Scientific Report

November 2011–September 2016

1. Research completed

1.1. **Dilation theorems and completely positive applications.** In the paper [12] one obtains dilations results for positive semidefinite type kernels with values operators in a VH space, which are invariant to the action of *-semigroups, and which produce representations of *-semigroups on the dilation spaces. The dilations produced by these kernels are either of linearization type (Kolmogorov decompositions), or on reproducing kernel spaces. Moreover, these theorems contain as particular cases, unifying them, both Stinespring type dilation theorems for B^*-algebras and Sz.-Nagy type theorems for *-semigroups, which have been for a long type considered as representing liniar, respectively nonlinear aspects.

In [2] one obtains existence criteria and parametrizations for completely positive applications which map a finite number of given matrices in other prescribesd matrices. The approach uses density matrices associated to linear functionals on matrix spaces, inspired by the linear functional Smith-Ward and the Hahn-Banach type theorem of Arveson. One gives necessary an sufficient conditions for existence of the solutions and one parametrizes the solutions through a closed convex set in an affine space. One obtains applications for quantic channels.

Continuing this research, one investigates in [3] techniques for the numerical implementation of the results in [2]. One describes two approaches, one of semidefinite programming and the other of convex minimization. Some numerical examples are studied.

In [4] one studies positive semidefinite kernels with values in the *-algebra of adjunctible operators on a VE-space which are invariant to the action of *-semigroups. One obtains a general dilation theorem and one determines the precise structure of the dilation. The theorem unifies at a non-topological level several dilation results.

The research has been continued in [5], where one considers positive semidefinite kernels with values in the *-algebra of operators continuously adjunctible on a VH-space (in the sense of Loynes) which are invariant to the action of *-semigroups. For these kernels one obtains necessary and sufficient boundedness conditions for the existence

of *-representations of *-semigroups on VJ-spaces of linearizations of the kernels. In some situations the boundedness conditions are automatically fulfilled, for instance in the case of Hilbert modules over local C^*-algebras. One obtains a direct proof of a very general Stinespring–Kasparov theorem.

1.2. **Classes of Hilbert modules.** In [11] one introduces a general concept of Hilbert triple, in which the embeddings are closed rather than continuous. This abstract model includes a large diversity of problems pertaining to Sobolev spaces, Hilbert spaces of holomorphic functions, and L^2 weighted spaces. One obtains an application to the Dirichlet problem associated to a class of degenerate PDEs.

In [14] one discusses the concept of Hilbert module over a local C^*-algebra, by using locally bounded operators on strict inductive limits of Hilbert spaces. This is shown to be a general operatorial model; as an application one obtains a direct construction of the outer tensor product of local C^* Hilbert modules.

In [15] one obtains some fundamental properties of local Hilbert spaces: topological properties, geometry of subspaces, linear functionals and dual spaces.

1.3. **Moment problems in several variables.** The results obtained refer to the truncated moment problem in several real variables and have been published in [1]. One obtains precise criteria for the existence of an absolutely continuous representation measure. The originality of the approach is given by the use of variational or convexity methods in the maximization of Boltzmann–Shannon entropy. Results are more precise when the support of the measur is compact. One also investigates the existence of Lebesgue integrable solutions.

1.4. **Noncommutative functions.** In the papers [18] şi [19] one obtains fluctuation properties of certain important classes of random matrices. In [18] one shows that the asymptotic behaviour of the fluctuations of orthogonally invariant random matrices is different from that of unitarily invariant ones and cannot be modeled in a commutative frame similar to the functional calculus of normal operators. In [22] one obtains the description of fluctuation moments for random matirices with entries in a noncommutative algebra.

The research is continued in [20], where one studies the second order freedom that appears in the study of unitary and unitarily invariant Haar random matrices. Surprisingly, one shows that a unitarily invariant random matrix is asymptotically free

relative to its transpose. Techniques in [18] are applied in [19] in the study of unitarily invariant matrices.

In another direction one investigates convergence properties of noncommutative power series, as opposed to the commutative case. Thus, in [21] one discusses elements of the theory of Hardy spaces of noncommutative functions in the noncommutative unit polydisc.

