Romanian Academy
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Doctoral thesis

Moments for Lie group representations

—Abstract—

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Contents

Chapter 1

Pre-Lie groups

This chapter contains general notions that are referred to throughout this work.

1.1 Preliminaries on the adjoint action of a topological group

Let G be any topological group with the set of neighborhoods of $1 \in G$ denoted by $\mathcal{V}_G(1)$. Define

$$\Lambda(G) = \{\gamma : \mathbb{R} \to G \text{ cont.} \mid (\forall s, t \in \mathbb{R}) \quad \gamma(s+t) = \gamma(s)\gamma(t)\}$$

and

$$(\forall n \geq 1) \quad \Lambda^n(G) := \underbrace{\Lambda(G) \times \cdots \times \Lambda(G)}_{n \text{ ori}}.$$

The elements of $\Lambda(G)$ are called *one-parameter subgroups* of G. We endow $\Lambda(G)$ with the topology of uniform convergence on compact subsets of $\mathbb{R}$. This topology is described by a neigborhood base at any point in $\Lambda(G)$ like this: for every $\gamma \in \Lambda(G)$ and $V \in \mathcal{U}$ (an open neighborhhod base at $1 \in G$) define

$$U_V(\gamma) := \{\beta \in \Lambda(G) \mid \gamma(-t)\beta(t) \in V, (\forall)t \in [-1, 1]\}.$$

The sets $U_V(\gamma)$ for $\gamma \in \Lambda(G)$ and $V \in \mathcal{U}$ form a neighborhood base of the topology of $\Lambda(G)$. For every $x \in G$ and $\gamma \in \Lambda(G)$ one defines $\gamma^x : \mathbb{R} \to G$, $\gamma^x(t) := x^{-1}\gamma(t)x$. We say that γ^x is the *conjugate of* γ *with respect to* $x \in G$ and we have $\gamma^x \in \Lambda(G)$.

1.2 Diferentiability on topological groups

Definition 1.2.1. Let G be any topological group, $\mathcal{Y}$ be any locally convex space, and X be any open subset of G. A continuous function $\varphi : X \to \mathcal{Y}$ is *differentiable of class* C^k (or k *times continuously differentiable*) if:

a) There exist $D^j\varphi : X \times \Lambda^j(G) \to \mathcal{Y}; j = 0, 1, \ldots, k$ with $D^0\varphi = \varphi$ and for every $x \in X$, $j = 0, 1, \ldots, k-1$ and $(\gamma_1, \ldots, \gamma_{j+1}) \in \Lambda^{j+1}(G)$ we have

$$D^{j+1}\varphi(x; \gamma_1, \ldots, \gamma_j, \gamma_{j+1}) := \frac{d}{dt}\Big|_{t=0} D^j\varphi(x\gamma_{j+1}(t); \gamma_1, \ldots, \gamma_j).$$

b) The derivative $D^j\varphi : X \times \Lambda^j(G) \to \mathcal{Y}$ is continuous for every $j \leq k$. We denote by $C^k(X, \mathcal{Y})$ the set of all functions $\varphi : X \to \mathcal{Y}$ which are k times continuously differentiable and

$$C^\infty(X, \mathcal{Y}) := \bigcap_{n \geq 1} C^n(X, \mathcal{Y}).$$

If $\mathcal{Y} = \mathbb{C}$ then $C^n(X) := C^n(X, \mathbb{C})$ for every $n = 1, 2, \ldots, \infty$.

Replacing $x\gamma_{j+1}(t)$ by $\gamma_{j+1}(t)x$ in the above definition, we obtain the notion of a function which is k *times continuously differentiable to the right* and the corresponding sets $C^k_{dr}(X, \mathcal{Y})$ and $C^\infty_{dr}(X, \mathcal{Y})$. The derivative to the right will be denoted by $D^j_r\varphi$.

