INSTITUTUL
DE
MATEMATICĂ

INSTITUTUL NAȚIONAL
PENTRU CREAȚIE
ȘTIINȚIFICĂ ȘI TEHNICĂ

COCYCLE REPRESENTATIONS OF SOLVABLE LIE GROUPS

by

HENRI MOSCOVICI and ANDREI VERONA

PREPRINT SERIES IN MATHEMATICS
No.6/1977

BUCUREȘTI

COCYCLE REPRESENTATIONS OF SOLVABLE LIE GROUPS

BY

HENRI MOSCOVICI* and ANDREI VERONA*

February, 1977

* Institute of Mathematics, Str. Academiei, 14, Bucharest -70109, Romania

INTRODUCTION

The aim of this paper is to extend to the case of cocycle representations the main results of the Auslander - Kostant - Pukanszky theory on unitary representations of solvable Lie groups. Essentially, their results provide complete answers to the following fundamental problems: (a) the characterization of type I - ness; (b) the parametrization of the unitary dual of a type I group; (c) the parametrization of the primitive ideal space of the group C^* - algebra (which seems to be the appropriate "dual object" for a non-type I group).. The answers to all these questions are given in terms of the coadjoint action of the group on the dual vector space of its Lie algebra.

We have found that the ideas developed by these authors can be used also in the treatment of α - representations (α being a 2 - cocycle) and the above mentioned problems get similar answers which this time involve, instead of the coadjoint action, an affine action of the group on the dual space of its Lie algebra, defined by α, which we call the "α - coadjoint action". Specifically, we prove that if S is a connected and simply connected solvable Lie group with Lie algebra $\mathfrak{s}$ and α is an analytic 2 - cocycle on S, then: (1) (S,α) is of type I (i.e. any factor α - representation of S is of type I) if and only if all the orbits of the α - coadjoint action are locally closed in $\mathfrak{s}^*$ and rational (Theorem 3.4.1); (2) if (S,α) is of type I then its α - dual $(S,\alpha)^\wedge$ (i.e. the set of all equivalence classes of irreductible α - representations of S) is parametrized by the orbits of S acting in a natural way on the space $\mathcal{B}^{\alpha}(\mathfrak{s}^*)$ of all pairs (s,η) with $s \in \mathfrak{s}^*$ and η an α - character on the "reduced stabilizer" of S at s (Theorem 3.4.2 (ii)); (3) the space Prim $C^*(S,\alpha)$ of al primitive ideals in the C^* - algebra associated to (S,α) is parametrized by the orbits of an equivalence relation P^{α} on $\mathcal{B}^{\alpha}(\mathfrak{s}^*)$ (Theorem 3.4.3.).

As one can see, besides the replacement of the coadjoint action by the α - coadjoint action, the only difference between our results (1), (2), (3) and the original results of Auslander - Kostant [1, Theorems V.3.2, V.3.3.]and Pukanszky [7, Theorem 1] consists in the fact that in our characterization of type I - ness the rationality condition and not the integrality one plays the essential rôle; the reason is that, the α - coadjoint action being no more linear but affine, the rationality of all orbits does not automatically imply their integrality (that this phenomenon really occurs, is showed in 3.5 by an example). A more subtle difference, having the same source, appears in the explicit construction of the irreducible α - representations (Theorem 3.4.2 (i)). Namely, the lack of the integrality property prevents us from applying the Auslander - Kostant procedure for constructing irreducible representations. Fortunately, this point can be handled by using the extension of the Auslander - Kostant construction which we gave in [5].

Some words about the organization of the material in this paper are now in order. In §1 we recall briefly the construction of unitary representations of a solvable Lie group, along the lines of the Auslander - Kostant method, as developed in [5]. To motivate the content of §2, let us record that if G is the central extension of S by R associated to α, the α - representations of S are in a one-to-one correspondence with the unitary representations of G whose restriction to the central subgroup R is a multiple of the character $t \mapsto e^{2\pi it}$. Thus, our investigation of the cocycle representations appears to be a special case of the study of the unitary representations of a connected and simply connected solvable Lie group G whose restriction to a connected, closed, central subgroup Z is a multiple of a given character λ. This more general setting is approached in §2, where we give a "relativized" version of the Auslander - Kostant - Pukanszky theory. Using this, we derive in §3 the main results in this paper which were already mentioned above.

Finally we mention that for an exponential Lie group S, the parametrization of $(S,\alpha)^\wedge$ in terms of the α - coadjoint action was previously obtained by T.Sund [9].

§1

Throughout this section we shall denote by G a connected and simply connected solvable Lie group with Lie algebra g. We fix also a closed, connected, central subgroup Z of G and a unitary character λ on Z. Further we denote by z the Lie algebra of Z and by ℓ the linear functional on z satisfying $d\lambda = 2\pi i \cdot \ell$. Clearly, $N = Z\cdot[G, G]$ is a closed, connected and nilpotent subgroup of G with Lie algebra $n = z + [g, g]$.

By g^* (resp. n^*, z^*) we mean the dual vector space of g (resp. n, z), while g^*_ℓ (resp. n^*_ℓ) stands for the linear variety of all $g \in g^*$ (resp. $f \in n^*$) such that $g|z = \ell$ (resp. $f|z = \ell$). The group G acts on g (resp. n) via the adjoint representation, and hence by duality on g^* (resp. n^*) via what is called the coadjoint representation, leaving g^*_ℓ (resp. n^*_ℓ) invariant. We shall write Ad(a) (resp. $Ad^*(a)$) for the linear transformation on g (resp. g^*) corresponding to $a \in G$.

The set of all unitary representations of G will be denoted Rep (G), the subset of the factor representations by Fac (G) and the subset of the irreducible representations by Irr (G). The collection of all normal representations (for the definition see [8, p.81]) is denoted by Facn (G). Further $Rep_\lambda(G)$ stands for the set of those $\pi \in$ Rep (G) such that $\pi|Z$ is a multiple of λ, and we put also $Fac_\lambda(G)$ = Fac (G) $\cap$ $Rep_\lambda(G)$, $Facn_\lambda(G)$ = Facn (G) $\cap$ $Rep_\lambda(G)$ and $Irr_\lambda(G)$ = Irr (G) $\cap$ $Rep_\lambda(G)$. Finally, $\hat{G}$ (resp. $\hat{G}^\lambda$) denotes the set of all quasi-equivalence clases in Fac (G) (resp. $Fac_\lambda(G)$), $\hat{G}_{norm}$ (resp. $\hat{G}^\lambda_{norm}$) the subset consisting of the quasi-equivalence classes in Facn (G) (resp. Facn_λ(G)), and $\hat{G}$ (resp. $\hat{G}^\lambda$) the set of all equivalence classes in Irr (G) (resp. Irr_λ(G)).

