Analysis and Potential
- On the Anniversary of Gheorghe Bucur -

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Abstracts

Non universality for the variance of the number of real roots of random trigonometric polynomials

Vlad Bally (Marne la Vallée, France, email: bally@univ-mlv.fr)

We study the Central Limit Theorem in total variation distance and we use it in order to obtain an invariance principle for the expected number of roots of a trigonometric polynomial with random coefficients. Surprisingly, when discussing the variance of the number of roots the above universality result breaks up: a constant depending on the forth moment of the random variables at hand comes on. Based on joint works with Lucia Caramellino and Guillaume Poly.

Optimal control approach to mean-field games system

Viorel Barbu (Iassy, Romania, email: vbarbu41@gmail.com)

The partial differential system arising in the mean-field games theory is treated as an Euler-Lagrange system associated with an optimal control problem governed by a linear Fokker-Planck equation with input controller in the drift term. In this way one obtains the existence and uniqueness of solution in appropriate Sobolev spaces.

Interactions in the Lorentz force equation

Cristian Bereanu (Bucharest, Romania, email: cristian.bereanu@imar.ro)

I will speak about the following result: assume that the Lorentz force equation has the fundamental Kepler electric potential. If the magnetic potential has good interactions with the Kepler electric potential, then the Lorentz force equation has a sequence of periodic solutions with fixed period. To prove our main result - using the Lustenik-Schnirelman category and Ekeland's variational principle-we develop a Lusternik-Schnirelman strategy for the nonsmooth Poincaré action functional associated to the Lorentz force equation.

A life time of Potential Theory in Bucharest

Lucian Beznea (Bucharest, Romania, email: lucian.beznea@imar.ro)

We present achievements of the Romanian Potential Theory School and emphasise several contributions of Professor Gheorghe Bucur in this frame.

Multiplicity results for singular and critical problem involving fractional elliptic operator

Mounir Bezzarga (Tunis, Tunisia, email: mounir.bezzarga@yahoo.fr)

We study the following elliptic problem:

$$\begin{cases} (-\Delta)^{\alpha} u = \frac{a(x)}{u^{\gamma}} + \lambda b(x)|u|^{2^*_\alpha - 2}u & \text{in } \Omega, \\ u = 0 & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases}$$

where $\alpha, \gamma \in (0,1)$, $2^*_\alpha = \frac{2N}{N-2\alpha}$, $(N > 2\alpha)$, Ω is a bounded domain with smooth boundary in $\mathbb{R}^N$, and the functions a, b are continuous on $\overline{\Omega}$. The existence and multiplicity of solutions are proved by using variational method and fibering maps on the Nehari manifold.

A potential theoretic construction of Hunt processes associated to generalized Mehler semigroups

Iulian Cîmpean (Bucharest, Romania, email: iulian.cimpean@imar.ro)

Generalized Mehler semigroups are formally generated by (possibly) non-Gaussian Ornstein-Uhlenbeck operators and form an important class of Markov semigroups in infinite dimensions. We intend to present a general potential theoretic method for the construction of Hunt Markov processes associated to such semigroups. Based on a joint work with Lucian Beznea (Bucharest) and Michael Röckner (Bielefeld).

Partially positive semidefinite maps on *-semigroupoids

Aurelian Gheondea (Ankara, Turkey and Bucharest, Romania, email: gheondea@theta.ro)

Motivated by Cuntz-Krieger-Toeplitz systems associated to undirected graphs and representations of groupoids, we obtain a generalisation of the Sz-Nagy's Dilation Theorem for operator valued partially positive semidefinite maps on $*$-semigroupoids with unit, with varying degrees of aggregation, firstly by $*$-representations with unbounded operators and then

we characterise the existence of the corresponding $*$-representations by bounded operators. Joint work with Bogdan Udrea, IMAR Bucharest.

A Trotter-Kato type result in the weak topology

Gabriela Marinoschi (Bucharest, Romania, email: gabimarinoschi@yahoo.com)

We present a Trotter-Kato type result in the weak topology. Let $\{A^\varepsilon\}_{\varepsilon>0}$ be a family of quasi m-accretive linear operators on a Hilbert space X. Under certain conditions, the result states that if the sequence of the powers of the resolvents of A^ε converges weakly to the corresponding powers of the resolvent of A, then the sequence of the semigroups generated by $-A^\varepsilon$ tends weakly to the semigroup generated by $-A$, uniformly with respect to t on compact intervals. An application regarding the solution existence to a parabolic-hyperbolic PDE is discussed.

About approximation and interpolation of neural networks

Ionel Popescu (Bucharest, Romania, emai: ioionel@gmail.com)

I will discuss approximation and interpolation of neural networks with an arbitrary activation function. The main result (still in work) is that as long as the activation function is not affine, then the deep neural networks are approximators and interpolators.

