IMAR Monthly Lecture

Frobenius-Separability type solutions of the set-theoretic Yang-Baxter equation

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Abstract: We investigate a special class of solutions of the set-theoretic Yang-Baxter equation, called Frobenius-Separability (FS) type solutions. In particular, we show that the category of solutions of the set-theoretic Yang-Baxter equation of FS type is equivalent to the category of pointed Kimura semigroups. As applications, all involutive, idempotent, nondegenerate, surjective, finite order, unitary or indecomposable solutions of FS type are classified. For instance, if $|X| = n$, then the number of isomorphism classes of all such solutions on X that are (a) left non-degenerate, (b) bijective, (c) unitary or (d) indecomposable and left-nondegenerate is: (a) the Davis number $d(n)$, (b) $\sum_{m|n} p(m)$, where $p(m)$ is the Euler partition number, (c) $\tau(n) + \sum_{d|n} \lfloor \frac{d}{2} \rfloor$, where $\tau(n)$ is the number of divisors of n, or (d) the Harary number. The automorphism groups of such solutions can also be recovered as automorphism groups $\mathrm{Aut}(f)$ of sets X equipped with a single endo-function $f \colon X \to X$. We describe all groups of the form $\mathrm{Aut}(f)$ as iterations of direct and (possibly infinite) wreath products of cyclic or full symmetric groups, characterize the abelian ones as products of cyclic groups, and produce examples of symmetry groups of FS solutions not of the form $\mathrm{Aut}(f)$.

Based on joint work with Alexandru Chirvasitu and Gigel Militaru.