}, \mathbb{R})$ such that $\widetilde{\varphi} \circ \iota = \varphi|_U$ and with the property that the Levi form of $\widetilde{\varphi}$ has at least $n - q + 1$ positive eigenvalues (> 0) in every point of $\widetilde{U}$.*

Definition 2. *A complex space X is said to be q-convex, if there exists a compact subset K of X and a smooth exhaustion function $\varphi : X \to \mathbb{R}$,* *which is q-convex on $X \setminus K$. If we can choose $K = \emptyset$, then X is said to be q-complete.*

In the last part of this chapter is presented the concept of a q-convex with corners function; this function is locally the maximum of a finite number of q-convex functions (see [12] and [13]).

The second chapter contains the specific results needed in the thesis.

To overcome the problem that neither the sum nor the maximum of two q-convex functions is not q-convex (as they might have different directions of positivity), M. Peternell [28] introduced the concept of convexity with respect to a linear set.

In what follows the notion of the Andreotti function of an analytical subset $A \subset X$ is presented, where X is a complex space. The Andreotti function will help us get some positive eigenvalues in the "normal direction" at the regular points of A.

This chapter ends with a brief presentation of (strictly) q-plurisubharmonic functions.

Chapter 3 contains the first original result of this thesis.

We begin with the remark that if $\pi : Z \to X$ is finite morphism of complex spaces, then Z is Stein if and only if X is Stein.

In [31], Stein proved that if X and Z are complex spaces and $\pi : Z \to X$ is an unramified covering such that X is Stein, then Z is Stein.

The last result was generalized to ramified coverings by Le Barz [21]. He proved that if $\pi : Z \to X$ is a locally semi-finite morphism such that X is Stein, then Z is Stein.

Ballico [3] improved Stein's theorem in a different direction: consider $\pi : Z \to X$ be an unramified covering of complex spaces. If X is q-complete, then Z is q-complete. Also, in [2], Ballico showed that if $\pi : Z \to X$ is a finite morphism of complex spaces such that X is q-complete and q-convex, respectively, then Z is q-complete and q-convex, respectively.

Colţoiu and Vâjâitu [11] proved that if $\pi : E \to B$ is a locally analytic fibration of complex spaces such that the fiber is a Stein curve and B is q-complete, then E is q-complete. The case when E is a topological covering of B was already done in [3].

Vâjâitu [34] generalized Ballico's results in [2] and showed the following theorem: let $\pi : Z \to X$ be a proper holomorphic map between finite dimensional complex spaces. If X is q-complete, then Z is $(q + r)$-complete, where r is the dimension of the fiber.

The purpose of this chapter is to prove a theorem (Theorem 2) which contains all the results mentioned before. This theorem is included in [18].

Following the ideas of Le Barz [21] we give the next definition.

Definition 3. *Let X and Z be two complex spaces. We say that a morphism $\pi : Z \to X$ is*

(a) semi-proper if Z is the disjoint union of some open spaces $(W^m)_{m\in\mathbb{N}}$ such that $\pi|_{W^m} : W^m \longrightarrow X$ is proper;

(b) locally semi-proper if for all $x \in X$, there exists a neighbourhood $U \ni x$ such that $\pi|_{\pi^{-1}(U)} : \pi^{-1}(U) \longrightarrow U$ is a semi-proper morphism.

Now we are ready to state the main result.

Theorem 2 ([18])**.** *Let X and Z be two complex spaces and $\pi : Z \to X$ a locally semi-proper morphism and $r = \max\{\dim \pi^{-1}(x) : x \in X\}$. If X is q-complete, then Z is $(q+r)$-complete.*

The proof of the above mentioned result uses the following criterion for the q-completeness of a complex space.