Proposition 1.2.2. *$C^\infty(X, \mathcal{Y}) = C^\infty_{dr}(X, \mathcal{Y})$ for every open subset X of the topological group G.*

Definition 1.2.3 ([BCR81],[HM07])**.** We say that G is a topological group *topological group with Lie algebra* if the topological space $\Lambda(G)$ has the structure of a topological Lie algebra over $\mathbb{R}$ whose algebraic operations satisfy the following conditions for all $t, s \in \mathbb{R}$ and $\lambda, \gamma \in \Lambda(G)$,

$$\begin{aligned}
(t\lambda)(s) &= \lambda(ts); \\
(\lambda + \gamma)(t) &= \lim_{n\to\infty} (\lambda(\frac{t}{n})\gamma(\frac{t}{n}))^n; \\
[\lambda, \gamma](t^2) &= \lim_{n\to\infty} (\lambda(\frac{t}{n})\gamma(\frac{t}{n})\lambda(-\frac{t}{n})\gamma(-\frac{t}{n}))^{n^2},
\end{aligned}$$

with uniform convergence on the compact subsets of $\mathbb{R}$.

We say that the topological group G is a *pre-Lie group* if it is a topological group with Lie algebra and for every nonconstant $\gamma \in \Lambda(G)$ there exists a real-valued function f of class C^∞ on some neighborhood of $1 \in G$ such that $Df(1; \gamma) \neq 0$.

Chapter 2

Enveloping algebras for topological groups

In this chapter we point out the advantages of one of the main technical novelties introduced in this thesis, namely the construction of a convenient topology on the space of differentiable functions on some topological group with values in a locally convex space $\mathcal{Y}$.

2.1 Preliminaries

Unless otherwise mentioned, we denote by G an arbitrary topological group.

2.2 Algebras of local operators

Definition 2.2.1. A *local operator* on G is any continuous linear operator $D\colon \mathcal{E}(G) \to \mathcal{E}(G)$ with the property

$$(\forall f \in \mathcal{E}(G)) \quad \operatorname{supp}(Df) \subseteq \operatorname{supp} f.$$

We will denote by $\operatorname{Loc}(G)$ the set of all local operators on G.

Definition 2.2.2. Denote

$$(\forall x \in G) \quad L_x\colon G \to G,\ L_x(y) = xy.$$

The *left-invariant local operators* on G are the elements of the set

$$\mathcal{U}(G) := \{D \in \mathrm{Loc}\,(G) \mid (\forall x \in G)(\forall f \in C^\infty(G)) \quad D(f \circ L_x) = (Df) \circ L_x\}.$$

2.3 Distributions with compact support

Definition 2.3.1. Denote

$$(\forall k \geq 1) \quad \Lambda^k(G) := \underbrace{\Lambda(G) \times \cdots \times \Lambda(G)}_{k \text{ times}}.$$

Pick any open set $V \subseteq G$. If $\mathcal{Y}$ is any locally convex space, then for every $k \geq 1$, any compact subsets $K_1 \subseteq \Lambda^k(G)$ and $K_2 \subseteq V$, and any continuous seminorm $|\cdot|$ on $\mathcal{Y}$ we define

$$p^{|\cdot|}_{K_1,K_2} \colon C^\infty(V, \mathcal{Y}) \to [0, \infty), \quad p^{|\cdot|}_{K_1,K_2}(f) = \sup\{|(D^\lambda_\gamma f)(x)| \mid \gamma \in K_1, x \in K_2\}.$$

For the sake of simplicity we will always omit the seminorm $|\cdot|$ on $\mathcal{Y}$ from the above notation, by writing simply p_{K_1,K_2} instead of $p^{|\cdot|}_{K_1,K_2}$.

We endow the function space $C^\infty(V, \mathcal{Y})$ with the locally convex topology defined by the family of these seminorms p_{K_1,K_2} and the locally convex space obtained in this way will be denoted by $\mathcal{E}(V, \mathcal{Y})$. If $\mathcal{Y} = \mathbb{C}$ then we write simply $\mathcal{E}(V) := \mathcal{E}(V, \mathcal{Y})$.