Nonlinear Fokker–Planck–Kolmogorov equations as gradient flows on the space of probability measures

Michael Röckner (Bielefeld, Germany, email: roeckner@math.uni-bielefeld.de)

We propose a general method to identify nonlinear Fokker–Planck–Kolmogorov equations (FPK equations) as gradient flows on the space of Borel probability measures on $\mathbb{R}^d$ with a natural differential geometry. Our notion of gradient flow does not depend on any underlying metric structure such as the Wasserstein distance, but is derived from purely differential geometric principles. Moreover, we explicitly identify the associated energy functions and show that these are Lyapunov functions for the FPK solutions. Our main result covers classical and generalized porous media equations, where the latter have a generalized diffusivity function and a nonlinear transport-type first-order perturbation.

Special session for PhD students

Wang's Harnack inequality for SDEs with multiplicative noise

Lecture for PhD students

Iulian Cîmpean (Bucharest, Romania, email: iulian.cimpean@imar.ro)

The aim of this seminar lecture is to introduce some fundamental ideas concerning Harnack inequality for stochastic differential equations, as a starting point for a series of several lectures that are intended to be presented in the Potential theory seminar in the forthcoming sessions. On brief, we are interested in learning and discussing the methods developed in [Feng-Yu Wang. "Harnack inequality for SDE with multiplicative noise and extension to Neumann semigroup on nonconvex manifolds." *Ann. Probab.* **39** (2011), 1449–1467].

Facets of stagnation in the PSO algorithm

Alexandra Andriciuc (Bucharest, Romania, email: andriciucalexandra@yahoo.com)

Particle Swarm Optimization is a swarm intelligence technique which shares many features with evolutionary algorithms. Although there is no centralized control, the local interactions between agents lead to the emergence of global behavior. The idea here is to mathematically analyze the behavior of a particle when there is no improvement over several time steps. If the system is seen as a black box that displays only the best positions found so far by the particles, it happens nothing seen from outside, as there certainly happens something inside, which is worthy to study.

Machine Learning algorithms and branching processes applications in medicine

Valentin Ioan Constantinescu (Bucharest, Romania, email: valentin.constantinescu@s.unibuc.ro)

The presentation aims to comprise the results obtained by applying machine learning algorithms to a medical database with real life observed data, in order to study diverse problems related to the cancer disease – for example: how accurately can a classification algorithm predict if there will be a reoccurrence of the disease for a patient. The algorithms are compared by analyzing the generalization error. The empirical study was conducted in collaboration with the C.I Parhon Institute of Endocrinology as part of an active working

group. In addition to the empirical results, the presentation touches briefly on some well-known theoretical results related to the application of branching processes in the study of cancer.

Considerations on a spinor representation of quantum computing

Răzvan Moraru (Bucharest, Romania, email: razvan.moraru@my.fmi.unibuc.ro)

Our aim is to offer a brief survey on the inherent relation between three classes of operators: braiding matrices, universal (entangling) quantum gates and the elements of the Clifford algebras - in particular Majorana field operators, any of them obeying to the Yang-Baxter equation. This series of correspondences potentially unveils a way to represent the quantum computing on the Clifford algebras, leading thus to a representation of quantum computing on fermionic fields.

Moving higher-order Taylor approximations method for smooth constrained minimization problems

Yassine Nabou (Bucharest, Romania, email: yassine.nabou@stud.acs.upb.ro)

We present a higher-order method for solving composite (non)convex minimization problems with smooth (non)convex functional constraints. At each iteration our method approximates the smooth part of the objective function and of the constraints by higher-order Taylor approximations, leading to a moving Taylor approximation method (MTA). We present convergence guarantees for MTA algorithm for both, nonconvex and convex problems. In particular, when the objective and the constraints are nonconvex functions, we prove that the sequence generated by MTA algorithm converges globally to a KKT point. Moreover, we derive convergence rates in the iterates when the problem's data satisfy the Kurdyka-Lojasiewicz (KL) property. Further, when the objective function is (uniformly) convex and the constraints are also convex, we provide (linear/superlinear) sublinear convergence rates for our algorithm. Finally, we present an efficient implementation of the proposed algorithm and compare it with existing methods from the literature. Based on a joint work with Ion Necoara (Bucharest).

Hardy-Rellich Inequalities in domains of $\mathbb{R}^N$ and applications

Teodor Ruginǎ (Bucharest, Romania, email: teorugina@yahoo.com)

I will give a brief introduction regarding the Hardy inequalities in different frameworks in $\mathbb{R}^N$ and connections to the Rellich inequality. The main focus will be on inequalities posed in a domain and with singular potential expressed as powers of distance to the boundary functions. I will present some recent results in this topic and also give some applications to the stochastic processes.

Positive solutions to semilinear Dirichlet problems with general boundary data

Alexandra Teodor (Bucharest, Romania, email: alexandravictoriateodor@gmail.com)

We give a probabilistic representation of the solution to a semilinear elliptic Dirichlet problem with general (discontinuous) boundary data. The boundary behaviour of the solution is in the sense of the controlled convergence initiated by A. Cornea. Uniqueness results for the solution are also provided. Based on a joint work with Lucian Beznea (Bucharest).