1.1. Let $g \in \mathfrak{g}^*$, G(g) be the isotropy subgroup of G at g with respect tot the coadjoint action, $\mathfrak{g}(g)$ be its Lie algebra, $X_g = \mathrm{Ad}^*(G).g$ be the orbit of g under G, and ν_g ge the G-invariant 2 - form on X_g induced by the 2 - cocycle $-dg$ on $\mathfrak{g}$. There exists a unique character χ_g on the identity component $G(g)_o$ of G(g) such that $d\chi_g = 2\pi i.g|\mathfrak{g}(g)$; its kernel Q_g is a normal subgroup of G(g). The set of all subgroups Γ of G(g) containing $G(g)_o$ is denoted by $S(g)$. For $\Gamma \in S(g)$, $\Gamma^\#$ is the inverse image in G(g) of the centralizer of Γ/Q_g in $G(g)/Q_g$. Further we define $A(g)$ as being the subset of all $\Gamma \in S(g)$ with $\Gamma \subset \Gamma^\#$, while $A_{max}(g)$ consists of those $\Gamma \in S(g)$ satisfying $\Gamma = \Gamma^\#$. By [5, Lemma 1.2], $\Gamma \in A(g)$ if and only if there exists a unitary character χ of Γ such that $\chi|G(g)_o = \chi_g$; the set of all such characters of Γ is denoted by $\hat{\Gamma}$.

Now let $f = g|\mathfrak{n}$, $M = G(f)$ be the isotropy subgroup of G acting on $\mathfrak{n}^*$, $\mathfrak{m} = \mathfrak{g}(f)$ be the Lie algebra of M, and $m = g|\mathfrak{m}$. Denoting by χ_f the unique character of the connected and simply connected subgroup $N(f) = N \cap G(f)$ whose differential is $2\pi i.f|\mathfrak{n}(f)$, and by Q_f the identity component of Ker χ_f, we observe that Q_f is normal in M, hence in the identity component M_o of M, and that M_o/Q_f is a connected, simply connected, nilpotent Lie group. The functional $m \in \mathfrak{m}^*$ vanishes on $\mathfrak{q}_f = \ker (f|\mathfrak{n}(f)$ and hence becomes a functional on the Lie algebra $\mathfrak{m}/\mathfrak{q}_f$ of M_o/Q_f, giving rise according to the Kirillov theory to an irreducible unitary representation of M_o/Q_f; its pull-back to M_o will be denoted $\rho_o(m)$.

Let $\mathcal{R}(g)$ (resp. $\mathcal{F}(g)$, $\mathcal{I}(g)$) denote the set of all unitary (resp. factor, irreducible) representations σ of $G(g)$ such that $\sigma|G(g)_o$ is a multiple of χ_g. One may associate to any $\sigma \in \mathcal{R}(g)$ a representation $\pi(\sigma)$ of G. This construction, due to Auslander and Kostant [1] and based on the Mackey little group method, will be briefly recorded here (after [4], [5], [10]).

One forms first the representation $\sigma \otimes \rho_o(m)$ of the direct product $G(g) \times M_o$ and one observes that it factorizes through a representation $(\sigma \otimes \rho_o(m))^\frown$ of the stabilizer $M_g = G(g)M_o$ of $\rho_o(m)$ in M (which acts naturally on $\hat{M}_o$). One considers next the representation $\tau(\sigma)$ of M induced by $(\sigma \otimes \rho_o(m))^\frown$ and one lifts it to a representation $\tau(\sigma)^\vee$ of the semi-direct product $M \times_s N$. Now, by forming the tensor product $\tau(\sigma)^\vee \otimes \nu(f)$, where $\nu(f)$ stands for the canonical extension (see [1, Proposition III.2.2. and Theorem III.3.1]) to $M \times_s N$ of the irreducible representation $\rho(f)$ of N associated via the Kirillov construction to $f \in n^*$, one get a representation of $M \times_s N$ which can be dropped down to a representation $(\tau(\sigma)^\vee \otimes \nu(f))^\frown$ of $K = MN$; note that K is the stabilizer of $\rho(f)$ in G acting on $\hat{N}$, and that this last representation when restricted to N is a multiple of $\rho(f)$. Finally, one defines $\pi(\sigma)$ as being the representation of G induced by $(\tau(\sigma)^\vee \otimes \nu(f))^\frown$.

1.1.1. Remark. An important feature of this construction, which is a consequence of the Mackey theory, is that the commuting rings of the representations σ and $\pi(\sigma)$ are algebraically isomorphic.

1.1.2. Remark. Another feature, which we state here for later use, is that $\pi(\sigma) \in Rep_\lambda(G)$ if and only if $g \in g^*_\ell$. Indeed, it is easy to see that $\pi(\sigma)|Z$ is a multiple of $\sigma|Z$, which in turn is a multiple of $\chi_g|Z$. But $\chi_g|Z = \lambda$ if and only if $g|z = \ell$.

1.2. There is a simple way for obtaining representations in $R(g)$ for a given $g \in g^*$, by inducing characters from various $\Gamma \in A(g)$ to $G(g)$. Indeed, if $\Gamma \in A(g)$ and $\chi \in \hat{\Gamma}$, then the representation $\sigma(g,\chi)$ of $G(g)$ induced by χ is obviously in $R(g)$. The corresponding representation $\pi(\sigma(g,\chi))$ of G will be denoted, more simply, $\pi(g,\chi)$. We recall after [5] that $\pi(g,\chi)$ is a factor representation if and only if $\Gamma^{\#\#} = \Gamma$, a type I factor representation if and only if $\Gamma^{\#}/\Gamma$ is finite, and is an irreducible representation if and only if $\Gamma = \Gamma^{\#}$. In addition, $\pi(g,\chi)$ can be described in terms of holomorphic induction, as follows.

Let h be a positive, n - admisible polarization at g (for these definitions, see [1], [4]), let $d = h \cap g$, and let D_o be the analytic subgroup corresponding to d. Then $D = D_o\Gamma$ is closed and there exists a unique character χ_D on D which extends χ and has the differential $2\pi i.g|d$. Let us denote by $K(G,D)$ the space of all continous functions $\psi: G \to C$, with compact support modulo D, satisfying

$$\psi(ad) = \Delta_D(d)\Delta_G(d)^{-1}\psi(a) , \quad a \in G, \quad d \in D,$$

where Δ_G and Δ_D are the modular functions on G and D respectively. There exists a positive G - invariant linear functional $\psi \mapsto \oint_{G/D} \psi(a)\, d\dot{a}$, which is unique up to a multiplicative constant. Consider now the space of all C^∞ - functions $\varphi: G \to C$, with compact support modulo D, which verify:

(i) $\varphi(ad) = \Delta_D(d)^{\frac{1}{2}}\Delta_G(d)^{-\frac{1}{2}}\ \chi_D(d)^{-1}\varphi(a),\quad a \in G,\ d \in D;$

(II) $x * \varphi = (\tfrac{1}{2}\ \mathrm{Tr}(x) - 2\pi i < g, x >)\varphi\ ,\quad x \in h,$

where

$$((x+iy)*\varphi)(a) = \frac{d}{dt}\ \varphi(a\ \exp tx)\Big|_{t=0} + i\ \frac{d}{dt}\ \varphi(a\ \exp ty)\Big|_{t=0}$$

for $x+iy \in g_C$, and $\mathrm{Tr}(x)$ stands for the trace of the operator on $g_C/h + \bar{h}$ induced by ad x, with $x \in h$. Since for any function φ satisfying (i), the function on G $a \mapsto |\varphi(a)|^2$ belongs to $K(G,D)$, it makes sense to put

$$\|\varphi\|^2 = \oint_{G/D} |\varphi(a)|^2\ d\mathring{a}\ ,$$

and we let $H(g,\chi,h)$ denote the completion of this space of functions with respect to the above norm. One defines finally the holomorphically induced representation $\rho(g,\chi,h)$ as being the representation of G by left translations on the Hilbert space $H(g,\chi,h)$.