We also denote by $\mathcal{E}'(G)$ the topological dual of $\mathcal{E}(G)$ endowed with the weak dual topology. This means that we have

$$\mathcal{E}'(G) = \{u \colon \mathcal{E}(G) \to \mathbb{C} \mid u \text{ is linear and continuous}\}$$

as a linear space, and this space of linear functionals is endowed with the locally convex topology defined by the family of seminorms $\{q_B \mid B \text{ finite } \subseteq \mathcal{E}(G)\}$, where for every finite subset $B \subseteq \mathcal{E}(G)$ we define the seminorm

$$q_B \colon \mathcal{E}'(G) \to \mathbb{C}, \quad q_B(u) := \max_{f \in B} |u(f)|.$$

The elements of $\mathcal{E}'(G)$ will be called *distributions with compact support* on G.

Definition 2.3.2. For every $f \in \mathcal{E}(G)$ we define $\check{f} \in \mathcal{E}(G)$ by

$$(\forall x \in G) \quad \check{f}(x) := f(x^{-1}).$$

For every $u \in \mathcal{E}'(G)$ we define $\check{u} \in \mathcal{E}'(G)$ by

$$(\forall f \in \mathcal{E}(G)) \quad \check{u}(f) := u(\check{f}).$$

For every $f \in \mathcal{E}(G)$ and $u \in \mathcal{E}'(G)$ we define their *convolution* as the function

$$f * u : G \to \mathbb{C}, \quad (f * u)(x) := \check{u}(f \circ L_x).$$

2.4 Exponential law for smooth functions on topological groups

Theorem 2.4.1. *Let G and H be any topological groups and $\mathcal{Y}$ be any locally convex space. Then for arbitrary $\varphi \in C^\infty(H \times G, \mathcal{Y})$, the corresponding function*

$$\widetilde{\varphi}\colon H \to C^\infty(G, \mathcal{Y}), \quad \widetilde{\varphi}(x)(g) := \varphi(x, g).$$

belongs to $C^\infty(H, \mathcal{E}(G, \mathcal{Y}))$. Moreover, the map

$$\mathcal{E}(H \times G, \mathcal{Y}) \to \mathcal{E}(H, \mathcal{E}(G, \mathcal{Y})), \quad \varphi \mapsto \widetilde{\varphi}$$

is an algebraic topological embedding of locally convex spaces.

2.5 Structure of invariant local operators

Proposition 2.5.1. *For every $f \in \mathcal{E}(G)$ and $u \in \mathcal{E}'(G)$ we have $f * u \in \mathcal{E}(G)$.*

Theorem 2.5.2. *Let G be any topological group and for every $u \in \mathcal{E}'(G)$ define the linear operator $D_u\colon C^\infty(G) \to C^\infty(G)$, $D_u f = f * u$*

Then the operator $\Psi\colon \mathcal{E}'_1(G) \to \mathcal{U}(G)$, $\Psi(u) = D_u$ is well defined, invertible, and its inverse is

$$\Psi^{-1}\colon \mathcal{U}(G) \to \mathcal{E}'_1(G), \quad (\Psi^{-1}(D))(f) = (D\check{f})(1) \text{ for all } f \in C^\infty(G) \text{ and } D \in \mathcal{U}(G).$$

Notes. The contents of this chapter are new and are based on the article [BNi14a] submitted for publication.

Chapter 3

Tangent groups of a 2-step nilpotent topological groups

This chapter is a first step in the study of moment maps of representations of topological groups, which will be completed in Chapter 5 by Theorem 5.3.2.

