Workshop on Stochastic Dynamics

within the project PN-III-P4-PCE-2021-0921

November 18-19, 2022, Bucharest, Romania

ABSTRACTS

The PSO algorithm – a particular version

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The PSO algorithm is a very popular one among the practitioners in ML thanks to the elegant coding manner and its interesting parallel with animal behavior. However, the mathematical literature is still lacking a solid background in this regard; for instance, one cannot say much about the convergence of the algorithm, given a precise choice for the hyperparameters.

Our purpose is to take advantage of the stochastic intervention from this algorithm - not approximating it with constants as in most previous algebraic works - in order to obtain qualitative results. Hence, classical probabilistic instruments are put to use, aiming to create a context that unifies the rigorous conditions with the go-to values from practice.

Joint work with Ionel Popescu and Iulian Cîmpean (Bucharest, Romania).

Uniqueness for nonlinear Fokker-Planck equations and for McKeanVlasov SDEs

Viorel Barbu (Iassy, e-mail: vb41@uaic.ro)

Under suitable assumptions the nonlinear Fokker-Planck equation $\rho_t - \Delta\beta(\rho) + div(Db(\rho)\rho = 0; \rho(0) = \rho_0$ has a unique mild solution. For bounded initial data also the distributional solution is unique. This implies the existence and uniqueness of probabilistically weak solutions to corresponding McKean-Vlasov SDEs.

These results were obtained in a joint work with Michael Röckner (Bielefeld University).

Keywords: Optimal control problem; optimal control; feedback control; stochastic differential equation; Kolmogorov equation; weak solution.

A variational condition for uniqueness of Doeblin measures

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Doeblin measures are known in the literature under many names: chains of infinite order, chains with complete connections, uniform martingales or g-measures. Intuitively, they can be regarded as a generalization of Markov chains to processes with infinite memory. In such a setting, the uniqueness of the stationary measure becomes a non-trivial problem. In this talk, I will review the previously known uniqueness criteria for Doeblin measures and present a new result obtained in collaboration with N. Berger, A. Johansson and A. Oberg. I will also give a conjecture regarding the threshold between the uniqueness and non-uniqueness regimes.

An excursion through the probabilistic representations of the fragmentation equation

Madalina Deaconu (Nancy, e-mail: madalina.deaconu@inria.fr)

The models of coagulation / fragmentation are used in many real problems as : in Astrophysics, the formation of stars and the creation of meteorites; in Geophysics, the accumulation of snow or the breakage of fragments in the avalanches. We study here the stochastic process which represents the evolution of the mass of a typical particle performing a fragmentation process.

In particular, we will introduce some possible probabilistic approaches and representations of the fragmentation equation. These interpretations allow the use of the probabilistic methodology to recover known results or to identify new behaviours. These probabilistic representations range from Markov chains to stochastic differential equations with jumps. We focus on how these processes are interconnected and we show also how these representations lead to new and easy to implement probabilistic numerical methods.

Hypocoercivity for second order in time reaction-diffusion and Cahn-Hilliard type equations with multiplicative noise

Benedikt Eisenhuth (Kaiserslautern, e-mail: eisenhuth@mathematik.uni-kl.de)

We analyze non-linear degenerate stochastic partial differential equations with multiplicative noise in terms of their infinitesimal generators. The focus is on second order in time stochastic Reaction-Diffusion and Cahn-Hilliard type equations with multiplicative noise.

We establish essential m-dissipativity of the associated generators in $L^2(\mu^\Phi)$ on smooth finitely based functions. Here Φ is an appropriate potential and μ^Φ is a measure with density $e^{-\Phi}$ w.r.t. an infinite dimensional Gaussian measure.

Using resolvent methods developed by Beznea, Boboc and Röckner we construct corresponding μ^Φ-standard right processes with infinite lifetime and weakly continuous paths providing weak solutions to such non-linear degenerate stochastic partial differential equations. Moreover, we identify the transition semigroup of the process with the C_0-semigroup $(T_t)_{t\geq 0}$ generated by the essential m-dissipative generator of the equation. Afterwards, we use the abstract Hilbert space hypocoercivity method, developed by Grothaus and Stilgenbauer to derive hypocoercivity of $(T_t)_{t\geq 0}$ and the transition semigroup, respectively.

A polynomial expansion for Brownian motion and the associated fluctuation process

Karen Habermann (Warwick, e-mail: karen.habermann@warwick.ac.uk)

We start by deriving a polynomial expansion for Brownian motion expressed in terms of shifted Legendre polynomials by considering Brownian motion conditioned to have vanishing iterated time integrals of all orders. We further discuss the fluctuations for this expansion and show that they converge in finite dimensional distributions to a collection of independent zero-mean Gaussian random variables whose variances follow a scaled semicircle.

We then link the asymptotic convergence rates of approximations for Brownian Lévy area which are based on the Fourier series expansion and the polynomial expansion of the Brownian bridge to these limit fluctuations. We close with a general study of the asymptotic error arising when approximating the Green's function of a Sturm-Liouville problem through a truncation of its eigenfunction expansion, both for the Green's function of a regular Sturm-Liouville problem and for the Green's function associated with the classical orthogonal polynomials.

Hypocoercivity for non-linear infinite-dimensional degenerate stochastic differential equations

Martin Grothaus (Kaserslautern, e-mail: martin.grothaus@gmail.com)

Motivated by problems from Industrial Mathematics we further developed the concepts of hypocoercivity. The original concepts needed Poincaré inequalities and were applied to equations in linear finite dimensional spaces. Meanwhile we can treat equations in manifolds or even infinite dimensional spaces. The condition giving micro- and macroscopic coercivity

we could relax from Poincaré to weak Poincaré inequalities. In this talk an overview and many examples are given.

