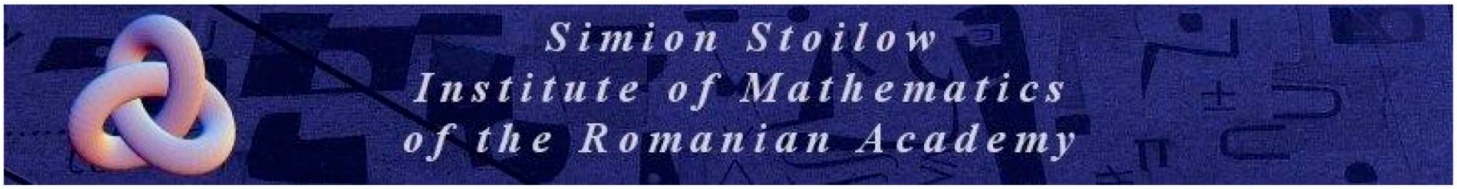

Mathematics and Machine Learning
Tuesday 26th September 2023, IMAR, Miron Nicolescu amphitheater

12:00-13:00 Arnulf Jentzen (The Chinese University of Hong Kong, Shenzhen & University of Münster) (IMAR monthly conference):

Overcoming the curse of dimensionality: from nonlinear Monte Carlo to the training of neural networks

Abstract: Partial differential equations (PDEs) are among the most universal tools used in modelling problems in nature and man-made complex systems. Nearly all traditional approximation algorithms for PDEs in the literature suffer from the so-called "curse of dimensionality" in the sense that the number of required computational operations of the approximation algorithm to achieve a given approximation accuracy grows exponentially in the dimension of the considered PDE. With such algorithms it is impossible to approximatively compute solutions of high-dimensional PDEs even when the fastest currently available computers are used. In the case of linear parabolic PDEs and approximations at a fixed space-time point, the curse of dimensionality can be overcome by means of Monte Carlo approximation algorithms and the Feynman-Kac formula. In this talk we present an efficient machine learning algorithm to approximate solutions of high-dimensional PDE and we also prove that deep artificial neural network (ANNs) do indeed overcome the curse of dimensionality in the case of a general class of semilinear parabolic PDEs. Moreover, we specify concrete examples of smooth functions which cannot be approximated by shallow ANNs without the curse of dimensionality, but which can be approximated by deep ANNs without the curse of dimensionality. In the final part of the talk we present some recent mathematical results on the training of neural networks.

14:00-14:40 Sorin Mitran (University of North Carolina, Chapel Hill)

Density Estimation, Information Geometry, and Neural Networks

Abstract: Statistical manifolds have been proposed by Amari as a possible framework for analysis of the information processing capabilities of both biological and artificial neural networks. An extension of this approach to dynamically reconstructed manifolds based on measured data is considered in lieu of the original consideration of analytically pre-defined manifolds. This leads to links with computational topology and minimal descriptions of generic non-Gaussian stochastic processes arising in machine learning. Sample biophysical applications are presented.

14:40-15:20 Lucian Beznea (IMAR and Politehnica Bucharest):

Stochastic solutions to evolution equations of non-local branching processes

Abstract: We give a probabilistic representation for the solution to a nonlinear evolution equation induced by a measure-valued branching process. The main application is to prove stochastic aspects for a nonlinear evolution equation related to the Neumann problem and the surface measure on the boundary, which corresponds to the reflecting Brownian motion as base movement, taking the killing rate given by the local time on the boundary. We use specific potential theoretical tools. The talk is based on joint works with Oana-Lupascu Stamate and Catalin Ioan Vrabie (Bucharest).

15:20-16:00 Iulian Cîmpean (University of Bucharest and IMAR):

From Monte Carlo to neural networks approximations of boundary value problems

Abstract: We present probabilistic and neural network approximations for solutions to Poisson equation subject to Holder continuous Dirichlet boundary conditions in general bounded domains in R^d. Our main results are two-folded: On the one hand we show that the solution to Poisson equation can be numerically approximated in the sup-norm by Monte Carlo methods, without the curse of high dimensions and efficiently with respect to the prescribed approximation error. On the other hand, we show that the obtained Monte Carlo solver renders a random ReLU deep neural network (DNN) that provides with high probability a small approximation error and low polynomial complexity in the dimension. This is joint work with L. Beznea, O. Lupascu, I. Popescu, and A. D. Zarnescu.

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