Automorphic forms and L-functions

Abstracts

Compatible systems of Galois representations of global function fields

Gebhard Böckle

Let K be a finitely generated infinite field over the finite prime field $\mathbb{F}_p$ with separable closure K^s and let X be a smooth projective variety over K. By Deligne the cohomology groups $H^i_{\text{ét}}(X_{K^s}, \mathbb{Q}_\ell)$ for varying primes $\ell \neq p$ form a ($\mathbb{Q}$-rational) compatible system of Galois representations of $\mathrm{Gal}(K^s/K)$ and its restriction to the geometric Galois group $G_K^{\text{geo}} = \mathrm{Gal}(K^s/K\mathbb{F}_p^s)$ is semisimple. Using mainly algebraic geometry, representation theory and Bruhat-Tits theory, Cadoret, Hui and Tamagawa showed recently that also the family of reductions $H^i_{\text{ét}}(X_{K^s}, \mathbb{F}_\ell)$ is semisimple as a representation of G_K^{geo} for almost all ℓ, the key case being that of a global function field K. This has important consequence for the image of G_K^{geo} for its action on the adelic module $H^i_{\text{ét}}(X_{K^s}, \mathbb{A}_{\mathbb{Q}})$.

In joint work with W. Gajda and S. Petersen, using automorphic methods as a main tool, we prove the analog of the above result for any E-rational compatible system of Galois representations of a global function field. In the talk I shall explain the context, indicate the applications and sketch how automorphic methods come to bear on the problem.

Quasimodular Siegel modular forms as p-adic modular forms

Siegfried Böcherer

There is a sophisticated theory of nearly holomorphic Siegel modular forms by Shimura. Using previous results by Nagaoka and myself on Rankin-Cohen operators and theta-operators we will present a proof that quasimodular forms (defined as constant terms or as holomorphic part of a nearly holomorphic Siegel modular form) are always p-adic.

The modular completion of certain (formal) generating series

Stephan Ehlen

We consider several "modular completions" of generating series of families of well-known modular forms. In this talk, I will focus on one particular case: Duke, Imamoglu and Toth considered the (formal) generating series $\sum_m G_m(\tau)e(mz)$, where for each m, $G_m(\tau)$ is the weakly holomorphic modular form of weight 3/2 whose Fourier coefficients are the traces of singular moduli of J_m, the weakly holomorphic modular form of weight 0 for the full modular group with principal part q^{-m} and vanishing constant term. They use this to construct a real-quadratic analogue of the Shimura lift. We construct a real analytic modular form $\Omega(\tau, z)$ of weights 3/2 and 2 which can be considered as a modular completion" of this generating series (which means that in a sense the generating series of the G_m is a formal mock-modular form) and relate it to well-known non-holomorphic theta functions. This is joint work with K. Bringmann and M. Schwagenscheidt.

Indefinite theta series

Jens Funke

In this talk, we discuss recent developments in the construction of theta series associated to indefinite quadratic forms from a geometric point of view. This is joint work with Steve Kudla.

Some Indefinite Theta Functions of Signature (1,3) and (1,2)

Jonas Kaszian

While theta functions for definite lattices yield well understood modular forms, indefinite lattices produce mock modular forms (usually of higher depth). We studied explicit examples of signature (1,3) and (1,2) with some degeneracies and with connections to Gromov-Witten theory and representation theory, respectively. This was joint work with Kathrin Bringmann, Larry Rolen and Antun Milas and is built on previous work by Sander Zwegers and others.