In [23] one obtains results concerning multiplication of noncommutative random variables which are c-free with respect to a pair (Φ, φ), where Φ is linear taking values in a Banach or C*-algebra, while φ is a scalar function. In particular, one constructs an analogue of Voiculescu's S transform. One also studies the relation between partial transpose and asymptotically free independence for Wishart ensembles.

1.5. **Stochastic Γ-corelated processes.** In [31] one extends a Gladyshev type theorem for periodical processes with continuous parameter to the operatorial case, by using a completely correlated action. For Γ-correlated processes with continuous parameter one analyses different conditions of operatorial continuity. One establishes the connections between periodically correlated processes with continuous parameter and harmonizable processes. An integral representation is also obtained for Γ-correlated processes with continuous parameter.

The paper [32] is a synthesis of different results that generalize mathematical prediction problems to the finite multidimensional case. In [30] one discusses geometrical concepts (like angle between past and future) for periodical Γ-correlated processes. The study of Γ-correlated processes is continued in [33, 35, 36].

A related direction is the study in [34] of the maximal function attached to an n-hipercontraction of class C_0.. This allows the construction of the functional model of the n-hipercontraction as the image in a Bergman space of the maximal function. As in the classical contractive case, one obtains controllability and observability operators on a related linear system.

1.6. **Multioperators on reproducing kernel spaces.** In [6] one obtains the characterization of contractively included subspaces of a reproducing kernel space of Nevanlinna–Pick type that are invariant to multiplications. This is an analogue of the classical Beurling Theorem. One also gives a characterization of complementary subspaces. The results generalize the Arveson–Drury situation.

In the recent paper [16] one obtains factorization results for certain classes of reproducing kernel Hilbert spaces. A consequence is a rigidity result for the structure of these spaces. The approach generalizes classical results for concrete function spaces, and has the advantage of using directly the basic properties of the kernels.

1.7. **Analytic functions and operators on function spaces.** In [7] one obtains a factorization theorem for analytic functions which map the upper half plane into itself, that generalizes classical results concerning meromorphic functions. The same idea is applied in [8] to solve an open problem concerning composition operators on the upper half plane.

In [9] one investigates the numerical range of a completely nonunitary contraction. One gives a final result to an old conjecture of Halmos, showing that the numerical range can be obtained as the intersection of the minimal unitary dilations. In case the defect spaces have dimension 1, one obtains precise relations between the geometrical structure of the numerical range and the properties of the functional model associated to the contraction.

Truncated Toeplitz operators on model spaces represent a new domain of study, offering interesting possibilities for multidimensional extensions. The paper [10] studies commutation properties of these operators, obtaining, in particular, characterization of normal, unitary, isometric or coisometric operators.

In the area of de Branges–Rovnyak spaces the paper [17] describes the class of operators that can be modeled by apces correponding to nonextremal function of the unit ball. [29] represents a short introduction to de Branges–Rovnyak spaces.

In [28] one solves an open problem of interest in the theory of operators on a Hilbert space, namely the coincidence of several equivalence relations: Schur coupling, matricial coupling, and equivalence after extension.

1.8. **Topological semigroups and connected problems.** In [25] one proves a general theorem concerning the existence of the uniform limit for the iterates of a Markov operator with the Feller property acting on a space of measurable functions. The more general case of topological semigroups endowed with a probability measure is studied in [26], where one shows the existence of a representation of the space of left continuous functions harmonic in a generalized sense on this semigroup as a space of continuous functions on a compact topological space. In [27] one gives a simple proof

of the extension of a Choquet–Deny theorem obtained by Szekely ąnd Zeng. In [24] one gives a nonprobabilistic proof for the integral representation of μ-harmonic functions on topological semigroups.

2. Papers

According to the references below, the outcome of the project consists in 36 papers, as follows:

- 16 papers published in ISI indexed journals: [1, 2, 4, 6, 7, 8, 9, 10, 11, 12, 17, 18, 22, 24, 25, 28]
- 2 pubished in non ISI journals: [30, 31].
- 6 published in collective volumes (conference proceedings, etc): [13, 29, 32, 33, 35, 36].
- 2 accepted papers: [15, 20].
- 10 submitted preprints: [3, 5, 14, 16, 19, 21, 23, 26, 27, 34].

