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A VISCOPLASTIC CONSTITUTIVE EQUATION FOR ROCKS

by

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§1 Introduction

Rocks have long been considered in mining practice as being linear elastic though it was well known that mechanical properties of rocks are much more complex (see OBERT and DUVALL [1967], BAKLASHOV and KARTOZYA [1975], ERJANOV et al. [1970], KARTASHOV [1973], GOLDSMITH and SACKMAN [1973], LAMA and VUTUKURI [1978]. The rheological properties of rocks are significant not only within geological time intervals but also within much shorter time intervals (days, months) of interest in mining industry.

The aim of the present paper is an attempt to establish a much more precise constitutive equation for rocks to be used in time intervals ranging from a few minutes (sometimes even shorter intervals) to several years. Rheological models for rocks, mainly linear viscoelastic models, were already proposed by many authors (see the literature already mentioned). It was thought however, that rocks are more complex and that their mechanical properties would rather be described by elastic viscoplastic nonlinear models for both shearing properties as well as for the volume compressibility. A tentative model is proposed based on several diagnostic tests. In the future additional tests of the same kind or other are still necessary to make precise some of the aspects mainly quantitative but also qualitative concerning the model. Meantime the model must be considered to be a first approximation pending further experimental data. Creep properties and deformation processes during loading were mainly considered as being the ones

involved in mining applications we had in mind. Other mechanical properties were marginally discussed.

§2 Standard experiments in compression

From the point of view followed here, the reason of doing such experiments was twofold. On one hand these are the first and the simplest tests which can be done in order to find the dominant mechanical properties of various rocks in quasistatic compression. Standard testing machines were used while the cylindrical specimens were 10 cm. long and 5 cm. in diameter, with unconfined lateral surface. In order to look for possible time effects, even these experiments have been done with various loading rates, controlled with the testing machine.

On the other hand, the rock response as revealed by such experiments will be used in order to estimate the deformation of the specimen during the first period of deformation in creep test, i.e. during the period when the testing machine used in creep experiments is loaded. Further compressive stresses and strains are defined as positive; in the experiments discussed here only positive stresses and strains are involved.

In fig.2.1 three stress-strain curves obtained in quasistatic experiments for dry schist are shown (porosity from 3 to 8%). The upper curve (full dots) corresponds to the loading rate 300 kgf $cm^{-2}min^{-1}$ (2942 N $cm^{-2}min^{-1}$), the middle one (triangles) to the loading rate 40 kgf $cm^{-2}min^{-1}$ (392 N cm^{-2} min^{-1}) and finally the lower one (circles) to a very small loading rate of 1.33 kgf $cm^{-2}min^{-1}$ (13 N $cm^{-2}min^{-1}$). It results from these curves that the stress-strain relations are nonlinear and are dependent on the loading rate. This loading rate effect is similar with the one reported by many authors