Theorem 3 ([10])**.** *Let X be a complex space and $\mathcal{M}$ a linear set over X. Let $\{X_i\}_{i\in\mathbb{N}}$ be an increasing sequence of open subsets of X such that $X = \bigcup_{i\in\mathbb{N}} X_i$ and there are functions $u_i : X_i \to \mathbb{R}$, $u_i \in \mathcal{B}(X_i, \mathcal{M}|_{X_i})$ and constants $C_i, D_i \in \mathbb{R}$, $C_i < D_i$, $i \in \mathbb{N}$ with the following properties:*

(a) $\{x \in X_i : u_i(x) < D_i\} \subset\subset X_i$ for every $i \in \mathbb{N}$

(b) $\{x \in X_{i+1} : u_{i+1}(x) < C_i\} \subset \{x \in X_i : u_i(x) < D_i\}$ for every $i \in \mathbb{N}$;

(c) for every compact set $K \subset X$ there is $j = j(K) \in \mathbb{N}$ such that

$$K \subset \{x \in X_{i+1} : u_{i+1}(x) < C_i\} \text{ for every } i \geq j.$$

Then there exists an exhaustion function $v \in \mathcal{B}(X, \mathcal{M})$. In particular, if $\operatorname{codim} \mathcal{M} \leq q-1$, then X is q-complete.

The next results contribute essentially in the proof of Theorem 2.

Lemma 1 ([28])**.** *Suppose that φ is a q-convex function on a complex space X. Then there exists a linear set $\mathcal{M}$ over X of codimension $\leq q-1$ such that φ is 1-convex with respect to $\mathcal{M}$.*

Lemma 2 ([28]). *Let $\iota : U \hookrightarrow \widetilde{U}$ be a local chart of the complex space X and $\varphi : U \to \mathbb{R}$ a smooth function. Then φ is 1-convex with respect to some linear set $\mathcal{M}$ if and only if for every compact subset $K \subset U$ there exists $\delta > 0$ and for each $x \in K$ there exist $\widetilde{\varphi} \in C^\infty(\widetilde{U}, \mathbb{R})$ such that $\widetilde{\varphi} \circ \iota = \varphi$ and*

$$L(\widetilde{\varphi}, \iota(x))\iota_*(\xi) \geq \delta \left\| \iota_*(\xi) \right\|^2$$

for all $\xi \in \mathcal{M}_x$.

Proposition 1 ([34]). *Let $\pi : Z \to X$ be a holomorphic map. Then there exists a decreasing chain of $p+1$ analytic subsets A_k of Z, where $p \leq \dim Z$, $Z = A_p \supset A_{p-1} \supset \cdots \supset A_1 \supset A_0 = \emptyset$ such that for every $k \in \{1, 2, \ldots, p\}$ we have $\dim A_{k-1} < \dim A_k$, Sing $(A_k) \subset A_{k-1}$ and*

$$\pi|_{A_k \setminus A_{k-1}} : A_k \setminus A_{k-1} \to X$$

has locally constant rank.

Lemma 3 ([21]). *Let X and Z be two complex spaces and $\pi : Z \to X$ a locally semi-proper morphism. Then there exists a locally finite covering $\{U_j\}_j$ of Z and a locally finite covering $\{V_l\}_l$ of X such that the following conditions hold:*

1. *for all j, there exists a positive integer m_j and a local chart $\iota_j : U_j \hookrightarrow \widetilde{U}_j$, where $\widetilde{U}_j$ is an open subset of $\mathbb{C}^{m_j}$;*
2. *for all l, there exists a positive integer n_l and a local chart $\tau_l : V_l \hookrightarrow \widetilde{V}_l$, where $\widetilde{V}_l$ is an open subset of $\mathbb{C}^{n_l}$;*
3. *for all j, there exists $l(j)$ such that we have $\pi(U_j) \subset V_{l(j)}$ and $\pi|_{U_j}$ extends to a holomorphic map $\widetilde{\pi} : \widetilde{U}_j \to \widetilde{V}_{l(j)}$;*

Also, there exists a C^∞ function $f : Z \to \mathbb{R}$ such that:

- *$\{z \in Z : f(z) < c_1\} \cap \{z \in Z : (\varphi \circ \pi)(z) < c_2\} \subset\subset Z$, $\forall c_1, c_2 \in \mathbb{R}$;*
- *for all j, there exists a map $g_j : V_{l(j)} \to \mathbb{R}$ such that $f|_{U_j} = g_j \circ \pi|_{U_j}$;*
- *g_j has a C^∞ extension, $\widetilde{g}_j : \widetilde{V}_{l(j)} \to \mathbb{R}$;*
- *for all compact sets $K \subset X$,*

$$\sup_{j \in \mathbb{N}} \left\{ \left| \frac{\partial^2 \widetilde{g}_j}{\partial z_r^{(l(j))} \partial \bar{z}_s^{(l(j))}} \right|_{|_{\tau_{l(j)}(V_{l(j)} \cap K)}} : V_{l(j)} \cap K \neq \emptyset, r, s = \overline{1, n_{l(j)}} \right\} < \infty.$$