3.1 A formula in differential calculus on topological groups

Proposition 3.1.1. *Let G be any topological group and $\mathcal{Y}$ be any locally convex space. Define $\pi\colon G\times G\to G$, $\pi(x,y)=xy$. For every $f\in C^k(G,\mathcal{Y})$, and $k\geq 1$ one has*

$$
\begin{aligned}
&D^k(f\circ\pi)((x,y);(\lambda_{11},\lambda_{12}),\dots,(\lambda_{k1},\lambda_{k2}))\\
&\qquad=\sum_{\ell=0}^{k}\sum_{\substack{i_1<\cdots<i_\ell\\ i_{\ell+1}<\cdots<i_k}} D^k f(xy;\lambda_{i_1 2},\dots,\lambda_{i_\ell 2},\lambda^y_{i_{\ell+1}1},\dots,\lambda^y_{i_k 1})
\end{aligned}
$$

where the above sum is performed according to the condition

$$
\{i_1,\dots,i_l\}\cup\{i_{l+1},\dots,i_k\}=\{1,\dots,k\}.
$$

Moreover it follows that if $f\in C^\infty(G,\mathcal{Y})$, then $f\circ\pi\in C^\infty(G\times G,\mathcal{Y})$.

3.2 2-step nilpotent topological groups vs. pre-Lie groups

Definition 3.2.1. Let G be any group. We denote

$$
[G,G]=\{xyx^{-1}y^{-1};x,y\in G\}
$$

and

$$Z(G) = \{g \in G; xg = gx, (\forall)x \in G\}$$

which is called the *center* of G and is a commutative subgroup of G.

We say that G is a *2-step nilpotent* gruoup if $[G, G] \subseteq Z(G)$.

We also define the commutator map

$$c : G \times G \to Z(G), \quad c(x, y) := xyx^{-1}y^{-1}.$$

Theorem 3.2.2 ([Ne06])**.** *Every 2-step nilpotent topological group is a topological group with Lie algebra.*

3.3 Tangent group of topological group with Lie algebra

Definition 3.3.1. Let G be any topological group with Lie algebra. The *tangent group* of G is $T(G) := G \ltimes \Lambda(G)$, which is the set $G \times \Lambda(G)$ endowed with the group operation $(x, \alpha)(y, \beta) = (xy, \alpha^y + \beta)$.

Proposition 3.3.2. *If G is any topological group with Lie algebra, then $T(G)$ is a topological group.*

Proposition 3.3.3. *If G is any 2-step nilpotent topological group, then $T(G) = G \ltimes \Lambda(G)$ is also a 2-step nilpotent topological group*

Theorem 3.3.4. *Let G be any 2-step nilpotent topological group. Then $T(G) = G \ltimes \Lambda(G)$ is a pre-Lie group if and only if for every $\alpha \in \Lambda(G)$, $\alpha \neq 0$, there exists a continuous linear functional $\psi : \Lambda(G) \to \mathbb{R}$ with $\psi(\alpha) \neq 0$.*

Notes. Proposition 3.1.1 was stated without any proof in [BCR81]. The proof we presented in this thesis is included in our article [Nic14]. All the results on 2-step nilpotent topolog[ical groups are new except for Teoremei 3.2.2, which was stated in [Ne06] with a mere sketch of the proof.

Chapter 4

Moments for finite-dimensional representations

In this chapter we establish some elementay properties of the momnt map for unitary representations of topological groups, in the case of the representations on finite-dimensional Hilbert spaces.

4.1 Basic properties of the moment map

Let G be any topological group, V be any complex Hilbert space with its unitary group $U(V) := \{T \in B(V); TT^* = T^*T = 1\}$. Consider any unitary representation $\rho : G \to U(V)$, that is, a group morphism for which the map $G \times V \to V, (g, y) \mapsto \rho(g)y$ is continuous. The space of vectors of class C^∞ for the representation ρ is

$$V_\infty := \{y \in V \mid \rho(\cdot)y \in C^\infty(G, V)\}.$$

The derivative of the representation $\rho : G \to U(V)$ is

$$d\rho : \Lambda(G) \to \mathrm{End}\,(V_\infty), \quad d\rho(\gamma)y = \lim_{t\to 0} \frac{\rho(\gamma(t))y - y}{t}$$

for all $\gamma \in \Lambda(G)$ and $y \in V_\infty$, and the moment map of ρ is

$$\Psi_\rho : V_\infty \setminus \{0\} \to \mathbb{R}^{\Lambda(G)}, \quad \Psi_\rho(y) = \frac{1}{i}\frac{(d\rho(\cdot)y, y)}{(y, y)}.$$