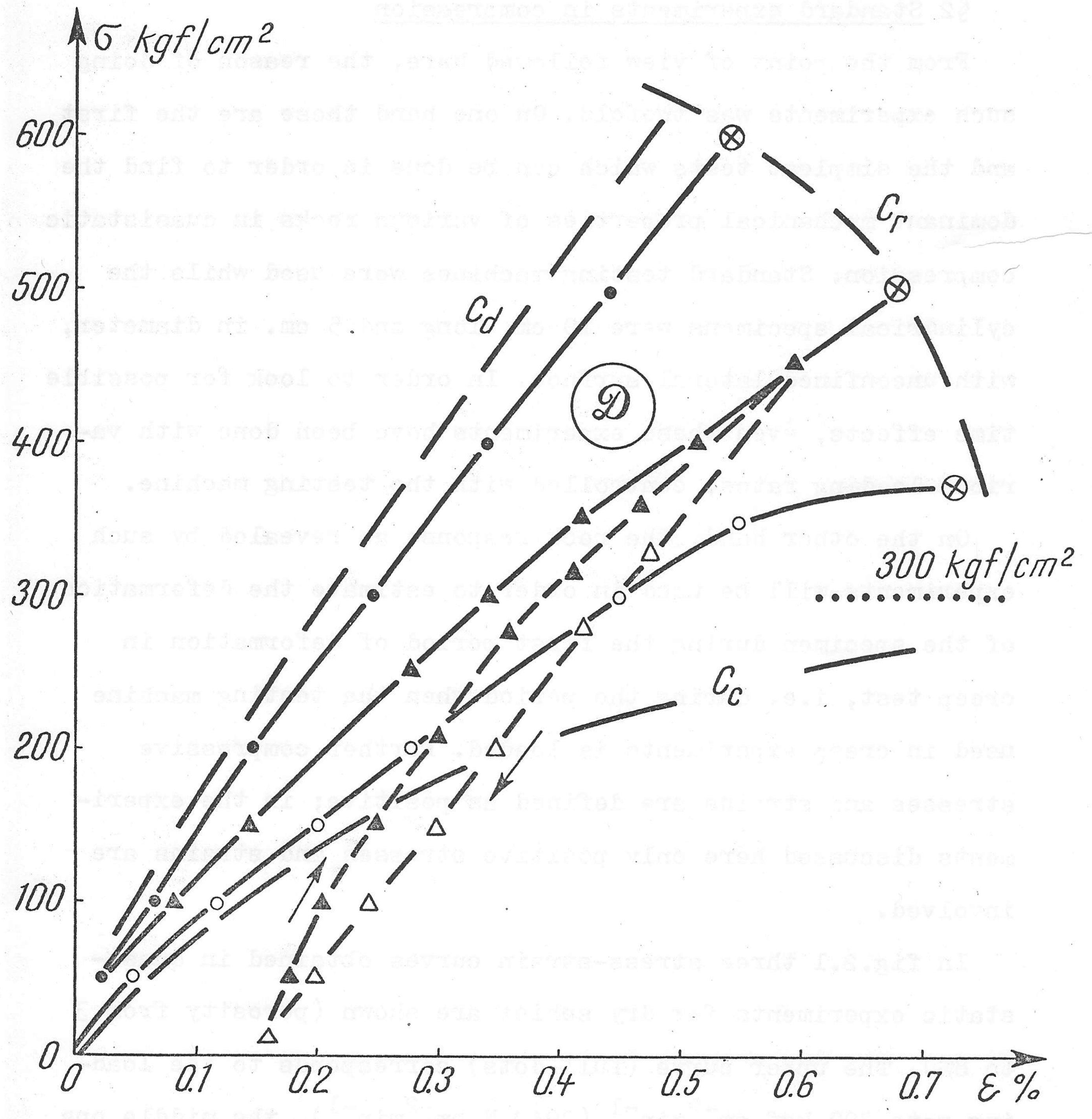

Fig.2.1 Stress-strain curves for dry schist and domains involved in the mathematical model

for various rocks (see for instance LAMA and VUTUKURI [1978], PERKINS et al. [1970], KUNTYSH and TEDER [1970]). Successive cycles of unloading followed by reloading have also been done, showing significant hysteresis loops and variation of the tangent modulus during both loading and unloading. In order to keep the figure simple only one such loop has been shown. The unloading produced starting from various stages of deformation have pointed out significant permanent strains. Generally the permanent component of the strain is, as order of magnitude, about one quarter to one third from the total deformation, and therefore it is quite significant.

The ultimate point on various curves shown on fig.2.1 corresponds to the fracture of the specimen. Generally with an increased rate of loading the stress at failure is higher, but the corresponding strain is smaller. The stress at failure determined with moderate rate of loadings (of the order of 40 kgf $cm^{-2}min^{-1}$ (392 N $cm^{-2}min^{-1}$)) is involved in the mathematical model and will be denoted by σ_r. This conventionally established magnitude for σ_r will be considered to be a typical constant for the particular kind of rock under consideration.

Another remark is that the stress-strain curve obtained with unconfined lateral surface of the specimen has the concavity directed towards the positive strain axis. Other rocks however, (sandstone) even in such kind of experiments possess stress-strain curves with an opposite curvature (directed towards the positive stress axis) while some other rocks possess stress-strain curves which are quite linear, though highly sensitive to the loading rate (limestone).