An arithmetic Riemann–Roch theorem on modular curves via heat kernel regularization

Jürg Kramer

The arithmetic Riemann–Roch theorem has been established by H. Gillet/C. Soulé as well as by G. Faltings for projective and generically smooth morphisms $f\colon \mathscr{X} \longrightarrow \mathscr{Y}$ of arithmetic varieties and hermitian vector bundles $\overline{\mathscr{E}} = (\mathscr{E}, \|\cdot\|)$ equipped with *smooth* hermitian metrics. In our talk, we will present a variant of an arithmetic Riemann–Roch theorem in a singular setting, namely, the case of the line bundle of modular forms of weight k (an even integer) on (regular, projective models of) modular curves equipped with the Petersson metric, which becomes logarithmically singular at the cusps. The proof starts from the known arithmetic Riemann–Roch formula for smooth hermitian metrics approximating the hyperbolic metric under consideration and then proceeds with an investigation of the degeneration behavior of the starting formula using heat kernel regularization techniques while the smoothened metric approaches the singular metric.

On multiple q-zeta values and period polynomials

Ulf Kühn

We present a class of q-analogues of multiple zeta values given by certain formal q-series with rational coefficients. After introducing a notion of weight and depth for these q-analogues of multiple zeta values we will state a dimension conjectures for the spaces of their weight- and depth-graded parts, which have a similar shape as the conjectures of Zagier and Broadhurst-Kreimer for multiple zeta values.

Harmonic Maass forms associated to real quadratic fields

Yingkun Li

It is well-known that definite quadratic forms give rise to theta series, which are holomorphic modular forms. In 1926, Hecke attached weight one holomorphic theta series to indefinite quadratic forms of signature (1, 1). This ingenious construction reminds one of the Rankin-Selberg unfolding method, yet predates it by a decade. In 2003, Bruinier and Funke introduced the notion of harmonic Maass forms, which have poles at the cusps and map to classical holomorphic modular forms under a suitable differential operator. In this talk, we will construct harmonic Maass forms that map to Heckes indefinite theta series. This is a joint work with P. Charollois.

On the arithmetic Grothendieck Riemann Roch Theorem

Răzvan Liţcanu

In this talk I will present some explicit results concerning a general form of the arithmetic Grothendieck-Riemann-Roch theorem, valid for general projective morphisms between regular arithmetic varieties. I will survey the axiomatic approach for defining the cohomological and analytic objects needed for formulating and proving this theorem and I shall explain how one can compute some numerical invariants involved.

Eisenstein series for Jacobi forms of lattice index

Andreea Mocanu

Jacobi forms are a mix of modular forms and abelian functions that arise in a natural way as functions of lattices (Jacobi theta functions are an example). Jacobi forms of lattice index play an important role in the mirror symmetry conjecture for K3 surfaces and they determine Lorentzian Kac-Moody Lie algebras of Borcherds type. In this talk, we introduce Eisenstein series for Jacobi forms of lattice index and we discuss some of their properties, such as their orthogonality to cusp forms and their Fourier expansion.

On Artin's L-functions

Florin Nicolae

Let $K/\mathbb{Q}$ be a finite Galois extension, $s_0 \in \mathbb{C} \setminus \{1\}$. We present a criterion for the Artin's L-functions associated to $K/\mathbb{Q}$ to be holomorphic at s_0.

Character sums and Multiple Dirichlet Series

Vicenţiu Paşol

We review MDS constructions. Over function fields, computation of character sums is necessary. We will show how the geometric interpretation of character sums helps us understand their structure, thus providing a tool for constructing MDS. This is joint work with Adrian Diaconu.

An arithmetic Riemann-Roch theorem on modular curves via zeta regularization

Anna von Pippich

Let $\Gamma \subset \mathrm{PSL}_2(\mathbb{R})$ be an arbitrary Fuchsian subgroup of the first kind. In this talk, we report on a generalization of the arithmetic Riemann–Roch theorem of Deligne and Gillet–Soulé to the case of the trivial sheaf on $\overline{\Gamma\backslash\mathbb{H}}$, equipped with the singular Poincaré metric. The proof combines metric degeneration and zeta regularization techniques with Mayer–Vietoris type formulas. As application we determine the special value at $s = 1$ of the derivative of the Selberg zeta function for $\Gamma = \mathrm{PSL}_2(\mathbb{Z})$. This is joint work with Gerard Freixas.