We endow the set $\mathbb{R}^{\Lambda(G)}$ of all functions $\Lambda(G) \to \mathbb{R}$ with the topology of uniform convergence on compact subsets of $\Lambda(G)$. Define

$$I_\rho^0 := \Psi_\rho(V_\infty \setminus \{0\})$$

and the *moment set* I_ρ is the closure of the set I_ρ^0.

Denote by $\Omega := \{v \in V \mid (v,v) = 1\}$ the unit sphere of V. Note that $I_\rho^0 = \Psi_\rho(\Omega)$. If the Hilbert space V is finite dimensional, the unit sphere Ω is compact and since Ψ_ρ is continuous, it follows that I_ρ^0 is compact, hence also closed in $\mathbb{R}^{\Lambda(G)}$, and in this case $I_\rho = I_\rho^0$. If the Hilbert space V is finite dimensional, then $V_\infty = V$ (see Corollary 5.2.2). All the Hilbert spaces in this chapter are assumed finite dimensional.

Theorem 4.1.1. *Let $\rho_j\colon G_j \to U(V_j)$ be finite-dimensional representations of topological groups for $j = 1,2$. Define $\iota\colon \mathbb{R}^{\Lambda(G_1)} \times \mathbb{R}^{\Lambda(G_2)} \to \mathbb{R}^{\Lambda(G_1)\times\Lambda(G_2)}$, $(\iota(f_1,f_2))(\gamma_1,\gamma_2) := f_1(\gamma_1) + f_2(\gamma_2)$ and $p_j\colon \mathbb{R}^{\Lambda(G_1)\times\Lambda(G_2)} \to \mathbb{R}^{\Lambda(G_j)}$ for $j = 1,2$ by $p_2(f)(\gamma_2) := f(1,\gamma_2)$ and $p_1(f)(\gamma_1) := f(\gamma_1, 1)$. Let $V = V_1 \otimes V_2$ and $\rho = \rho_1 \otimes \rho_2, \rho : G_1 \times G_2 \to U(V)$ be the representation of $G_1 \times G_2$ given by $\rho(g_1,g_2)v_1 \otimes v_2 = \rho_1(g_1)v_1 \otimes \rho_2(g_2)v_2$. Then one has $\iota(I_{\rho_1}^0 \times I_{\rho_2}^0) \subseteq I_\rho^0$ si $p_j(I_\rho^0) \subseteq \mathrm{conv}\,(I_{\rho_j}^0)$ for $j = 1,2$.*

Theorem 4.1.2. *Let $\rho_1 : G \to U(V_1)$ and $\rho_2 : G \to U(V_2)$ be finite-dimensional unitary representations of the topological group G. Let $V = V_1 \otimes V_2$ and $\rho : G \to U(V)$ be the representation of G defined by $\rho(g)v_1 \otimes v_2 := \rho_1(g)v_1 \otimes \rho_2(g)v_2$. Then*

$$I_{\rho_1}^0 + I_{\rho_2}^0 \subseteq I_\rho^0 \subseteq \mathrm{conv}\,(I_{\rho_1}^0) + \mathrm{conv}\,(I_{\rho_2}^0).$$

4.2 Liniarity properties

We say that the topological group G has the *Trotter property* if $\Lambda(G)$ has a structure of real vector space with

$$(\forall \alpha,\beta \in \Lambda(G)) \quad (\alpha+\beta)(t) = \lim_{n\to\infty} (\alpha(\frac{t}{n})\beta(\frac{t}{n}))^n$$

uniformly for t in any compact subset of $\mathbb{R}$.