§3 Cree tests

The creep tests have been performed using specimens of the sizes as mentioned before, on which successively increasing loads (in steps) were applied. The specimen was first loaded with a certain constant stress and the strain was recorded for several days or weeks. Generally at the end of a finite, well determined time period, the strain remains constant. When it became evident that the strain will increase no more, an additional load was applied to the specimen and so on. Typical strain-time curves obtained in such kind of creep tests are shown on fig.3.1 and fig.3.2, again for schist. The last points shown on these curves correspond to the failure of the specimen. Since for the same type of rock there is a broad range of strength characteristics expressed by various values of the stress at failure, in order to describe the creep test it is more useful to report the variation of the strain to the ratio $\sigma_{effective}/\sigma_r$ rather than to the effective stress $\sigma_{effective}$. This ratio will be denoted by $\Delta = \sigma_{eff}/\sigma_r$ and will be called loading ratio. Here σ_r is obtained in standard compressive tests with medium loading rate, as described above and is a typical constant for the rock under consideration. The curve from fig.3.1 is showing that at each increase of the loading ratio we get an instantaneous increase of the strain, followed by a slow increase of the strain due to creep.

A very important aspect in the deformation by creep is the following. If the loading ratio applied to an unstressed and undeformed specimen is not surpassing a certain limit, then after several days or weeks the strain will become constant and will stay constant no matter how long is the time used to perform the experiment. Let us denote by Δ_s the highest value