According to [5, Lemma 2.2], $\pi(g,\chi)$ and $\rho(g,\chi,h)$ are unitarily equivalent; their equivalence class will be denoted $\rho(g,\chi)$.

1.3. Fix now $f \in n^*$ and let g_f^* denote the linear variety of all $g \in g^*$ with $g|n = f$. Assume that for any $g \in g_f^*$ the orbit X_g is locally closed in g^*. If $\pi \in \mathrm{Fac}(G)$ and $\pi|N$ is carried by the orbit $G.\rho(f)$ in $\hat{N}$, then using [10, 4.1] one can see that the construction involving Mackey's machinery described in 1.1 can be reversed and provides an element $g \in g_f^*$ and a representation $\sigma \in F(g)$ such that $\pi(g,\chi)$ is unitarily equivalent to π. For our purposes, the following result, based on this remark, will be especially useful. Before stating it let us introduce one more definition: an orbit X_g in $\mathfrak{g}^*$ will be called <u>rational</u> if the cohomology class $[\nu_g] \in H^2(X_g, R)$ is rational.

1.3.1. LEMMA. <u>Assume that any orbit $X_g \subset \mathfrak{g}^*_f$ is locally closed and rational, and let $\pi \in Irr(G)$ be such that $\pi|N$ is carried by the orbit $G.\rho(f)$ in $\hat{N}$. Then there exists $g \in \mathfrak{g}^*_f$, $\Gamma \in A_{max}(g)$ and $\chi \in \hat{\Gamma}$ such that π is unitarily equivalent to $\pi(g, \chi)$.</u>

<u>Proof</u>. By what we have said above, it suffices to prove that if $g \in \mathfrak{g}^*_f$, then each $\sigma \in I(g)$ is equivalent to a representation of $G(g)$ induced by a character $\chi \in \hat{\Gamma}$ for some $\Gamma \in A_{max}(g)$. Consider an arbitrary $\Gamma \in A_{max}(g)$. As it is known [6, p.465], the rationality of the cohomology class $[\nu_g]$ amounts to the finitude of the index of $G(g)^{\#}$ in $G(g)$. Since $G(g)^{\#}$ is clearly contained in Γ, Γ must be of finite index in $G(g)$ too. It follows that the abelian normal subgroup Γ/Q_g is regularly embedded in $G(g)/Q_g$. Furthermore, since $\Gamma \in A_{max}(g)$, the stabilizer of any $\chi \in \hat{\Gamma}$ in $G(g)$ acting on $\hat{\Gamma}$ is Γ itself. Therefore, the Mackey machinery works and leads us to the desired conclusion.

§2

Besides the notations already fixed in §1, we need a few others which we are going to introduce now.

First we denote by $\mathcal{B}(g^*)$ the set of all pairs $p = (g, \chi)$ with $g \in g^*$ and $\chi \in \widehat{G(g)}^{\#}$ and by $\mathcal{B}(g^*_\ell)$ the subset of $\mathcal{B}(g^*)$ consisting of those pairs $p = (g, \chi)$ with $g \in g^*_\ell$. The group G acts in an abvious way on $\mathcal{B}(g^*)$, leaving $\mathcal{B}(g^*_\ell)$ invariant.

Next, we write $\tilde{G}$ for the Ad-algebraic hull of G, i.e. the smallest algebraic group of automorphisms of g containing Ad(G), and we note that $\tilde{G}$ acts naturally on g (resp. n) and hence by duality on g^* (resp. n^*) and leaves g^*_ℓ (resp. n^*_ℓ) invariant.

Finally, we denote by J_λ the two-sided ideal in $C^*(G)$ which is the intersection of all kernels in $C^*(G)$ of the representations of G whose restriction to Z is a multiple of the character λ, and we put $C^*(G|\lambda) = C^*(G)/J_\lambda$. There is a natural bijection between $Rep_\lambda(G)$ and the set of all non-degenerate representations of $C^*(G|\lambda)$. We shall say that G <u>is of type I (mod λ)</u> if all representations in $Rep_\lambda(G)$ are of type I, or equivalently $C^*(G|\lambda)$ is a type I C^* - algebra.

2.1. To prove our results in this section, we need the following lemmas, which are slightly modified versions of Proposition and Lemme on p.5 in [2].

2.1.1. LEMMA. <u>Let π be an irreducible representation of G whose restriction to N is carried by the orbit $\tilde{G}.\rho(f)$, where</u>

$f \in n^*$. The following assertions are equivalent:

(i) π is normal;

(ii) if $g \in g^*$ and $g|n \in \tilde{G}.f$, then X_g is locally closed and rational.

2.1.2. LEMMA. Assume that $f \in n^*$ satisfies condition (ii) above. Then any factor representation of G whose restriction to N is carried by $\tilde{G}.\rho(f)$ is of type I.

2.1.3. LEMMA. The restriction to N of a normal irreducible representation of G is carried by a transitive quasi-orbit.

These claimes can be proved exactly as in [2, loc.cit.] after noticing that all Pukanszky's results used there are also valid when $L = [G, G]$ is replaced by N.

2.2. We are now in a position to prove the main results of this section.

2.2.1. THEOREM. G is of type I (mod λ) if and only if, for any $g \in g_\ell^*$, X_g is locally closed and rational.

Proof. Assume G of type I (mod λ). Then $C^*(G|\lambda)$ is a type I C^* - algebra, hence any $\pi \in Irr_\lambda(G)$ is normal. Now let $g \in g_\ell^*$, $\Gamma \in A_{max}(g)$ and $\chi \in \hat{\Gamma}$. By [5, Theorem 2.1 (3)] and Remark 1.1.2, $\pi(g, \chi) \in Irr_\lambda(G)$, therefore it is normal. Using now Lemma 2.1.1., we get that X_g is locally closed and rational.

To prove the converse assertion, let $\pi \in Fac_\lambda(G)$. As it is known, its restriction to N is carried by some orbit $G.\rho(f)$ in $\hat{N}$, with $f \in n_\ell^*$. By our hypothesis and Lemma 2.1.2, it follows that π is of type I.