Proposition 4.2.1. *If he topological group G has the Trotter property and $\rho : G \to U(V)$ is a continuous unitary representation of G, then for every $v \in V_\infty \setminus \{0\}$ the functional $\Psi_\rho(v) : \Lambda(G) \to \mathbb{R}$ is linear.*

Notes. The results of this chapter are new, and are generalizations of some properties of the moment sets of finite dimensional representations of compact Lie groups studied in [Wi92].

Chapter 5

Convex geometry o moments for representations of solvable groups

In this final chapter of this thesis we obtain our main result (Theorem 5.3.2), which establishes the convexity property of moment sets of all representations of solvable separable locally compact groups.

5.1 Preliminariies

Proposition 5.1.1. *If G is any topological group and $\mathcal{Y}$ is any locally convex space, then the following assertions hold.*

1. *Fix any open set $V \subseteq G$. If for all $k \geq 1$, compact sets $K_1 \subseteq \Lambda^k(G)$ and $K_2 \subseteq V$, and continuous seminorm $|\cdot|$ on $\mathcal{Y}$ one defines $q^{|\cdot|}_{K_1,K_2} \colon C^\infty(V,\mathcal{Y}) \to [0,\infty)$ by*

$$q^{|\cdot|}_{K_1,K_2}(f) := \begin{cases} \sup\{|(D^R_\gamma f)(x)| \mid \gamma \in K_1, x \in K_2\} & \text{if } K_1 \neq \emptyset, \\ \sup\{|f(x)| \mid x \in K_2\} & \text{if } K_1 \neq \emptyset, \end{cases}$$

   then one thus obtains a family of seminorms that determines the topology of $C^\infty(V,\mathcal{Y})$ introduced in Definition 2.3.1.

2. *For all $g \in G$ and $\gamma \in \Lambda(G)$, the operators*

$$D_\gamma, D^R_\gamma, \lambda(g), \rho(g) \colon C^\infty(G,\mathcal{Y}) \to C^\infty(G,\mathcal{Y})$$

   are well-defined and continuous, where

$$(\lambda(g)f)(x) := f(gx) \text{ and } (\rho(g)f)(x) := f(xg) \text{ for all } x \in G \text{ and } f \in C^\infty(G,\mathcal{Y}).$$

One endows $\mathcal{H}_\infty$ with the locally convex topology for which the injective linear map $A\colon \mathcal{H}_\infty \to C^\infty(G,\mathcal{H})$, $\quad v \mapsto \pi(\cdot)v$ is a homeomorphism onto its image, where the image of the above map is regarded as a subspace of the locally convex space $C^\infty(G,\mathcal{H})$ endowed with the topology introduced in Definition 2.3.1.

Proposition 5.1.2. *Let G be any locally compact group G and $\pi\colon G \to U(\mathcal{H})$ be any representation extended to*

$$\pi\colon L^1(G) \to \mathcal{B}(G), \quad \pi(f) = \int_G f(x)\pi(x)\mathrm{d}x.$$

If $f \in \mathcal{C}_0^\infty(G)$ and $v \in \mathcal{H}$, then $\pi(f)v \in \mathcal{H}_\infty$ and

$$(\forall \gamma \in \Lambda(G)) \quad \mathrm{d}\pi(\gamma)(\pi(f)v) = \pi(D^R_\gamma f)v. \tag{5.1.1}$$

Moreover, for every smooth δ-family $\{f_W\}_{W\in\mathcal{W}}$ one has $\overline{\{\pi(f_W)v \mid W \in \mathcal{W},\ v \in \mathcal{H}\}} = \mathcal{H}_\infty$.