3. Dissemination

The results have been presented to several international conferences, which have also been an opportunity to have talks with foreign mathematicians on themes related to the projects. Also, the members of the team have done collaboration visits in different foreign countries, some of which have resulted in joint papers.

3.1. **Participations to international conferences.**

2012.

- 24th International Conference on Operator Theory, Timişoara, July 26–30. Communications by C. Ambrozie, A. Gheondea, I. Valuşescu
- 9th Advanced Course on Operator Theory and Complex Analysis, Sevilla (Spania), June 12–14. Communication by D. Timotin.
- 13th Romanian–Finnish Seminar, Ploieşti, 26–30 iunie. Communications by Ambrozie, Gheondea, Prunaru, Timotin.
- Workshop on Operator Theory, Complex Analysis, and Applications, Lisabona (Portugalia) July 11–13. Communication by D. Timotin.
- Conference on Spectral Theory and Differential Operators, Graz (Austria), August 27–31. Communication by A. Gheondea.

- International Conference for Functional Analysis, Timişoara, October 12–14. Communication by I. Valuşescu.
- Summer Conference of the Canadian Mathematical Society, Regina (Canada), June 2–4. Communication by Mihai Popa.
- Matrices and Operators, Bangalore (India), December 27–30. Communication by D. Timotin.

2013.

- Recent advances in Operator Theory and Operator Algebras, Bangalore (India), January 7–11. Communication by D. Timotin.
- Anniversary Conference of the Faculty of Sciences of Bucharest, August 29–September 1. Communication by A. Gheondea.
- Spectral Problems for Operators and Matrices, Zagreb (Croaţia), September 16–20. Communication by A. Gheondea.
- Structured Function Systems and Applications week, Oberwolvach, February 23–March 2. Germania.Communication by C. Ambrozie.
- Focus Program on Noncommutative Distributions in Free Probability Theory, Toronto (Canada), July 22–26. Communication by Mihai Popa.
- A.M.S. National Meeting, San Diego (USA), January 10–13. Communication by Mihai Popa.
- Sz.-Nagy Centennial Conference, Szeged (Ungaria), June 24–28. Communication by Dan Timotin.
- Joint International Meeting of the American Mathematical Society and the Romanian Mathematical Society, Alba Iulia, June 27–30. Dan Timotin was a plenary speaker. Communication by C. Ambrozie.
- Conference on the Shift Operator, Montreal (Canada), August 26–30. Communication by D. Timotin.
- Conference on Applied and Industrial Mathematics, Bucureşti, September 19–22. A ţinut o expunere I. Valuşescu.
- Conference on Applied and Industrial Mathematics, Bucureşti, 19–22 septembrie. Communication by I. Valuşescu.
- Classical and Functional Analysis, Azuga, October 28–29. Communication by I. Valuşescu.

- International Workshop on Operator Theory and Applications, Bangalore (India), December 16–20. Communication by D. Timotin.

2014.

- Workshop on Operator Theory, Complex Analysis, and Applications, Lisbon (Portugal), June 19–21. Communication by D. Timotin.
- International Workshop on Operator Theory and Applications, Amsterdam (Netherlands), July 14–18. Communication by I. Valuşescu.
- 25th International Conference on Operator Theory, Timişoara, June 30–July 5. Communications by M. Popa and I. Valuşescu.
- 16th Workshop on non-commutative analysis, Bedlewo (Poland), July 16-12. Communication by Mihai Popa.
- A.M.S. Joint Mathematics Meetings, Baltimore (USA), January 15–18. Communication by Mihai Popa.
- Classical and Functional Analysis, Buşteni, September 3–6. Communication by I. Valuşescu.
- Recent Advances in Operator Theory and Operator Algebras, Bangalore (India), December 9–19. Communication by D. Timotin.

2015.