2.2.2. THEOREM. Assume G of type I (mod λ). (i) Let $p = (g, \chi) \in \mathcal{B}(\mathfrak{g}_\ell^*)$, $\Gamma \in A_{max}(g)$ and let $\chi' \in \hat{\Gamma}$ be an extension of χ (which surely exists by [5, Lemma 1.2 (ii)]). Then the equivalence class of irreducible representations $\rho(g, \chi')$ depends only upon p, and will be denoted accordingly ξ (p).
(ii) The assignment $p \mapsto \xi(p)$ induces a bijection of $\mathcal{B}(\mathfrak{g}_\ell^*)/G$ on to $\hat{G}^\wedge$.

Proof. The first claim can be checked by arguing as in [5, 3.2].

Now let $\pi \in Irr_\lambda(G)$; it is normal, hence by Lemma 2.1.3. its restriction to N is carried by an orbit $G.\rho(f)$. Thus, we are in a position to apply Lemma 1.3.1 which ensures us that there exist $g \in \mathfrak{g}_f^* \subset \mathfrak{g}_\ell^*$, $\Gamma \in A_{max}(g)$ and $\chi' \in \hat{\Gamma}$ such that π is equivalent to $\pi(g, \chi')$. Putting $\chi = \chi'|G(g)^\#$ and $p = (g, \chi)$, we see that π is in the class $\xi(p)$. This proves the surjectivity of $\xi : \mathcal{B}(\mathfrak{g}_\ell^*) \to \hat{G}^\lambda$.

Now let $p = (g, \chi) \in \mathcal{B}(\mathfrak{g}_\ell^*)$ and $a \in G$. Recall that $a.p = (Ad^*(a)g, \chi^a)$, where $\chi^a(c) = \chi(a^{-1}ca)$ for $c \in G(Ad^*(a)g)^\# = aG(g)^\# a^{-1}$. Let $\Gamma \in A_{max}(g)$ and $\chi' \in \hat{\Gamma}$ be an extension of χ. Then $\Gamma^a = a\Gamma a^{-1} \in A_{max}(Ad^*(a)g)$ and $\chi'^a \in \hat{\Gamma^a}$ extends χ^a. In view of [5, 3.1], $\pi(g, \chi')$ is equivalent to $\pi(ad(a)g, \chi'^a)$, which means that $\xi(ap) = \xi(p)$. Conversely, if $\xi(p_1) = \xi(p_2)$ with $p_i = (g_i, \chi_i) \in \mathcal{B}(\mathfrak{g}_\ell^*)$ $(i = 1,2)$, then after choosing $\Gamma_i \in A_{max}(g_i)$ and $\chi_i' \in \hat{\Gamma}_i$ which extend χ_i $(i = 1,2)$, whe obtain two equivalent representations $\pi(g_1, \chi_1')$ and $\pi(g_2, \chi_2)$. Again by [5, loc. cit.], there exists $a \in G$ such that $g_1 = Ad^*(a)\, g_2$ and $\chi_1'|\Gamma_1 \cap \Gamma_2^a = \chi_2'^a|\Gamma_1 \cap \Gamma_2^a$. But this clearly implies $p_1 = a.p_2$.

2.3. We close this section by rephrasing in our context the main results of Pukanszky in [7] and [8].

The equivalence relation on $\mathcal{B}(g^*)$ introduced by Pukanszky in [6, ch.II] will be denoted here by $\mathcal{P}$; as it is easily seen, $\mathcal{B}(g_\ell^*)$ is left invariant by $\mathcal{P}$. Given $\mathcal{O} \in \mathcal{B}(g^*)/\mathcal{P}$ one forms as in [6 , loc.cit.] the normal factor representation $\rho(\mathcal{O}) = \int_{\mathcal{O}}^{\oplus} \rho(p)\, d\mu_{\mathcal{O}}(p)$. The kernel of $\rho(\mathcal{O})$ in $C^*(G)$ will be denoted $J(\mathcal{O})$ and the quasi-equivalence class of $\pi(\mathcal{O})$ by $\zeta(\mathcal{O})$.

2.3.1. Remark. The map $J : \mathcal{B}(g^*)/\mathcal{P} \to \mathrm{Prim}\, C^*(G)$, which according to [7, Theorem 1] is a bijection, establishes a bijection between $\mathcal{B}(g_\ell^*)/\mathcal{P}$ and $\mathrm{Prim}\, C^*(G|\lambda)$ too, this latter space being view as the subspace of $\mathrm{Prim}\, C^*(G)$ consisting of all primitive ideals which contain J_λ.

Indeed, if $\mathcal{O} \subset \mathcal{B}(g_\ell^*)$ then $\pi(\mathcal{O})|Z = \int_{\mathcal{O}}^{\oplus} \pi(p)|Z\, d\mu_{\mathcal{O}}(p)$ is a multiple of λ since every $\pi(p)|Z$ is so; therefore $J(\mathcal{O}) \supset J_\lambda$. Conservely, assume that $J(\mathcal{O}) \supset J_\lambda$. Since $J(\mathcal{O}) = \mathrm{Ker}\, \pi(p)$ for any $p \in \mathcal{O}$ [7, p.93], $\pi(p)|Z$ will be a multiple of λ, hence $p \in \mathcal{B}(g_\ell^*)$. Thus $\mathcal{O} \subset \mathcal{B}(g_\ell^*)$.

2.3.2. Remark. In a similar way one checks that the map $\zeta : \mathcal{B}(g^*)/\mathcal{P} \to \hat{G}_{norm}$, which by [8, Theorem 3] is bijective, establishes a bijection between $\mathcal{B}(g_\ell^*)/\mathcal{P}$ and $\hat{G}^\lambda_{norm}$.

§3

From now on we shall denote by S a connected and simply connected solvable Lie group, and by $\mathfrak{s}$ its Lie algebra.

3.1. For the convenience of the reader we shall record here some known facts about 2-cocycles on a Lie group.

Let $\alpha \in Z^2(S,T)$ be a Borel 2-cocycle on S with values in the circle group T, and let

$$1 \longrightarrow T \xrightarrow{i^\alpha} S^\alpha \xrightarrow{p^\alpha} S \longrightarrow e$$

be the corresponding group extension; precisely, $S^\alpha = T \times S$ with the multiplication rule $(t_1,a_1)(t_2,a_2) = (t_1 t_2 \alpha(a_1,a_2), a_1.a_2)$, $i^\alpha(t) = (t,e)$, $p^\alpha(t,a) = a$. It is known that there exists a well-determined Lie group structure on S^α such that i^α and p^α are Lie homomorphisms. Moreover, when α is analytic, the analytic structure on S^α is exactly the product structure of T and S.