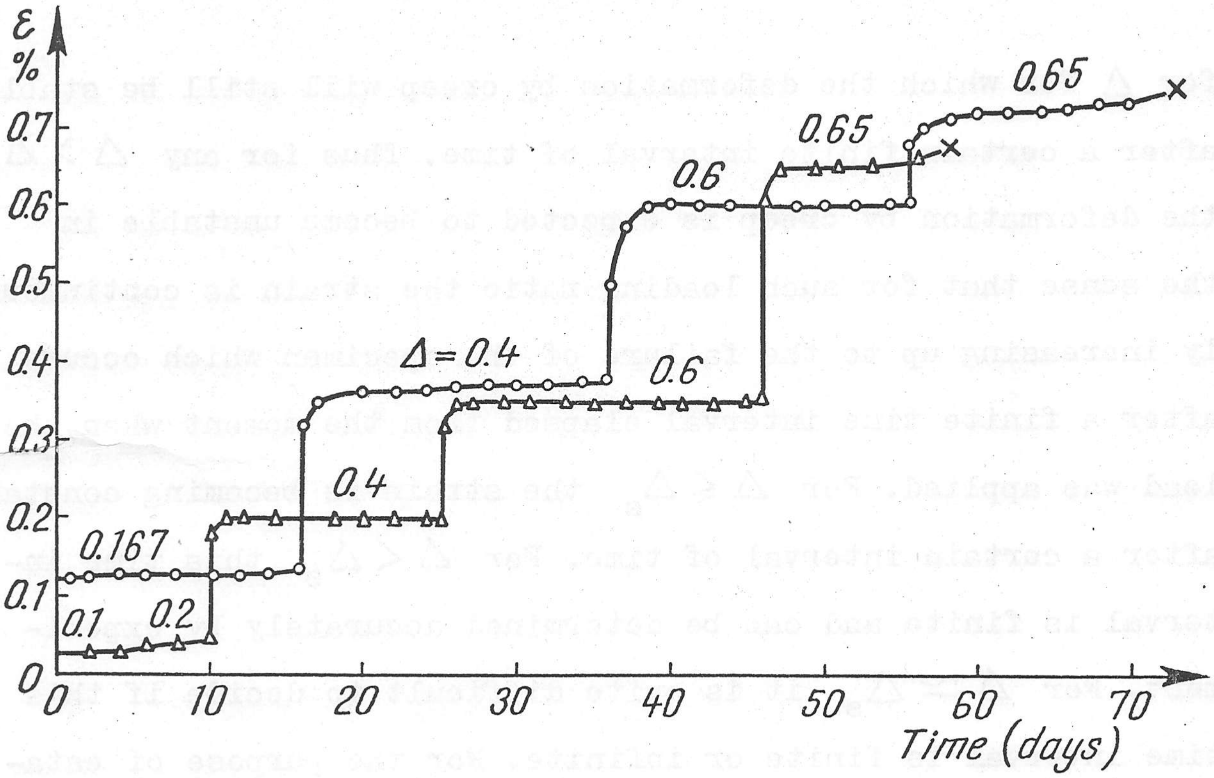

Fig.3.1 Strain-time curves for schist during creep tests

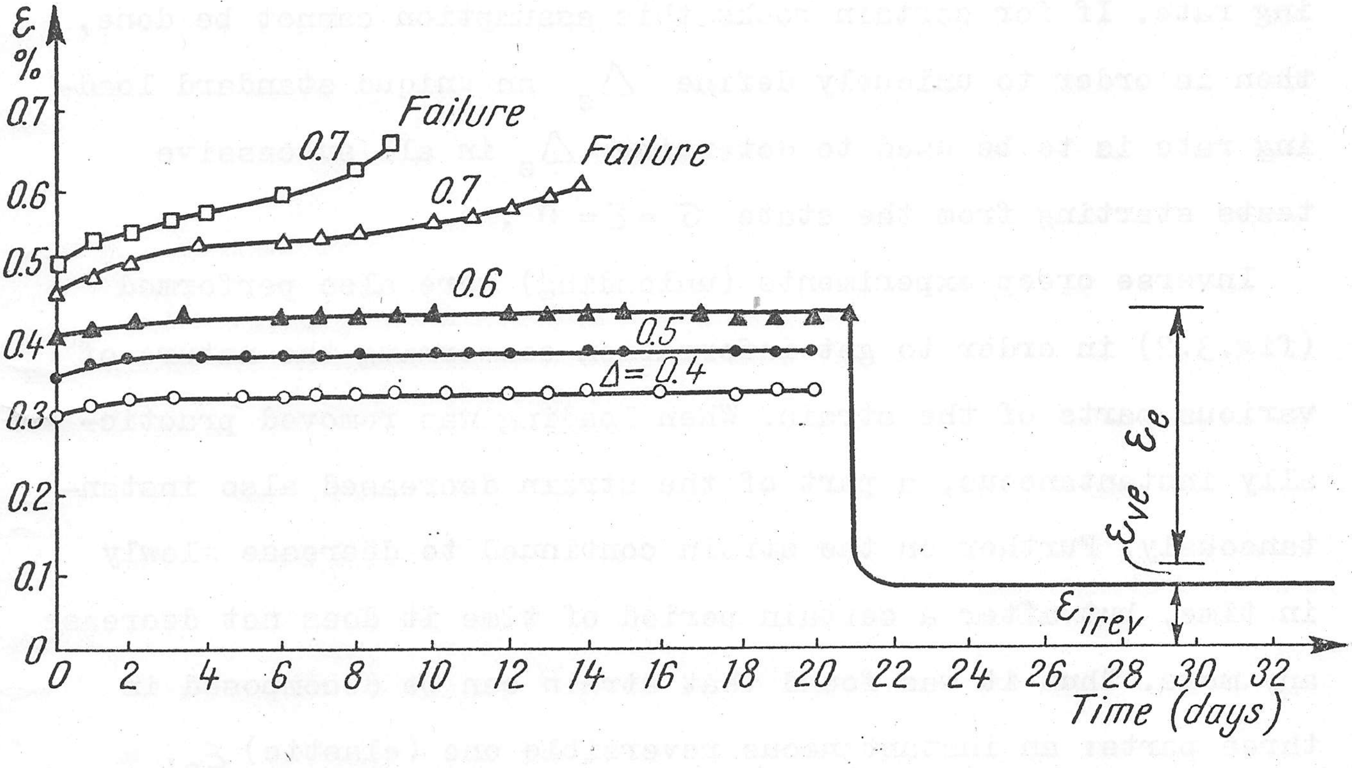

Fig.3.2 Strain-time curves for schist during creep tests and one inverse creep test

for Δ for which the deformation by creep will still be stable after a certain finite interval of time. Thus for any $\Delta > \Delta_s$ the deformation by creep is expected to become unstable in the sense that for such loading ratio the strain is continuously increasing up to the failure of the specimen which occurs after a finite time interval elapsed from the moment when the load was applied. For $\Delta \leqslant \Delta_s$ the strain is becoming constant after a certain interval of time. For $\Delta < \Delta_s$ this time interval is finite and can be determined accurately by experiment. For $\Delta \simeq \Delta_s$ it is quite difficult to decide if this time interval is finite or infinite. For the purpose of establishing a model further Δ_s will be chosen so that for any $\Delta \leqslant \Delta_s$ the time of stabilization by creep be finite. Thus Δ_s is depending on σ_{eff} and σ_r but it will be assumed that Δ_s does not depend on initial strain nor on the loading rate. If for certain rocks this assumption cannot be done, then in order to uniquely define Δ_s an unique standard loading rate is to be used to determine Δ_s in all successive tests starting from the state $\sigma = \varepsilon = 0$.