- 8th Congres of Romanian Mathematicians, Iaşi, June 26–July 1. Communications by A. Gheondea and I. Valuşescu.
- International Workshop on Operator Theory and Applications, Tbilisi (Georgia), July 6–11. Communication by I. Valuşescu.
- C*-Algebras, Random Matrices, Free Probability, and Beyond, Kingston (Canada), October 16-17. Communication by Mihai Popa.
- Classical and Functional Analysis, Buşteni, July 17–19. Communication by I. Valuşescu.
- International Conference on Theory and Applications of Mathematics and Informatics, Alba Iulia, September 17–20. Communication by I. Valuşescu.
- Complex Geometry and Operator Theory, Bangalore, December 1–3. Communication by D. Timotin.

2016.

- International Conference of Complex Analysis and Related Topics, The 14th Romanian-Finnish Seminar, June 20–24, Bucureşti. Communications by A. Gheondea and Ilie Valuşescu.
- 26th International Conference on Operator Theory, July 27–June 2, Timişoara. Communications by A. Gheondea. M. Popa, and I. Valuşescu.
- The 5th Summer Workshop of Operator Theory, July 5–9, Krakow (Poland). Communication by A. Gheondea.
- 19th Colloquiumfest, Saskatoon (Canada), August 30-31. Communication by Mihai Popa.
- International Workshop on Operator Theory and Operator Algebras, Lisbon (Portugal), July 5–8. Communication by I. Valuşescu.
- 7th European Congress of Mathematics, Berlin (Germany), July 18–22. Communication by I. Valuşescu.

3.2. **Collaboration visits.**

- July 9–17, 2012: Aurelian Gheondea, Krakow. Cooperation with P. Cojuhari.
- July 3–17, 2013: Aurelian Gheondea, Krakow. Cooperation with P. Cojuhari.
- March 17 martie–April 4, 2013: C.-G. Ambrozie, Bilkent University (Ankara). Cooperation with A. Gheondea.
- July 2–12, 2014: Aurelian Gheondea, Krakow. Cooperation with P. Cojuhari.
- August 8–23, 2014: D. Timotin, Seoul (Korea), participation to International Congress of Mathematicians; various discussions with participants.
- November 14–30, 2015: D. Timotin, Bangalore. Cooperation with J. Sarkar.
- March 19–26, 2016: D. Timotin, Metz. Cooperation with Wing Suet Li.
- May 2–15, 2016: D. Timotin, Bordeaux. Cooperation with Elizabeth Strouse and M. Zarrabi.

References

[1] Ambrozie, C.-G., Multivariate truncated moments problems and maximum entropy, *Ann. Math. Phys.* **3** (2013), 145–161. .

[2] Ambrozie, C.-G., Gheondea, A., An interpolation problem for completely positive maps on matrix algebras: solvability and parametrization, *Linear and Multilinear Algebra* 63 (2015), 826–851.

[3] Ambrozie, C.-G., Gheondea, A., Interpolation for completely positive maps: numerical solutions, arxiv:1401.8239v2.

[4] Ay, S., Gheondea, A., Representations of *-semigroups associated to invariant kernels with values adjointable operators, *Linear Algebra and Applications* 486 (2015), 361–388.

[5] Ay, S., Gheondea, A., Representations of $*$-semigroups associated to invariant positive semidefinite kernels with values continuously adjointable operators on VH-spaces, *J. Math. Anal. Appl.*, submis; arXiv: 1507.06840 [math.OA].

[6] Benhida, Ch., Timotin, D., Contractively included subspaces of Pick spaces, *Complex Analysis and Operator Theory* 9 (2015), 245–264.

[7] Bercovici, H., Timotin, D., Factorizations of analytic self-maps of the upper half-plane, *Ann. Acad. Sci. Fenn. (Math.)* **37** 2012, 649–660.

[8] Bercovici, H., Timotin, D., A note on composition operators in a half-plane, *Arch. Math.*, publicat online pe 22.11.2012.

[9] Bercovici, H., Timotin, D., The numerical range of a contraction with finite defect numbers, *Journal of Math. Analy. and Appl.* 417 (2014), 42–56.

[10] Chalendar, I., Timotin, D., Commutation relations for truncated Toeplitz operators, *Operators and Matrices* 8 (2014), 877–888.