Now let $\tilde{S}^\alpha$ denote the universal covering group of S^α, $q^\alpha : \tilde{S}^\alpha \to S^\alpha$ be the canonical projection, and $\tilde{p}^\alpha : \tilde{S}^\alpha \to S$ defined as $\tilde{p}^\alpha = p^\alpha \circ q^\alpha$. A little computation, involving the homotopy exact sequence of the fibre bundle $\tilde{p}^\alpha : \tilde{S}^\alpha \to S$ shows that Ker $(\tilde{p}^\alpha)$ is a connected and simply connected Lie group. Moreover, $q \mid \mathrm{Ker}\,(\tilde{p}^\alpha) : \mathrm{Ker}\,(\tilde{p}^\alpha) \to T$ is a covering homomorphism, hence Ker $(\tilde{p}^\alpha)$ is isomorphic to R. We shall choose an identification between them such that the restriction of q^α to R=Ker $(\tilde{p}^\alpha)$ becomes $t \mapsto e^{2\pi i t}$. We get thus the commuting diagram:

$$\begin{array}{ccccccccc}
0 & \longrightarrow & R & \longrightarrow & \tilde{S}^\alpha & \xrightarrow{\tilde{p}^\alpha} & S & \longrightarrow & e \\
 & & \downarrow e^{2\pi i ?} & & \downarrow q^\alpha & & \downarrow = & & \\
1 & \longrightarrow & T & \longrightarrow & S^\alpha & \longrightarrow & S & \longrightarrow & e
\end{array}$$

According to a classical result of Malcev [3], there exists an analytic cross-section $\sigma : S \to \tilde{S}^\alpha$. Then $\tilde{\alpha} : S \times S \to R$, $\tilde{\alpha}(a,b) = \sigma(a)\sigma(b)\sigma(ab)^{-1}$ is an analytic 2-cocycle on S and the extensions

$$0 \longrightarrow R \longrightarrow S^{\tilde{\alpha}} \xrightarrow{\tilde{p}^\alpha} S \longrightarrow e$$

$$0 \longrightarrow R \longrightarrow S^{\tilde{\alpha}} \xrightarrow{p^{\tilde{\alpha}}} S \longrightarrow e$$

are isomorphic. It follows that $\beta \in Z^2(S,T)$ given by $\beta(a,b) = e^{2\pi i \tilde{\alpha}(a,b)}$ is an analytic 2-cocycle equivalent to α.

As usually, by an α - representation of S we mean a Borel map ρ of S into the unitary group of a separable Hilbert space which satisfies the law $\rho(a)\rho(b) = \alpha(a,b)\rho(ab)$ and $\rho(e) = \mathrm{Id}$. Our aim in this section being the study of α - representations of S and bearing in mind the fact that equivalent coycles give rise to equivalent representation theories, by what we said above, there will be no loss of generality in assuming from the beginning that $\alpha \in Z^2(S, T)$ is of the form $\alpha(a,b) = e^{2\pi i \tilde{\alpha}(a,b)}$ with $\tilde{\alpha} \in Z^2(S,R)$ analytic.

The cocycle $\alpha \in Z^2(S, T)$ with the above property being fixed from now on, we shall denote $S^{\tilde{\alpha}}$ by G, and $p^{\tilde{\alpha}}$ by p; further we denote by $\sigma : S \to G$ the canonical cross-section $\sigma(a) = (o,a)$. Since G, when viewed as an analytic manifold, is precisely $R \times S$, the Lie algebra $\mathfrak{g}$ of G can be (and will be) identified to $R \times \mathfrak{s}$. Then, the bracket operation on $\mathfrak{g} = R \times \mathfrak{s}$ is of the form

$$[(t_1,x_1), (t_2, x_2)] = (\omega_\alpha(x_2, x_1), [x_1, x_2]),$$

with $\omega_\alpha \in Z^2(\mathfrak{s}, R)$. A direct computation gives the following formula expressing ω_α in terms of α:

$$\omega_\alpha(x,y) = \frac{\partial^2}{\partial s \partial t}(\tilde{\alpha}(\exp sy, \exp tx) - \tilde{\alpha}(\exp tx, \exp sy))\Big|_{t=s=0}$$

3.2. We maintain the above notation. In particular, $\mathfrak{g} = R \times \mathfrak{s}$ as vector spaces, which enables us to identify $\mathfrak{g}^*$ to $R \times \mathfrak{s}^*$. Given $s \in \mathfrak{s}^*$ we shall denote by $g(s)$ the functional $(1,s) \in \mathfrak{g}^*$.

Since R is central in $\mathfrak{g}$, the coadjoint action of G on $\mathfrak{g}^*$ factorizes through an action of S on $\mathfrak{g}^*$ which, in addition, preserves the hyperplanes of the from $\{t\} \times \mathfrak{s}^*$ in $\mathfrak{g}^*$ and induces on each of them an affine action of S, depending upon $t \in R$. Actually, we are interested only in the action of S on $\{1\} \times \mathfrak{s}^*$ which we shall view as an affine action on $\mathfrak{s}^*$, via the affine bijection $s \longmapsto g(s)$. This action, which will be called the <u>α - coadjoint action</u> of S and will be denoted Ad^*_α, is given explicitely by the formula

$$\mathrm{Ad}^*(o,a)g(s) = g(\mathrm{Ad}^*_\alpha(a)s), \quad a \in S, \quad s \in \mathfrak{s}^*.$$

The isotropy subgroup of S at $s \in \mathfrak{s}^*$ with respect to this action is denoted $S_\alpha(s)$. It is not difficult to check that its Lie algebra $\mathfrak{s}_\alpha(s)$ coincides with the set of those $x \in \mathfrak{s}$ such that $\omega_\alpha(x,y) = \langle s,[x,y]\rangle$ for all $y \in \mathfrak{s}$. Clearly, $G(g(s)) = p^{-1}(S_\alpha(s)) = R \times S_\alpha(s)$, $G(g(s))_o = p^{-1}(S_\alpha(s)_o) = R \times S_\alpha(s)_o$ and $\mathfrak{g}(g(s)) = (dp)^{-1}(\mathfrak{s}_\alpha(s)) = R \times \mathfrak{s}_\alpha(s)$. The orbit through $s \in \mathfrak{s}^*$ under the α - coadjoint action of S will be denoted Y_s; it possesses a canonical symplectic structure which is given by the unique $\mathrm{Ad}^*_\alpha(S)$-invariant 2 - form ϑ^α_s, induced by the 2 - cocycle $\omega_\alpha + ds$ on $\mathfrak{s}^*$. When the cohomology class $[\vartheta^\alpha_s] \in H^2(Y_s, R)$ is rational, Y_s will be called a <u>rational orbit</u>.

Now given $s \in \mathfrak{s}^*$ we define $\eta_s : S_\alpha(s)_o \to T$ by $\eta_s(c) = \chi_{g(s)}(o,c)$, where $\chi_{g(s)}$ has the same meaning as in 1.1. Then η_s is an α - character of $S_\alpha(s)_o$, that is $\eta_s(a)\eta_s(b) = \alpha(a,b) \cdot \eta_s(ab)$, its differential at $e \in S_\alpha(s)_o$ is given by $(d\eta_s)_e(x) = 2\pi i \langle s, x \rangle$, $x \in \mathfrak{s}_\alpha(s)$, and the relationship between η_s and $\chi_{g(s)}$ is expressed by the formula: $\chi_{g(s)}(t, a) = e^{2\pi i t}\eta_s(a)$, $(t,a) \in G(g(s))_o$. For later use it is worth mentioning that, as in the case of ordinary characters, an α - character on a connected Lie group is uniquely determined by its differential at the unit element.