Lemma 4 ([34])**.** *Let $\pi : Z \to X$ be a holomorphic map between reduced complex spaces with $r = \max\{\dim \pi^{-1}(x) : x \in X\}$. Then there exists $\mathcal{N}$ a linear set of codimension $\leq r$ over Z such that for any relatively compact open subset U of Z, there exists a finite covering $\{V_l\}_l$ of $\overline{\pi(U)}$ by relatively compact open subsets and smooth functions $\psi_l : U_l \to \mathbb{R}_+$ such that ψ_l is 1-convex with respect to $\mathcal{N}$ over $U_l \cap U$, where $U_l = \pi^{-1}(V_l)$.*

The chapter ends with a converse theorem of that of Le Barz, namely if X is n-dimensional Stein space, then there exists $f : X \to \mathbb{C}^n$ locally semi-finite morphism. The idea that emerges from the proof of the above mentioned result is that if $f : X \to \mathbb{C}^n$ is a locally semi-finite morphism, where X is a n-dimensional Stein space, then f is almost proper. The converse of the last statement is studied and it is showed that if $f : X \to Y$ is a locally semi-finite morphism, with Y being Stein, then f is not necessarily almost proper. The example that is considered is the covering with an infinite number of sheets of the punctured disk in $\mathbb{C}$, covering given by the logarithm function.

In the last chapter are included the original results from the papers [19] and [20].

By the solution of the Levi problem if $Y \subset \mathbb{C}^n$ is an open subset which is locally Stein, i.e., every point $x \in \mathbb{C}^n$ has a neighbourhood V such that $V \cap Y$ is Stein, then Y is itself Stein (Oka [26] and [27], Bremermann [4], Norguet [25]). From Oka's characterization of Stein domains in $\mathbb{C}^n$ we have that in this case $-\log d$ is plurisubharmonic on Y, where d represents the euclidean distance to the boundary of Y. More generally, K. Oka considered unbranched Riemann domains $p : Y \to \mathbb{C}^n$ over $\mathbb{C}^n$ and he showed that Y is Stein if and only if $-\log d$ is plurisubharmonic on Y. This implies that if p is a Stein morphism (i.e. each point $x \in \mathbb{C}^n$ has a neighbourhood $V = V(x)$ such that $p^{-1}(V)$ is Stein), then Y is Stein.

Grauert and Docquier [14] generalized Oka's result in the context of Stein manifolds. In particular, they proved that if $p : Y \to X$ is an unbranched Riemann domain, p is a Stein morphism and X is Stein, then Y is Stein.

In [1], Andreotti and Narasimhan showed that if $Y \subset X$ is an open subset of a Stein space X with isolated singularities and Y is locally Stein, then Y is (globally) Stein. The general case of the above result, for arbitrary singularities, is called the "local Steinness problem" or the "Levi problem on singular spaces" and it is still an open problem. For a survey concerning the Levi problem on Stein spaces see [7] and [30].

Colţoiu and Diederich improved the above mentioned results and they proved the following theorem.

Theorem 4 ([8])**.** *Let X and Y be two complex spaces with isolated singu-*

larities and $p : Y \to X$ an unramified Riemann domain. Suppose that X is Stein and that p is a Stein morphism, i.e., every point $x \in X$ has an open neighbourhood $V = V(x)$ such that $p^{-1}(V)$ is Stein. Then Y is Stein.

The goal of this chapter is to improve Colţoiu and Diederich's theorem in two directions:

(1) modifying the hypothesis about X being Stein (X will now be q-complete);

(2) modifying the hypothesis about $p : Y \to X$ being a Stein morphism (p will now be locally q-complete with corners).

The results detailed below are obtained from the above remarks and will be proved in the subsequent chapters. The theorems have been included in the papers [19] (Theorem 5) and [20] (Theorem 6), that have been accepted for publication.