On local and global solutions for a class of stochastic shallow water models

Oana Lang (London, e-mail: o.lang15@imperial.ac.uk)

In this talk I will talk about the analytical properties of a class of stochastic shallow water (SRSW) models which include SRSW models derived using the Location Uncertainty (Mémin, 2014) approach. We prove that there exists a unique maximal strong solution and a unique global weak solution, using a method which relies on a Cauchy approximating sequence. Our method is very convenient as although the strong solution exists in a higher order Sobolev space, we show that it suffices to prove the Cauchy property in L^2. Comparisons with other types of SRSW models will also be made.

This is joint work with Dan Crisan and Étienne Mémin.

Well-posedness for the Cahn-Hilliard-Navier-Stokes equations driven by gradient type noise

Ionuţ Munteanu (Iassy, e-mail: ionut.munteanu@uaic.ro)

In this work we study the problem of existence and uniqueness of solutions for the Cahn-Hilliard-Navier-Stokes system perturbed by divergence type noise. We apply a rescaling argument to transform the stochastic system into a random deterministic one. The rescale operators possess very good properties which allow us to show that the rescaled Navier-Stokes equation has a unique solution by appealing to m-accretive operators theory, and to show the existence for the Cahn-Hilliard equation via a fixed point argument.

For the two-dimensional case we prove global in time well-posedness, while, for the three-dimensional case, we obtain only a local in time result, but with the maximal existence time non-random. We emphasize that, the literature on the subject of existence of solutions for the stochastic Navier-Stokes equations provide results that involve only random maximal time existence (stopping-time). Hence, our deterministic existence-time result is itself another important obtain of the present work.

Stochastic Integration from a Functional-Analytic Point of View

Alexandru Mustăţea (Bucharest, e-mail: Alexandru.Mustatea@imar.ro)

We present a construction of the concept of stochastic integration in Riemannian manifolds from a purely functional-analytic point of view. We show that there are infinitely many such integrals, and that any two of them are related by a simple formula.

We also find that the Stratonovich and Itô integrals known to probability theorists are two instances of the general concept constructed herein.

A modified SIRD model for Covid19 spread prediction using ensemble neural networks

Marian Petrică (Bucharest, e-mail: marianpetrica11@gmail.com)

In this talk, we propose an analysis of Covid19 evolution and prediction on Romania combined with the mathematical model of SIRD, an extension of the classical model SIR, which includes the deceased as a separate category. In addition, one of the parameters of our model includes the proportion of infected and tested versus infected.

Since there are many factors which have an impact on the evolution of the pandemic, we decide to treat the estimation and the prediction based on the previous 7 days of data, particularly important here being the number of deceased.

We perform the estimation and prediction using neural networks in two steps. Firstly, by simulating data with our model, we train several neural networks which learn the parameters of the model. Secondly, we use an ensemble of ten of these neural networks to forecast the parameters from the real data of Covid19 in Romania.

Many of these results are backed up by a theorem which guarantees that we can recover the parameters from the reported data.

Pure branching and total mass processes

Adela Popescu (Bucharest, e-mail: adepopescu@yahoo.com)

We will show that for a branching process the number of particle in the system is a discrete branching process.

Spectral gap estimates for Brownian motion with sticky reflecting boundary diffusion

Max von Renesse (Leipzig, e-mail: renesse@uni-leipzig.de)

Introducing an interpolation method we estimate the spectral gap for Brownian motion on general domains with sticky-reflecting boundary diffusion associated to the first nontrivial eigenvalue for the Laplace operator with corresponding Wentzell-type boundary condition. In the manifold case our proofs involve novel applications of the celebrated Reilly formula.

Stochastic solution for a nonlinear Dirichlet problem with discontinuous boundary data

Alexandra Teodor (Bucharest, e-mail: alexandravictoriateodor@gmail.com)

We give a probabilistic representation of the solution of a nonlinear Dirichlet problem associated to a branching process with general boundary data, in the sense of the controlled convergence initiated by A. Cornea.

The talk is based on a joint work with. L. Beznea.

Two results on the optimal control for stochastic partial differential equations (SPDE)

Wilhelm Stannat (Berlin, e-mail: rstannat@math.tu-berlin.de)

The talk will consist of two parts:

In the first part we present an extension of Peng's maximum principle for semilinear SPDEs in one space-dimension with non-convex control domains and control-dependent diffusion coefficients to the case of general cost functionals with Nemytskii-type coefficients (ref [1]). The analysis is based on a new approach to the characterization of the second order adjoint state as the solution of a function-valued backward SPDE.

In the second part we present preliminary results on the approximation of optimal feedback controls for SPDEs using a general type of approximation that allows us to reduce the control problem to an optimization over deterministic controls. Similar to ref [2] we derive necessary optimality conditions, prove the existence of an optimal control and derive explicit convergence rates for the reduced problem. We also provide numerical illustrations using radial basis functions in the approximation.

The talk is based on joint work with A. Vogler and L. Wessels.

References:

[1] W. Stannat, L. Wessels, Peng's Maximum Principle for Stochastic Partial Differential Equations, *SIAM J. Control Optim.*, **59** (2021), 3552–3573.
[2] W. Stannat, L. Wessels, Deterministic control of stochastic reaction-diffusion equations, *Evolution Equations & Control Theory*, **10**(2021), 701–722.