Let $S^\alpha(s)$ denote the set uf all subgroup Σ of $S_\alpha(s)$ containing $S_\alpha(s)_o$. For $\Sigma \in S^\alpha(s)$ we define $\Sigma^\#$ as being the set of all $a \in S_\alpha(s)$ such that $\alpha(a,b)\alpha(a^{-1}, b^{-1})\alpha(ab, a^{-1}b^{-1})\eta_s(aba^{-1}b^{-1}) = \alpha(a, a^{-1})\alpha(b, b^{-1})$ for all $b \in \Sigma$. Clearly, if $\Sigma \in S^\alpha(s)$, $p^{-1}(\Sigma) = R \times \Sigma$ is an element of $S(g(s))$ which we denote by $\Gamma(\Sigma)$ and $\Gamma(\Sigma)^\# = R \times \Sigma^\#$; it follows that $\Sigma^\# = p(\Gamma(\Sigma)^\#)$, in particular $\Sigma^\# \in S^\alpha(s)$. Define now $A^\alpha(s)$ (resp. $A^\alpha_{max}(s)$) to be the subset of $S^\alpha(s)$ consisting of all Σ such that $\Sigma \subset \Sigma^\#$ (resp. $\Sigma = \Sigma^\#$). Obviously, $\Sigma \in A^\alpha(s)$ (resp. $A^\alpha_{max}(s)$) if and only if $\Gamma(\Sigma) \in A(g(s))$ (resp. $A_{max}(g(s))$). Given $\Sigma \in S^\alpha(s)$ we let $\hat{\Sigma}$ be the set of all α - characters of Σ which extend η_s; then $\hat{\Sigma}$ is nonvoid if and only if $\Sigma \in A^\alpha(s)$. For $\eta \in \hat{\Sigma}$, we define $\chi(\eta) \in \widehat{\Gamma(\Sigma)}$ by $\chi(\eta)(t,a) = e^{2\pi it}\eta(a)$.

3.3. We intend now to adapt the procedure of holomorphic induction (see 1.2) in order to obtain α - representations of S starting with functionals $s \in \mathfrak{s}^*$ and α - characters on subgroups in $A^\alpha(s)$. To this end we need first an adequate notion of polarization.

We shall say that a Lie subalgebra k of $\mathfrak{s}_C$ is an α - polarization at $s \in \mathfrak{s}^*$ if: (1) k is a maximally isotropic subspace of $\mathfrak{s}_C$ for the bilinear alternating form $\omega_\alpha + ds$; (2) $k + \bar{k}$ is a Lie subalgebra of $\mathfrak{s}_C$; (3) k is Ad $S_\alpha(s)$-invariant. When $k \cap [\mathfrak{s}, \mathfrak{s}]_C$ is a maximally isotropic subspace of $[\mathfrak{s}, \mathfrak{s}]_C$, relative to the restriction of $\omega_\alpha + ds$ to $[\mathfrak{s}, \mathfrak{s}]_C$, k will be called <u>admisible</u>. When $-i(\omega_\alpha + ds)(x,\bar{x}) \geq 0$ for any $x \in k$, k will be called <u>positive</u>,

Since the pull-back to g of $\omega_\alpha + ds$ coincides with $dg(s)$, it follows easily that k is an (admisible, resp.positive) α - polarization of $\mathfrak{s}$ at s if and only if $h(k) = (dp)^{-1}(k)$ $(= R \times k)$ is an (n - admisible, resp. positive) polarization of g at g(s), where $n = R + [g, g]$. This remark and the known facts concerning the polarizations ensure the existence of admisible, positive α - polarizations at any $s \in s^*$.

We fix now an $s \in s^*$, a positive, admissible α - polarization k at s, a subgroup $\Sigma \in A^{\alpha}(s)$ and an α - character $\eta \in \hat{\Sigma}$. Let $e = k \cap s$, E_o be the analytic subgroup of S corresponding to e, and $E = E_o\Sigma$. Clearly $D_o = p^{-1}(E_o) (= R \times E_o)$ is the analytic subgroup corresponding to $d = h(k) \cap g$, and $D = p^{-1}(E) (=R \times E)$ coincides with $D_o\Gamma(\Sigma)$. It follows that E is closed and there exists a unique α - character η_E on E which extends η and has the differential $(d\eta_E)_e = 2\pi is|e$. Consider the space of all C^{∞} - functions $\varphi: S \to C$, with compact support modulo E, which verify:

(i) $\varphi(ae)=\Delta_E(e)^{\frac{1}{2}}\Delta_S(e)^{-\frac{1}{2}}\eta_E(e)^{-1}\alpha(a,e)\varphi(a)$, $a \in S$, $e \in E$;

(ii) $(x*\varphi)(a)=(x*\alpha(a,?)(e)-2\pi i<s,x>+1/2\ Tr(x))\varphi(a)$, $x \in k$;

(iii) $||\varphi||^2 = \oint_{S/E} |\varphi(a)|^2 d\dot{a} < \infty$

where the symbols $*$, Tr and $\oint_{S/E}$ have the same meaning as in 1.2, with G, D replaced by S, E. We let $H_{\alpha}(s, \eta, k)$ denote the completion of the space of functions considered above, and then we define $\rho_{\alpha}(s, \eta, k)$ as being the α - representation of S on $H_{\alpha}(s,\eta,k)$ given by the formula:

$$(\rho_{\alpha}(s, \eta, k)(a)\varphi)(b) = \alpha(a,a^{-1}b)\varphi(a^{-1}b), \quad a,b \in S$$

3.3.1. LEMMA. <u>Let s, Σ, η and k be as above and let $g = g(s)$, $\Gamma = \Gamma(\Sigma)$ $\chi = \chi(\eta) \in \hat{\Gamma}$ and $h = h(k)$. Then $\rho_{\alpha}(s, \eta, k)$ and $\rho(g, \chi, h) \circ \sigma$ are unitarily equivalent α - representations of S.</u>

<u>Proof</u>. For $\varphi \in H_{\alpha}(s, \eta, k)$ define $T\varphi \in H(g, \chi, h)$ by

$(T\varphi)(t,a) = e^{-2\pi it}\varphi(a)$. Then $T: \mathcal{H}_\alpha(s, \eta, k) \to \mathcal{H}(g, \chi, h)$ is an isomorphism of Hilbert spaces which intertwines $\rho_\alpha(s, \eta, k)$ and $\rho(g, \chi, h) \circ \sigma$.

Owing to this lemma we may extend to the case of cocycle representations Theorem 2.1 in [5].

3.3.2. THEOREM. Let $s \in \mathfrak{s}^*$, $\Sigma \in A^\alpha(s)$ and $\eta \in \hat{\Sigma}$.

(1) The equivalence class of the α - representation $\rho_\alpha(s, \eta, k)$ does not depend on the choice of a positive, admissible α - polarization k at s; accordingly, it will be denoted in the sequel $\rho_\alpha(s,\eta)$.