5.2 Moment sets of representations of topological groups

Proposition 5.2.1. *Let $\phi\colon G_1 \to G_2$ be any morphism of topological groups and define $P\colon \mathbb{R}^{\Lambda(G_2)} \to \mathbb{R}^{\Lambda(G_1)}$, $P(f) := f \circ \Lambda(\phi)$. For any representation $\pi\colon G_2 \to U(\mathcal{H})$ denote by $\mathcal{H}_\infty(\pi)$ and $\mathcal{H}_\infty(\pi\circ\phi)$ the spaces of smooth vectors of the representations π and $\pi\circ\phi$, respectively. Then $\mathcal{H}_\infty(\pi) \subseteq \mathcal{H}_\infty(\pi\circ\phi)$ and $P(I^0_\pi) \subseteq I^0_{\pi\circ\phi}$. If moreover $\mathcal{H}_\infty(\pi)$ is dense in $\mathcal{H}_\infty(\pi\circ\phi)$, then $\overline{P(I_\pi)} = I_{\pi\circ\phi}$. If in addition I_π is convex , then also $I_{\pi\circ\phi}$ is.*

Corollary 5.2.2. *Let G be any topological group. If $\pi\colon G \to U(\mathcal{H})$ is any representation with $\dim\mathcal{H} < \infty$, then $\mathcal{H}_\infty = \mathcal{H}$.*

5.3 Main result

Definition 5.3.1. A topological group G is called *solvable* if for every neighborhood V of $1 \in G$ there exists an integer $k \geq 0$ with $G^{(k)} \subseteq V$. Here $G = G^{(0)} \supseteq G^{(1)} \supseteq \cdots$ is the descending derived series of G defined by the condition that $G^{(k+1)}$ is the closed subgroup of G generated by the set $\{xyx^{-1}y^{-1} \mid x, y \in G^{(k)}\}$.

Theorem 5.3.2. *Let G be any solvable separable locally compact group. Then the closed moment set of every representation of G is convex.*

Note bibliografice. The results of this chapter are new and are included in our article [BNi14b], which was submitted for publication.

Bibliography

[Ak95] S.S. Akbarov, Smooth structure and differential operators on a locally compact group. (Russian) *Izv. Ross. Akad. Nauk Ser. Mat.* **59** (1995), no. 1, 3–48; translation in *Izv. Math.* **59** (1995), no. 1, 1–44.

[AL92] D. Arnal, J. Ludwig, La convexité de l'application moment d'un groupe de Lie. *J. Funct. Anal.* **105** (1992), no. 2, 256–300.

[Be06] D. Beltiţă, *Smooth homogeneous structures in operator theory*. Chapman & Hall/CRC Monographs and Surveys in Pure and Applied Mathematics, 137. Chapman & Hall/CRC, Boca Raton, FL, 2006.

[BB11] I. Beltiţă, D. Beltiţă, On differentiability of vectors in Lie group representations. *J. Lie Theory* **21** (2011), no. 4, 771–785.

[BNi14a] D. Beltiţă, M. Nicolae, On universal enveloping algebras in a topological setting. *Preprint* arXiv:1402.0186 [math.FA] (trimis spre publicare).

[BNi14b] D. Beltiţă, M. Nicolae, Moment convexity of solvable locally compact groups. *Preprint* arXiv:1408.4554 [math.RT] (trimis spre publicare).

[Bos76] H. Boseck, Über Darstellungen lokal-kompakter topologischer Gruppen. *Math. Nachr.* **74** (1976), 233–251.

[BCR81] H. Boseck, G. Czichowski, K.P. Rudolph, *Analysis on topological groups —General Lie theory*. Teubner-Texte zur Mathematik, 37. BSB B. G. Teubner Verlagsgesellschaft, Leipzig, 1981.

[BR80] H. Boseck, K.P. Rudolph, An axiomatic approach to differentiability on topological groups. *Math. Nachr.* **98** (1980), 27–36.

