

Scientific Report

November 2013–November 2014

1. RESEARCH COMPLETED

1.1. Noncommutative functions. Asymptotic behavior of fluctuations of orthogonally invariant random matrices have been studied in [7] (published). [8] continues this direction, by studying second order real freedom appearing in the study of Haar unitary and unitarily invariant random matrices.

In [9] one develops the theory of Hardy spaces of noncommutative functions in the noncommutative unit polydisc.

In [10] one obtains a description of fluctuation moments for random matrices with entries in a noncommutative algebra. The paper has been accepted for publication.

1.2. Markov semigroups. Papers [11] and [12] have been published. The first gives a nonprobabilistic proof for the boundary integral representation of μ -harmonic functions on topological semigroups. The second proves a general theorem concerning the existence of the uniform limit for iterates of a Markov operator with the Feller property.

1.3. Completely positive applications on matrix spaces. In [1] (accepted for publication) have been obtained existence criteria for completely positive maps on matrix spaces which have prescribed values on a finite set.

In [2] one investigates concrete method to implement these techniques. This is done through semidefinite programming, as well as through convex minimization.

1.4. Γ -correlated stochastic processes. In [15] one discusses the problem of the angle between past and future, of the Friedrichs angle as well as other geometrical concepts in the framework of periodic Γ -correlated stochastic processes.

[16] represents a survey of several results dealing with the generalization of mathematical prediction from finite to infinite dimensions. Stationary as well as nonstationary processes are considered.

1.5. Beurling-type theorems for reproducing kernel spaces. The final version of [3] has been accepted for publication. One characterizes therein contractively included subspaces in Nevalinna–Pick spaces that are either invariant to multiplication, or complementary to such subspaces. This is an analogue of the classical Beurling Theorem.

1.6. de Branges–Rovnyak spaces. In [6] one describes the class of operators that can be modeled through de Branges–Rovnyak spaces corresponding to nonextreme functions of the unit ball.

[14] represents a short introduction to de Branges–Rovnyak spaces; it will be published in the proceedings of a conference held in August 2013 at Centre de Recherches Mathématiques (Montreal).

1.7. Equivalences for operators on Hilbert spaces. In [13] one solves an interesting open problem in operator theory on Hilbert spaces, namely the coincidence of several important equivalence relations: simple equivalence, equivalence after extension, Schur coupling, and matricial coupling.

2. ONGOING RESEARCH

- (1) Interpolation theorems for different reproducing kernel spaces.
- (2) Study of complete positivity for operators on ordered spaces.
- (3) Study of multiplicativity properties for free random variables.
- (4) Dilation theorems for positive semidefinite measurable kernels and their connections to representations on Hilbert space direct integrals.
- (5) Dilation theory for commutative families of operators on Hilbert spaces.
- (6) Study of Hardy spaces of noncommutative functions.
- (7) Connections between the maximal function attached to a contraction and linear systems; investigations of the hypercontraction case.

3. PAPERS

Synthesis of papers for November 2013–November 2014:

- 7 published papers: [4, 6, 7, 11, 12, 13, 15].
- 4 accepted papers: [1, 3, 5, 10]
- 4 preprints: [2, 8, 9, 14].

- 1 paper in the Proceedings of a conference: [16].

4. DISSEMINATION

Members of the team have participated to several international conferences, presenting the results obtained in the project and discussing with other mathematicians questions related to the theme of the project.

- International Workshop on Operator Theory and Applications, Bangalore (India), December 16–20. Presentation by D. Timotin.
- Workshop on Operator Theory, Complex Analysis, and Applications, Lisbon (Portugal), June 19–21. Presentation by D. Timotin.
- International Workshop on Operator Theory and Applications, Amsterdam (The Netherlands), July 14–18. Presentation by I. Valuşescu.
- 25th International Conference on Operator Theory, Timișoara, June 30 iunie–July. Presentations by M. Popa and I. Valuşescu.
- 16th Workshop on non-commutative analysis, Bedlewo (Polonia), July 16–12. Presentation by Mihai Popa.
- A.M.S. Joint Mathematics Meetings, Baltimore (USA), January 15–18. Presentation by Mihai Popa.
- Classical and Functional Analysis, Bușteni, September 3–6. Presentation by I. Valuşescu.

Other actions:

— July 2–12: visit of A. Gheondea to AGH University in Krakow, continuing cooperation on themes of the project with prof. P. Cojuhari.

— August 8–23: participation of D. Timotin to the International Congress of Mathematicians; discussions on subjects related to the project with several specialists present to the congress.

— During the 25th International Conference on Operator Theory, Timișoara, June 30 iunie–July, a section is organized on the theme of the project (multidimensional operator theory). Besides members of the team, to this section have participated six foreign invited mathematicians and an invited Romanian.

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