Theorem 5 ([19])**.** *Let X and Y be complex spaces with isolated singularities and $p : Y \to X$ an unbranched Riemann domain. Assume that X is a q-complete space and that p is a Stein morphism, i.e., each point $x \in X$ has a neighbourhood $V = V(x)$ such that $p^{-1}(V)$ is Stein. Then Y is also q-complete.*

Theorem 6 ([20])**.** *Let X and Y be complex spaces with isolated singularities and $p : Y \to X$ an unbranched Riemann domain. Assume that X is Stein and that p is locally q-complete with corners, i.e., each point $x \in X$ has a neighbourhood $V = V(x)$ such that $p^{-1}(V)$ is q-complete with corners. Then Y is q-complete with corners.*

If X and Y are smooth, then Theorem 5 was showed by Vâjâitu in [33].

If p is the inclusion map, then Theorem 6 was proved by Vâjâitu in [32]. Also, if X and Y are smooth, Vâjâitu showed that if X is r-complete with corners and p is locally q-complete with corners, then Y is $(q+r-1)$-complete with corners (see [33]).

Two reductions are made in both proofs: $\mathrm{Sing}(X) = \{x_1, x_2, \ldots, x_k\}$ is a finite set and $p(Y) \subset\subset X$. Also, we distinguish between two cases:

(a) $x_1, x_2, \ldots, x_k \notin p(Y)$;

(b) $x_1, \ldots, x_l \notin p(Y)$ and $x_{l+1}, \ldots, x_k \in p(Y)$.

As in the case of Theorem 2, we need a couple of criteria for testing the q-completeness and the q-completeness with corners of a complex space. We denote by $\mathrm{F}_q(X)$ the set of all q-convex functions with corners on X.

Proposition 2 ([33]). *Let Y be a complex space and $\mathcal{N}$ a linear set over Y of codimension $\leq q-1$. Suppose that there exists a function $\Phi : Y \to \mathbb{R}$ such that $\Phi \in \mathcal{B}(Y, \mathcal{N})$ and for every $c \in \mathbb{R}$ the sublevel set $\{\Phi < c\}$ is $\mathcal{N}$-complete (i.e., there is an exhaustion function $u_c \in \mathcal{B}(\{\Phi < c\}, \mathcal{N}|_{\{\Phi<c\}})$). Then Y is q-complete.*

Proposition 3 ([33]). *Let X be a complex space and $\Phi \in F_q(X)$ such that for every $c \in \mathbb{R}$ the set $X_c := \{\Phi < c\}$ is q-complete with corners. Then X is q-complete with corners.*

The fundamental results that are used in the proofs are Lemma 1 (for Theorem 5) and the following theorem.

Theorem 7 ([10]). *Let X be a 1-convex complex space. Then X carries a strongly plurisubharmonic exhaustion function $\Phi : X \to [-\infty, \infty)$. Moreover, Φ can be chosen $-\infty$ exactly on the exceptional set S of X and real analytic outside S.*

The ideas that are used in the subsequent are inspired by [8]. We recall here a classical patching technique for plurisubharmonic functions with bounded differences (using the results of M. Peternell [29] and Matsumoto [22]) and Hörmander's [17] regularization method needed to avoid the difficulties that appear when trying to build a function with a Levi form bounded from below in the vertical direction. This ideas, as well as Theorem 7, are present in the proofs of both theorems.

For the proof of Theorem 6 we need specific results.

Theorem 8 ([5]). *Let X be a complex manifold and $f : X \to \mathbb{R}$ a continuous strongly q-plurisubharmonic function. Then for an arbitrary continuous function $\delta : X \to (0, \infty)$ there exists a function $\widetilde{f} \in F_q(X)$ such that $|\widetilde{f} - f| < \delta$.*

Lemma 5 ([6]). *Consider X to be a complex space, $A \subset X$ an analytic subset and $f \in F_q(A)$. Then for every $\eta \in C^0(A, \mathbb{R})$, $\eta > 0$ there is an open neighbourhood U of A in X and $\widetilde{f} \in F_q(U)$ such that $\left| \widetilde{f}|_A - f \right| \leq \eta$.*

Lemma 6 ([28]). *Let X be a complex space and $A \subset X$ an analytic subset. Then there exists $h \in C^\infty(X, \mathbb{R})$, $h \geq 0$ such that:*

(a) $\{h = 0\} = A$;

(b) *for every $x \in X$ there exists an open neighbourhood U of x and a smooth function $\sigma : U \to \mathbb{R}$ such that $\log(h|_{U \setminus A}) + \sigma|_{U \setminus A}$ is plurisubharmonic.*

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