[BR87] O. Bratteli, D.W. Robinson, *Operator algebras and quantum statistical mechanics*. 1. Second edition. Texts and Monographs in Physics. Springer-Verlag, New York, 1987.

[Di64] J. Dixmier, *Les C^*-algèbres et leurs représentations*. Cahiers Scientifiques, Fasc. XXIX Gauthier-Villars & Cie, Éditeur-Imprimeur, Paris, 1964.

[Eh56] L. Ehrenpreis, Some properties of distributions on Lie groups. *Pacific J. Math.* **6** (1956), 591–605.

[Fo45] R.H. Fox, On topologies for function spaces. *Bull. Amer. Math. Soc.* **51** (1945), 429–432.

[Gl02a] H. Glöckner, Infinite-dimensional Lie groups without completeness restrictions. In: A. Strasburger, J. Hilgert, K.-H. Neeb, W. Wojtyński (eds.), "Geometry and analysis on finite- and infinite-dimensional Lie groups" (Będlewo, 2000), Banach Center Publ., 55, Polish Acad. Sci., Warsaw, 2002, pp. 43–59.

[HM07] K.H. Hofmann, S.A. Morris, *The Lie theory of connected pro-Lie groups.* A structure theory for pro-Lie algebras, pro-Lie groups, and connected locally compact groups. EMS Tracts in Mathematics, 2. European Mathematical Society (EMS), Zürich, 2007.

[KM97] A. Kriegl, P.W. Michor, *The convenient setting of global analysis.* Mathematical Surveys and Monographs, 53. American Mathematical Society, Providence, RI, 1997.

[LW11] D.H. Leung, Y.S. Wang, Local operators on C^p. *J. Math. Anal. Appl.* **381** (2011), no. 1, 308–314.

[Mag81] L. Magnin, Some remarks about C^∞ vectors in representations of connected locally compact groups. *Pacific J. Math.* **95** (1981), 391–400.

[Mi90] P.W. Michor, The moment mapping for unitary representations. *Ann. Global Anal. Geom.* **8** (1990), no. 3, 299–313.

[MZ55] D. Montgomery, L. Zippin, *Topological transformation groups.* Interscience Publishers, New York-London, 1955.

[Ne00] K.-H. Neeb, *Holomorphy and convexity in Lie theory.* de Gruyter Expositions in Mathematics, 28. Walter de Gruyter & Co., Berlin, 2000.

[Ne06] K.-H. Neeb, Towards a Lie theory of locally convex groups. *Jpn. J. Math.* **1** (2006), no. 2, 291–468.

[Ne10] K.-H. Neeb, On differentiable vectors for representations of infinite dimensional Lie groups. *J. Funct. Anal.* **259** (2010), no. 11, 2814–2855.

[Nic14] M. Nicolae, On differential calculus on pre-Lie groups. *Preprint*, 2014.

[NS12] K.-H. Neeb, H. Salmasian, Differentiable vectors and unitary representations of Fréchet-Lie subgroups. *Math. Z.* **275** (2013), no. 1–2, 419–451.

[Pe60] J. Peetre, Rectification à l'article "Une caracterisation abstraite des operateurs differentiels". *Math. Scand.* **8** (1960), 116–120.

[Ru91] W. Rudin, *Functional analysis.* Second edition. International Series in Pure and Applied Mathematics. McGraw-Hill, Inc., New York, 1991.

[WD73] J.C. Wells, C.R. DePrima, Local automorphisms are differential operators on some Banach space. *Proc. Amer. Math. Soc.* **40** (1973), 453–457.

[Wi89] N.J. Wildberger, Convexity and unitary representations of nilpotent Lie groups. *Invent. Math.* **98** (1989), no. 2, 281–292.

[Wi92] N.J. Wildberger, The moment map of a Lie group representation. *Trans. Amer. Math. Soc.* **330** (1992), no. 1, 257–268.