Inverse creep experiments (unloading) were also performed (fig.3.2) in order to get information concerning the nature of various parts of the strain. When loading was removed practically instantaneous, a part of the strain decreased also instantaneously. Further on the strain continued to decrease slowly in time, but after a certain period of time it does not decrease any more. Thus it was found that strain can be decomposed in three parts: an instantaneous reversible one (elastic) ε_e, a non-instantaneous reversible one (viscoelastic) ε_{ve} and finally a permanent one ε_p. It is interesting to mention that generally the deformation of the specimen obtained during the first stage of deformation, i.e. during the time when the load is applied to the machine, contains a significant permanent component. This is obvious if comparison is done between this initial instantaneous component of the strain and ε_e obtained during instantaneous unloading. Recalling the results mentioned in §2 we come to the conclusion that this initial permanent component is of viscoplastic nature.

A mathematical model to describe the main experimentally found characteristics presented above, can be obtained as follows. We first consider in the $\sigma - \varepsilon$ plane (fig.2.1) a domain $\mathcal{D}$ bounded by three curves, all three obtained by experiments. Curve C_c is the boundary of stabilization by creep; points on this boundary are the points of maximum strain which can be reached with various stresses (relatively small) during creep tests. The experimental data used to determine C_c suggest that this boundary has an horizontal asymptote $\sigma = a = \text{const.}$, and therefore the stresses involved in the experiments determining C_c do not surpass the stress corresponding to this asymptote. If such an asymptote does not exist then $\sigma = a$ is the ultimate stress still producing a stabilization of deformation by creep. Curve C_d is the curve of instantaneous response of the rock, i.e. this would be the response of the rock if the fastest (from the point of view of the kind of experiments under consideration) possible loading is applied to the specimen. For instance if creep behaviour is to be described, then a fast loading obtained with a standard testing machine would be such a fast loading, generally much faster than the one which could be obtained when a creep machine is loaded.

Finally the last curve C_r is the boundary where the fracture of the specimen is produced. These three curves are bounding the domain $\mathscr{D}$ of all possible stress-strain states which can be reached in loading processes by creep tests. We observe here that for stresses in the neighbourhood of the $\sigma = a$ both curves C_r and C_c are difficult to be found by experiments since very long time intervals are necessary. Therefore it is not certain that the two curves C_r and C_c have a certain common horizontal asymptote but the experimental data we used would rather suggest this idea. The experimental data for concrete obtained by RUSCH [1960] would also rather suggest this idea.

We consider now the general form of the constitutive equation (initially proposed for metals by CRISTESCU [1963])

$$\dot{\varepsilon} = \left(\frac{1}{E} + F(\varepsilon, \sigma, \operatorname{sgn}\dot{\sigma})\right)\dot{\sigma} + \Psi(\varepsilon, \sigma) \tag{3.1}$$

with

$$F(\varepsilon, \sigma, \operatorname{sgn}\dot{\sigma}) = \begin{cases} \Phi(\varepsilon, \sigma) & \text{if} \quad \dot{\sigma} \geq 0 \\ 0 & \text{if} \quad \dot{\sigma} < 0 \end{cases} \tag{3.2}$$

where $\Phi(\varepsilon, \sigma)$ describes the "fast" or "instantaneous" properties of the rock and is a non-negative function of class C^1 on $0 \leq \varepsilon$, $0 \leq \sigma \leq E\varepsilon$. $\Psi(\varepsilon, \sigma)$ describes the "slow" properties of the rock (creep, relaxation). E is an elastic modulus determined in "fast" quasistatic loading tests. We recall that for rocks E determined in quasistatic loading tests is dependent on the loading rate. Here E = const. is defined by conventionally choosing a certain "fast" loading rate for the machine. In the model however, the cases when E depends on the stress states can also be considered, if necessary. The dots above the letters in formulas above mean time derivative. It will be assumed that strains and rotations are small. The "instantaneous" response of the rock starting from the state $\varepsilon = \sigma = 0$ will be approximated by a curve

$$\varepsilon = \alpha \left(\frac{\sigma}{E_0}\right)^{\beta} \qquad (3.3)$$

with $\alpha > 0$ and $\beta > 0$ dimensionless characteristic constants of that particular rock and $E_0 =$ const. a particular value of E, for instance the value of E obtained in dynamic tests (by low amplitude wave propagation). Then function $\Phi(\varepsilon,\sigma)$ will be defined as

$$\Phi(\varepsilon,\sigma) = \alpha\beta\left(\frac{\sigma}{E_0}\right)^{\beta-1} - \frac{1}{E}\,, \qquad \forall(\varepsilon,\sigma)\in\mathcal{D}\,. \qquad (3.4)$$