(2) $\rho_\alpha(s,\eta)$ is primary if and only if $\Sigma^{\#\#} = \Sigma$; when this is so, $\rho_\alpha(s,\eta)$ is of type I if and only if $\Sigma^{\#}/\Sigma$ is finite.

(3) $\rho_\alpha(s,\eta)$ is irreducible if and only if $\Sigma \in A^\alpha_{max}(s)$; when this is so, $\rho_\alpha(s,\eta)$ is normal if and only if Y_s is locally closed in $\mathfrak{s}^*$ and rational.

(4) Let $\Sigma' \in A^\alpha(s)$ be such that $\Sigma \subset \Sigma'$ and let $\eta' \in \hat{\Sigma'}$ be an extension of η. Then

$$\rho_\alpha(s,\eta) \simeq \int^{\oplus}_{(\Sigma'/\Sigma)^\frown} \rho_\alpha(s, \eta'.\check{\nu})d\nu,$$

where $\check{\nu}$ stands for the pull back to Σ' of the character ν of the abelian group Σ'/Σ, and $d\nu$ is the Haar measure on the character group $(\Sigma'/\Sigma)^\frown$.

3.4. We are going to transfer now the results in §2 to the context of cocycle representations. To this end, a few preliminary comments are in order. First we note that the rôle of the central subgroup Z of G is played by $\mathbb{R}$, and the character λ (resp. the functional ℓ) is explicitely given: $\lambda(t) = e^{2\pi it}$

(resp. $\langle \ell, t \rangle = t$). Next we observe that $C^*(G|\lambda)$ is just $C^*(S,\alpha)$ and the map $\pi \to \pi\circ\sigma$ establishes a bijection of $\mathrm{Rep}_\lambda(G)$ onto the set $\mathrm{Rep}(S,\alpha)$ of all α - representations of S; this bijection preserves the factoriality, the type, the normality, the irreducibility of representations and also the relations of equivalence and quasi-equivalence.

Now let $\mathcal{B}^\alpha(\mathfrak{s}^*)$ be the set of all pairs $q = (s,\eta)$ with $s \in \mathfrak{s}^*$ and $\eta \in S_\alpha(s)^\#$. The group S acts on $\mathcal{B}^\alpha(\mathfrak{s}^*)$ by $(a,(s,\eta)) \to (\mathrm{Ad}^*_\alpha(a)s, \eta^a)$, where $\eta^a(b) = \alpha(a,a^{-1})^{-1}\,\alpha(a^{-1},b)\alpha(a^{-1}b,a)\eta(a^{-1}ba)$, $b \in S_\alpha(\mathrm{Ad}^*_\alpha(s))^\#$. Clearly the assignment $q = (s,\eta) \to p(q)=(g(s),\chi(\eta))$ establishes a bijection of $\mathcal{B}^\alpha(\mathfrak{s}^*)$ onto $\mathcal{B}(\mathfrak{g}^*_\ell)$, which induces a bijection between the orbit spaces $\mathcal{B}^\alpha(\mathfrak{s}^*)/S$ and $\mathcal{B}(\mathfrak{g}^*_\ell)/G$. By transporting the equivalence relation $\mathcal{P}$ on $\mathcal{B}(\mathfrak{g}^*_\ell)$ via the above bijection, we get an equivalence relation $\mathcal{P}^\alpha$ on $\mathcal{B}^\alpha(\mathfrak{s}^*)$. Of course, $\mathcal{P}^\alpha$ can be introduced independently of $\mathcal{P}$, but we see no special reason for carrying out this point here.

Remark also that $(dp)^*: \mathfrak{s}^* \to \mathfrak{g}^*$, the transposed map of the projection $dp: \mathfrak{g} \to \mathfrak{s}$, induces for each $s \in \mathfrak{s}^*$ an isomorphism of symplectic spaces between $(Y_s, \mathcal{V}^\alpha_s)$ and $(X_{g(s)}, \mathcal{V}_{g(s)})$ (in particular Y_s is rational if and only if $X_{g(s)}$ is rational), and that Y_s is locally closed in $\mathfrak{s}^*$ if and only if $X_{g(s)}$ is locally closed in $\mathfrak{g}^*$.

Finally let us note that a class $\mathcal{O}^\alpha \in \mathcal{B}^\alpha(\mathfrak{s}^*)/\mathcal{P}^\alpha$ may be identified, via the bijection between $\mathcal{B}^\alpha(\mathfrak{s}^*)$ and $\mathcal{B}(\mathfrak{g}^*_\ell)$, to a class $\mathcal{O} \in \mathcal{B}(\mathfrak{g}^*_\ell)$. Thus, by imitating the construction in [6, ch.II], one may form the direct integral $\rho_\alpha(\mathcal{O}^\alpha) = \int^\oplus_{\mathcal{O}^\alpha} \rho_\alpha(q)d\mu_{\mathcal{O}^\alpha}(q)$, where $\mu_{\mathcal{O}^\alpha}$ is the pull back of the measure $\mu_{\mathcal{O}}$; the quasi-equivalence class of $\rho_\alpha(\mathcal{O}^\alpha)$ will be denoted by $\zeta_\alpha(\mathcal{O}^\alpha)$ and its kernel in

$C^*(S,\alpha)$ by $J_\alpha(0^\alpha)$. As a matter of fact $\rho_\alpha(0^\alpha) = \rho(0)\circ\sigma$, hence $\zeta_\alpha(0^\alpha)$ (resp. $J_\alpha(0^\alpha)$) corresponds to $\zeta(0)$ (resp. $J(0)$) through the natural bijection between $(S,\alpha)^\frown_{norm}$ (resp. Prim $C^*(S,\alpha)$) and $G^\frown_{norm}$ (resp. Prim $C^*(G|\lambda)$).

The above remarks and the results in §2 enable us to state the main results of this paper.

