Scientific Report

December 2014–November 2015

1. Research completed

1.1. Completely positive applications and semigroup representations.
In [2] one studies positive semidefinite kernels with values in the *-algebra of adjointable operators on a VE-space, which are invariant to the action of *-semigroups. One obtains a general dilation theorem: for this type of kernels one may obtain representations of the *-semigroup either on the VE-spaces which linearize the kernel, or on the corresponding reproducing kernel VE-space. One obtains thus the precise structure of the dilation. The theorem unifies at the non-topological level certian dilation results.

The research has been pursued in [3], where one considers positive semidefinite kernels with values in the *-algebra of continuously adjointable operators on a VH-space. In the general case one must assume certain boundedness conditions, which are automatically satisfied in some particular situations. This general approach allows a direct proof of a general dilation theorem of Stinespring–Kasparov type for completely positive applications on locally C*-algebras and with values adjointable operators on Hilbert modules over locally C*-algebras.

1.2. Multidimensional operator models.
In [8] one discusses the concept of concrete Hilbert model over a locally C*-algebra by means of locally bounded operators on strict inductive limits of Hilbert spaces. One shows that this concept represents an operator model for all locally C*-Hilbert modules. As an application one obtains a direct construction of the exterior tensor product of locally C*-Hilbert modules.

1.3. The maximal function of an n-hypercontraction.
In [13] one studies the maximal function associated to an n-hypercontraction of class $C_0.$. One uses it in order to characterize the functional model of the n-hypercontraction as the image in a Bergman space of the associated maximal function. As in the case of contractions, the problem is discuss in the context of a certain related linear system, where controllability and observability operators play an important role.

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1.4. Γ-correlated stochastic processes. In [12, 14, 15] one continues the study of Γ-correlated stochastic processes. The first paper continues a research from the previus year and contains mainly results concerning the nonstationary case of Γ-correlated processes. [14] presents the adaptation of results to the case of Γ-stationary processes with continuous time parameter, while [15] presents a new method for obtaining the predictable part of a periodic Γ-correlated process.

1.5. Noncommutative functions. .

In the paper [10] one obtains therein results concerning the multiplication of random noncommutative variables which are c-free with respect to a pair (Φ, φ), where Φ is a linear application with values in a Banach algebra or a C*-algebra, while φ is scalar. In particular, one constructs in this framework an analogue of Voiculescu's S-transform.

In another direction, one studies the connection between the partial transpose and the free asymptotical independence for Wishart sets.

2. Ongoing research

(1) Interpolation theorems for Nevanlinna–Pick spaces and other reproducing kernel spaces.
(2) Study of complete positivity for operators on ordered spaces.
(3) Study of independence properties for free random variables.
(4) Dilation theorems for positive semidefinite measurable kernels and their connections to representations on Hilbert space direct integrals.
(5) Study of Hardy spaces of noncommutative functions.

3. Papers

Synthesis of papers for November 2013-November 2014:

- 8 published papers: [1, 2, 4, 5, 7, 6, 9, 11].
- 2 submitted papers: [3, 7].
- 2 preprints: [10, 13].
- 3 papers in Proceedings of conferences: [12, 14, 15].

4. DISSEMINATION

Members of the team have participated to several international conferences, presenting the results obtained in the project and discussing with other mathematicians questions related to the theme of the project.

- Recent Advances in Operator Theory and Operator Algebras, Bangalore (India), December 9–19 . Presentation by D. Timotin.
- 8-th Congress of Romanian Mathematicians, Iaşi, June 26–July 1. Presentations by A. Gheondea ąnd I. Valuşescu.
- International Workshop on Operator Theory and Applications, Tbilisi (Georgia), July 6–11. Presentation by I. Valuşescu.
- C*-Algebras, Random Matrices, Free Probability, and Beyond, Kingston (Canada), October 16–17. Presentation by Mihai Popa.
- Classical and Functional Analysis, Buşteni, July 17–19. Presentation by I. Valuşescu.
- International Conference on Theory and Applications of Mathematics and Informatics, Alba Iulia, September 17–20. Presentation by I. Valuşescu.

Other actions:

— November 2–12: visit of D. Timotin to Indian Statistical Institute, Bangalore (India): cooperation with prof. J. Sarkar.

— During the 8-th Congress of Romanian Mathematicians, Iaşi, June 26–July 1, a subsection was organized on the theme of the project (multidimensional operator theory). Besides members of the team, to this section have participated two Romanian specialists in the domain.

BIBLIOGRAFIE

[1] Ambrozie, C.-G., Gheondea, A., An interpolation problem for completely positive maps on matrix algebras: solvability and parametrization, *Linear and Multilinear Algebra* 63 (2015), 826–851.

[2] Ay, S., Gheondea, A., Representations of *-semigroups associated to invariant kernels with values adjointable operators, *Linear Algebra and Applications* 486 (2015), 361–388.

[3] Ay, S., Gheondea, A., Representations of *-semigroups associated to invariant kernels with values continuously adjointable operators, submitted to la *Journal of Mathematical Analysis and Applications*, arXiv: 1502.00883v2.

[4] Benhida, Ch., Timotin, D., Contractively included subspaces of Pick spaces, *Complex Analysis and Operator Theory* 9 (2015), 245–264.

[5] Chalendar, I., Timotin, D., Commutation relations for truncated Toeplitz operators, *Operators and Matrices* 8 (2014), 877–888.

[6] Cojuhari, P., Gheondea, A, Triplets of closely embedded Hilbert spaces, *Integral Equations Operator Theory* 21 (2015), 1–33.

[7] Gheondea, A., Reproducing Kernel Krein Spaces, in *Operator Theory*, D.A. Alpay (Ed.), Vol. 1, pp. 311–343, Berlin, Springer Verlag 2015.

[8] Gheondea, A., Operator models for Hilbert locally C*-modules, submitted to *Journal of the London Mathematical Society*, arXiv: 1507.07643v1.

[9] Popa, Mihai; Jiao, Yong, On fluctuations of traces of large matrices over a non-commutative algebra, *J. Operator Theory* 73 (2015), 71–90.

[10] Popa, M., Vinnikov, V., Wang, J-C., On the multiplication of operator valued c-freerandom variables, preprint.

[11] Timotin, D., A short introduction to de Branges-Rovnyak spaces, în *Invariant Subspaces of the Shift Operator*, American Mathematical Society, 2015, 21–38.

[12] Valuşescu I., Some remarks on the infinite-variate prediction II, *Proceedings of the conference Classical and Functional Analysis*, Buşteni, 3–6 sept. 2014, 33–52.

[13] Valuşescu I., On a model for the maximal function of an n-hypercontraction. Preprint IMAR nr. 3/2015.

[14] Valuşescu I., Continuous parameter Γ-stationary processes. Proceedings of the conference Classical and Functional Analysis, Buşteni, iulie 2015. Transilvania University Press, Brasov, 2015, 79–99.

[15] Valuşescu I., Stationary dilation of a nonstationary Γ-correlated process. Proceedings of the International Conference on Theory and Applications of Mathematics and Informatics, Alba Iulia, 17-20 September 2015, *Acta Univ. Apulensis*, special issue, 2015, 121–134.

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