The coefficient function $\Psi(\varepsilon,\sigma)$ is found by experiments so that $\Psi(\varepsilon,\sigma) = 0$ would be the equation of the curve of stabilization by creep, i.e. the equation of curve C_c. In other words it will be assumed that exists a curve $(\varepsilon, \sigma = \sigma_c(\varepsilon)) \in \mathcal{D}$ with the properties

$$\begin{aligned} \Psi(\varepsilon,\sigma) &> 0 \quad \text{if} \quad \sigma_c(\varepsilon) < \sigma \\ \Psi(\varepsilon,\sigma) &= 0 \quad \text{if} \quad \sigma_c(\varepsilon) \geq \sigma\,. \end{aligned} \qquad (3.5)$$

Since the loadings surpassing a certain limit $\sigma = a$ produce an unstable deformation by creep (i.e. finally producing fracture), the equation of the boundary C_c was chosen in the form

$$\sigma = a\left[1 - \exp(-b\,\varepsilon)\right] \qquad (3.6)$$

where $b > 0$ is a dimensionless constant, while $\sigma = a$ is the highest value of the stress still producing stabilization of the deformation by creep. This criterion to choose the value of a is ensuring, but a slight higher value for a can also be used if necessary in specific applications.

Since even for a certain kind of rock one can find distinct categories of mechanical properties which can be roughly characterized by the conventionally defined fracture resistance σ_r it is more convenient to introduce in (3.6) the maximal loading ratio Δ_s

$$a = \Delta_s \sigma_r \tag{3.7}$$

which would still produce a stable creep, while σ_r is obtained with standard testing machines. Now (3.7) can be written as

$$\frac{\sigma}{\sigma_r} = \Delta_s \left[1 - \exp(-b\,\varepsilon) \right] \ . \tag{3.8}$$

Generally the coefficient b is also somehow dependent on σ_r but it is of lesser importance for the discussion which follows.

Sometimes it is useful to determine the boundary of stabilization by creep using tests intermediate between creep tests and standard tests. These are the tests in which continuously increasing loading is applied but with a very small loading rate. Let us assume that

$$\sigma = \sigma_r \, f(\varepsilon) \tag{3.9}$$

is the stress-strain curve obtained with such a procedure. It was found that this curve is close to the creep stabilization curve and sometimes even slightly lower than the later one. Then (3.9) can well be used for the creep stabilization boundary. For many rocks (3.9) can be well approximated by a straight line

$$\frac{\sigma}{\sigma_r} = h\,\varepsilon \ . \tag{3.10}$$

Thus the coefficient function $\Psi(\varepsilon,\sigma)$ entering the constitutive equation was chosen in the form

$$\Psi(\varepsilon,\sigma) = \begin{cases} \frac{k(\sigma)}{E}\left\{\sigma - \sigma_r\Delta_s\left[1 - \exp(-b\,\varepsilon)\right]\right\}^m & \text{if} \\ \qquad \sigma > \sigma_r\Delta_s\left[1 - \exp(-b\,\varepsilon)\right] \\ 0 \quad \text{if} \quad 0 \leq \sigma \leq \sigma_r\Delta_s\left[1 - \exp(-b\,\varepsilon)\right], \end{cases} \tag{3.11}$$

with $m > 0$ a constant (further only the value $m=1$ will be used) or in the form

$$\Psi(\varepsilon,\sigma) = \begin{cases} \frac{k(\sigma)}{E}\left\{\exp\frac{\sigma - \sigma_r\Delta_s\left[1 - \exp(-b\,\varepsilon)\right]}{n} - 1\right\} & \text{if} \\ \qquad \sigma > \sigma_r\Delta_s\left[1 - \exp(-b\,\varepsilon)\right] \\ 0 \quad \text{if} \quad 0 \leq \sigma \leq \sigma_r\Delta_s\left[1 - \exp(-b\,\varepsilon)\right], \end{cases} \tag{3.12}$$

with $n = \text{const.}$, or various other forms (see CRISTESCU and SULICIU [1976]). Finally if the stabilization boundary is taken in the form (3.7) we can write

$$\Psi(\varepsilon,\sigma) = \begin{cases} \frac{k(\sigma)}{E}\left[\sigma - \sigma_r h\,\varepsilon\right] & \text{if} \quad \sigma > \sigma_r h\,\varepsilon \\ 0 & \text{if} \quad 0 \leq \sigma \leq \sigma_r h\,\varepsilon \end{cases} \tag{3.13}$$

or again another variant can be used. In these formulas since $E/k(\sigma)$ can be considered to be a "viscosity coefficient" of the rock in creep tests, $k(\sigma)$ is in fact governing the variable value of this coefficient. To keep the model simple here it will be assumed that k depends on stress alone.