3.4.1. THEOREM. <u>(S,α) is of type I if and only if, for any $s \in \mathfrak{s}^*$, Y_s is locally closed and rational.</u>

3.4.2. THEOREM. <u>Assume (S,α) of type I.</u>

(i) <u>Let $q = (s,\eta) \in \mathcal{B}^\alpha(\mathfrak{s}^*)$, $\Sigma \in A^\alpha_{max}(s)$ and $\eta' \in \hat{\Sigma}$ be an α - character extanding η. Then the equivalence class of irreducible α - representations $\rho_\alpha(s,\eta')$ depends only upon q, and will be denoted accordingly $\xi_\alpha(q)$.</u>

(ii) <u>The assignment $q \mapsto \xi_\alpha(q)$ induces a bijection of $\mathcal{B}^\alpha(\mathfrak{s}^*)/S$ onto $(S,\alpha)^\wedge$.</u>

3.4.3. THEOREM. <u>The maps $J_\alpha : \mathcal{B}^\alpha(\mathfrak{s}^*)/P^\alpha \to$ Prim $C^*(S,\alpha)$ and $\zeta_\alpha : B(\mathfrak{s}^*)/P^\alpha \to (S,\alpha)^\frown_{norm}$ are bijective.</u>

3.5. In this closing section we shall exhibit the example promised in the introduction.

Let S be the connected and simply connected Lie group with Lie algebra $\mathfrak{s} = \sum_{i=1}^{6} \mathbb{R}e_i$, the only non-vanishing brackets between the e_i's being: $[e_1, e_5] = 2\pi e_2$, $[e_2, e_5] = -2\pi e_1$, $[e_3, e_6] = 2\pi e_4$, $[e_4, e_6] = -2\pi e_3$. Consider now $\omega \in Z^2(\mathfrak{s}, \mathbb{R})$, $\omega(e_i, e_j) = 1/2\, \delta_{i5}\delta_{j6}$, $1 \le i < j \le 6$, and choose $\alpha \in Z^2(S, \mathbb{T})$ of the form $\alpha(a,b) = e^{2\pi i \tilde{\alpha}(a,b)}$ with $\tilde{\alpha} \in Z^2(S, \mathbb{R})$ analytic, such that $\omega = \omega_\alpha$. Denote by $\{f_1,\dots,f_6\}$ the basis of $\mathfrak{s}^*$ dual to $\{e_1,\dots,e_6\}$.

By a direct computation one checks that, for $s = t_1f_1+\dots+t_6f_6 \in \mathfrak{s}^*$ $S_\alpha(s) = S_\alpha(s)^{\#}(=S_\alpha(s)_0)$ if $t_1 = t_2 (=0)$ or $t_3 = t_4 = 0$ and is isomorphic to Z_2^2 otherwise. On the other hand, since the central extension $\mathfrak{g}$ of $\mathfrak{s}$ corresponding to ω is isomorphic to Dixmier's Lie algebra, all the orbits if the α - coadjoint action are locally closed. It follows that (S,α) is type I, although only "few" orbits are integral.

REFERENCES

1. L.Auslander and B.Kostant, *Polarization and unitary representations of solvable Lie groups*, Inventiones math., 14 (1971), 255-354.

2. J.-I. Charbonnel, *La formule de Plancherel pour un groupe de Lie résoluble connexe*, Thèse 3e cycle, Université Paris VIII, 1975.

3. A.Malcev, *On the simple connectedness of invariant subgroups of Lie groups*, Dokl.Acad. Nauk SSSR, 34 (1942), 10-13.

4. C.C. Moore, *Representations of solvable and nilpotent groups and harmonic analysis on nil and solvmanifolds*, in Proceedings of symposia in pure mathematics, vol.26, 3 - 44.

5. H. Moscovici and A. Verona, *Holomorphically induced representations of solvable Lie groups*, Preprint No.23 (1976), INCREST, Bucharest.

6. L.Pukanszky, *Unitary representations of solvable Lie groups*, Ann.scient.École Norm.Sup.,4 (1971), 457 - 608.

7. L. Pukanszky, *The primitive ideal space of solvable Lie groups*, Inventiones math., 22 (1973), 75 - 118.

8. L. Pukanszky, *Characters of connected Lie groups*, Acta mathematica, 133 (1974), 81 - 137.

9. T. Sund, *Multiplier representations of exponential Lie groups*, Preprint.

10. M. Vergne, *Representations unitaires des groupes die Lie résolubles*, Lectures Notes in Math. 431, Springer-Verlag, 205-227.

INSTITUTUL
DE
MATEMATICĂ

INSTITUTUL NATIONAL
PENTRU CREAȚIE
ȘTIINȚIFICĂ ȘI TEHNICĂ

PREPRINT SERIES IN MATHEMATICS
1976 ISSUES

1. S. Strătilă, D. Voiculescu, L. Zsido : *ON CROSSED PRODUCTS.*
2. I. Suciu and I. Valușescu : *FACTORIZATION OF SEMI - SPECTRAL MEASURES.*
3. Zoia Ceaușescu : *ON INTERTWINING DILATIONS.*
4. D. Burghelea and R. Lashof : *STABILITY OF CONCORDANCES AND THE SUSPENSION HOMEORPHISM.*
5. Zoia Ceaușescu and F. H. Vasilescu : *TENSOR PRODUCTS AND TAYLOR'S JOINT SPECTRUM.*
6. Gr. Arsene and S. Strătilă : *A BIBLIOGRAPHY ON OPERATOR ALGEBRAS AND RELATED TOPICS I.*
7. Otto Liess : *THE FUNDAMENTAL PRINCIPLE OF EHRENPREIS - PALAMODOV.*
8. Henri Moscovici and Andrei Verona : *QUANTIZATION AND PROJECTIVE REPRESENTATIONS OF SOLVABLE LIE GROUPS.*
9. A. Halanay and T. Morozan : *STABILIZATION BY LINEAR FEEDBACK OF LINEAR DISCRETE STOCHASTIC SYSTEMS.*
10. Dan Burghelea : *ON THE DECOMPOSITION OF THE AUTOMORPHISMS GROUP OF* $M \times S^1$.
11. N. Popa : *PERMANENCE PROPERTIES OF TENSOR PRODUCTS OF BANACH LATTICES.*
12. C. Vârsan : *LOCAL CONTROLABILITY ALONG A SINGULAR TRAJECTORY.*
13. C. Apostol, C. Foiaș and C. Pearcy : *QUASIAFFINE TRANSFORMS OF COMPACT PERTURBATION OF NORMAL OPERATORS.*
14. Zoia Ceaușescu and Ciprian Foiaș : *ON INTERTWINING DILATIONS III.*
15. R.G.Douglas and C. Foiaș : *A HOMOLOGICAL VIEW IN DILATION THEORY.*
16. Dan Burghelea : *AUTOMORPHISMS OF MANIFOLDS.*
17. I. Suciu and I. Valușescu : *ESSENTIAL PARAMETERS IN PREDICTION.*
18. Gr. Arsene and Zoia Ceaușescu : *ON INTERTWINING DILATIONS IV.*
19. Dan Voiculescu : *ON A THEOREM OF M. D. CHOI AND E. G. EFFROS.*
20. A. Halanay and S. Mirică : *THE TIME - OPTIMAL FEEDBACK CONTROL FOR SINGULARITY PERTURBED LINEAR SYSTEMS.*
21. A. Halanay and S. Mirică : *TIME - OPTIMAL FEEDBACK USING STATE ESTIMATOR.*
22. A. Bacopoulos, G. Godini and I. Singer : *ON BEST APPROXIMATION IN VECTOR - VALUED NORMS.*
23. Henri Moscovici and Andrei Verona : *HOLOMORPHICALLY INDUCED REPRESENTATIONS OF SOLVABLE LIE GROUPS.*
24. Ion Colojoară : *SUR LE THEOREME DE STONE.*
25. Constantin Bănică : *LE LIEU REDUIT ET LE LIEU NORMAL D'UN MORPHISME.*