[11] Cojuhari, P., Gheondea, A, Triplets of closely embedded Hilbert spaces, *Integral Equations Operator Theory* 21 (2015), 1–33.

[12] Gheondea, Aurelian, Dilations of some VH-spaces operator valued invariant kernels, *Integral Eq. Operator Theory*, publicat online pe 9.11.2012.

[13] Gheondea, A., Reproducing Kernel Krein Spaces, in *Operator Theory*, D.A. Alpay (Ed.), Vol. 1, pp. 311–343, Berlin, Springer Verlag 2015.

[14] Gheondea, A., Operator models for Hilbert locally C*-modules, submis la *Journal of the London Mathematical Society*, arXiv: 1507.07643v1.

[15] Gheondea, A., On locally Hilbert spaces, *Opuscula Mathematica*, acceptat.

[16] Kumari, R., Sarkar, J., Sarkar, S., Timotin, D., Factorization of kernels and reproducing kernel Hilbert spaces, arxiv xxxx

[17] Mashreghi, J., Timotin, D., Nonextreme de Branges–Rovnyak apaces as models for contractions, *Integral Equations Operator Theory* 80 (2014), 137–152.

[18] Mingo, James; Popa, Mihai, Real Second Order Freeness and Haar Orthogonal Matrices, *J. Math. Physics* 54 (2013), 35pp.

[19] Mingo, James; Popa, Mihai, On the Relation between Complex and Real Second Order Free Independence, preprint.

[20] Mingo, James; Popa, Mihai, Freeness and the transposes of unitarily invariant random matrices, acceptat la *J. Functional Analysis*.

[21] Popa, Mihai; Vinnikov, Victor, H^2-Spaces of Non-Commutative Functions, preprint.

[22] Popa, Mihai; Jiao, Yong, On fluctuations of traces of large matrices over a non-commutative algebra, *J. Operator Theory* 73 (2015), 71–90.

[23] Popa, M., Vinnikov, V., Wang, J-C., On the multiplication of operator valued c-free random variables, preprint.

[24] Prunaru, B., A Poisson boundary for topological semigroups, *Archiv der Mathematik* 102 (2014), 449–454.

[25] Prunaru, B., Asymptotic stability for a class of Markov semigroups, *Mathematical Reports* 15 (2013), 489–495.

[26] Prunaru, B., A construction of Poisson spaces for topological semigroups, preprint.

[27] Prunaru, B., A short proof of a Choquet–Deny theorem for abelian topological semigroups, preprint.

[28] Timotin, D., Schur coupling and related equivalence relations for operators on a Hilbert space, *Linear Algebra Appl.* 452 (2014), 106–119.

[29] Timotin, D., A short introduction to de Branges-Rovnyak spaces, în *Invariant Subspaces of the Shift Operator*, American Mathematical Society, 2015, 21–38.

[30] Valuşescu I., Some geometrical aspects of Γ-correlated processes, *Anal. Univ. Oradea* XXI (2014), 113–123.

[31] Valuşescu I., Notes on continuous parameter periodically Γ-correlated processes, *An. Univ. Timişoara* **50** (2012), 115–125.

[32] Valuşescu I., Some remarks on the infinite-variate prediction, *Proceedings of the conference Classical and Functional Analysis*, Azuga, 28-29 sept. 2013, 74–102.
[33] Valuşescu I., Some remarks on the infinite-variate prediction II, *Proceedings of the conference Classical and Functional Analysis*, Buşteni, 3–6 sept. 2014, 33–52.
[34] Valuşescu I., On a model for the maximal function of an n-hypercontraction. Preprint IMAR nr. 3/2015.
[35] Valuşescu I., Continuous parameter Γ-stationary processes. Proceedings of the conference Classical and Functional Analysis, Buşteni, iulie 2015. Transilvania University Press, Brasov, 2015, 79–99.
[36] Valuşescu I., Stationary dilation of a nonstationary Γ-correlated process. Proceedings of the International Conference on Theory and Applications of Mathematics and Informatics, Alba Iulia, 17-20 September 2015, *Acta Univ. Apulensis*, special issue, 2015, 121–134.

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