On a conjecture of Gross on special values of L-functions

Cristian Popescu

I will report on my recent work on an equivariant refinement of a conjecture of Gross linking special values of p-adic and global L-functions. I will make connections with recent results of Dasgupta and Kakde on the original, non-equivariant Gross conjecture.

Operators algebras and endomorphisms of spaces of invariant vectors under a discrete group

Florin Rădulescu

We consider pairs consisting of a countable discrete group G and an almost normal subgroup Γ and unitary representation π that has the property that π restricted to Γ is a multiple of the left regular representation of Γ. The Murray von Neumann dimension theory gives a multiplicity with a continuum set of values. For example if G acts by measure preserving transformations on an infinite measure space, with Γ having a fundamental domain, then the multiplicity is infinite. On the other hand representations in the analytic discrete series of $PSL(2, \mathbb{R})$, when resticted to $PSL(2, \mathbb{Z})$ have a fractional multiplicity (not always an integer). We analyze the space of Γ-invariant vectors and the action of G on this space. The left regular representation, which acts on $l^2(\Gamma)$, has no invariant vectors, but it is easy to imagine a theory of Γ invariant vectors, which "live" outside the Hilbert space. The representation of G extends (by admitting also vectors that are invariant by finite index subgroups). This is easily extended to integer multiplicity. The case of fractional dimension is more complicated, and we analyze this from an operator algebra perspective, obtaining a correspondence on the two distinct representations of G, and their Plancherel distributional traces.

Kronecker limit formulas for parabolic, hyperbolic and elliptic Eisenstein series via Borcherds products

Markus Schwagenscheidt

The classical Kronecker limit formula describes the constant term in the Laurent expansion at the first order pole of the non-holomorphic Eisenstein series associated to the cusp at infinity of the modular group. Recently, the meromorphic continuation and Kronecker limit type formulas were investigated for non-holomorphic Eisenstein series associated to hyperbolic and elliptic elements of a Fuchsian group of the first kind by Jorgenson, Kramer and von Pippich. In joint work with Anna von Pippich and Fabian Völz, we realized averaged versions of all three types of Eisenstein series for $\Gamma_0(N)$ as regularized theta lifts of a single type of Poincaré series, due to Selberg. Using this realization and properties of the Poincaré series we derive the meromorphic continuation and Kronecker limit formulas for the above Eisenstein series. The corresponding Kronecker limit functions are then given by the logarithm of the absolute value of the Borcherds product associated to a special value of the underlying Poincaré series.

The triple product L-function: formulas and applications

Mike Woodbury

Given cusp forms $f = \sum a_n q^n$, $g = \sum b_n q^n$ and $h = \sum c_n q^n$ where $q = e^{2\pi i z}$ with z a variable in the complex upper half plane, one can consider the triple product L-function $L(s, f \times g \times h) = \sum a_n b_n c_n n^{-s}$. There is a distinguished history of this L-function and in particular its central value in relation to trilinear forms. We focus on using a formula of Ichino together with recent formulas for so-called local trilinear forms. This work has applications to analytic number theory, p-adic L-functions and physics.

From knots to algebraic numbers

Don Zagier

In recent years, there has been intense interest in so-called quantum invariants of knots and their asymptotic properties, a typical example being the celebrated Volume Conjecture for the Kashaev invariant. But it turns out there are also very interesting arithmetic properties of these invariants, including a surprising near-modular transformation property. Even though manythese are only conjectural, one can check them numerically to high precision, and when one does this, algebraic numbers of a special sort (roots of units in certain number fields) appear by magic. This led, in joint work with Frank Calegari and Stavros Garoufalidis, to a new (non-conjectural) construction of units starting from elements in so-called Bloch groups, and as a side product also to a solution of Nahms conjecture on the modularity of certain q-hypergeometric series.