Therefore the mathematical model to describe the slow deformation of rocks is fully determined in the form (3.1). The viscosity coefficient can be determined by measuring the strain at various times during creep tests and using the constitutive equation. It will be given in Poise.

The numerical coefficients were determined for several rocks. For instance for schist several classes of strength characterized by the stress at fracture σ_r were determined. For one of these classes stresses up to $\sigma = 300$ kgf cm^{-2}(2942 N cm^{-2}) are still producing a stable creep while stresses above this value do not. Since for this kind of rocks the mean value for σ_r is $\sigma_r = 500$ kgf cm^{-2}(4903 N cm^{-2}) it yields $\Delta_s = 0.6$ and further a = 300 kgf cm^{-2}(2942 N cm^{-2}) and b = 300. Using formula (3.1) with (3.11) and m = 1 we can find a mean value for k from creep tests. Generally for such rock k is of the order of magnitude 0.2 - 0.07 d^{-1} (d stands for "day") and it is smaller for higher applied stress. This will imply for the "viscosity coefficient" Ek^{-1} in such kind of creep tests with final constant strain, an order of magnitude of 10^{12} Poise. Even for stationary creep we have found a viscosity coefficient of the same order of magnitude. This is a value smaller than 10^{17}- 10^{18} Poise reported by MAXIMOV et al. for stationary creep of argillaceous schist (see VYALOV [1978]). VOLAROVITCH [1977] reports for granite, gabbro and limestone tested at rates of strains of the order 10^{-4}- 10^{-8} sec^{-1} values for the viscosity coefficient in the range 10^{12}- 10^{14} Poise, i.e. of the same order of magnitude as found in our experiments. If formula (3.13) is used in conjunction with (3.1) we obtain h = 69.3 and further k is somewhere between 0.2 d^{-1} and 0.09 d^{-1}. Thus curve C_c is determined by both approaches. From tests performed with standard testing machines it yields for the other constants approximately the values $E = 167{,}000$ kgf cm^{-2} ($1{,}637{,}769$ N cm^{-2}), $\alpha = 4.2$ and $\beta = 1.2$ and therefore the curve C_d is also determined. Thus the constitutive equation (3.1) is fully determined.

In a similar way numerical coefficients to be used in the constitutive equations of the form (3.1) were determined for several other rocks (rock salt: BARONCEA et al. [1977], dolomite, limestone, sandstone, gnaiss etc.). Some of these coefficients vary significantly from one rock to another, and sometimes even within the same kind of rocks. From all these coefficients the elastic modulus E is the one which varies the most from one kind of rock to another. The order of magnitude for the other coefficients is not changing too much when passing from one rock to other.

Finally the third curve C_r necessary to fully define the boundary of the domain $\mathcal{D}$ is determined experimentally using both standard testing machines (and various loading rates) and creep tests (with various loading stresses). It was found that for higher stresses and/or higher rates of loadings the strain at fracture is smaller. On the other hand the fracture points determined by standard testing machines are furnishing higher strains (for the same stress) than those furnished by creep tests. Curve C_r in conjunction with the constitutive law (3.1) is of great importance for practical applications as for instance the prediction of the failure of an undergroud structure etc.

§4 Volume compressibility

It is well known that in what concerns the volume compressibility the rocks (and soils) have quite distinct properties from other materials, as for instance metals, in the sense that volume is compressible and this compressibility is partially permanent (VOLAROVITCH et al [1974], LEVYKIN and VAVAKIN [1978], VYALOV [1978], STEPHENS et al.[1970]).

In order to investigate the volume compressibility of various rocks a special device was made. (Variants of such kind of devices are described for instance by VOLAROVITCH et al. [1974] and STEPHENS et al. [1970]). Small rock samples, 20 mm. in diameter and about the same height were compressed with a piston inside a thick walled hard steel cylinder. Thus these are experiments with confined lateral surface. The diameter of the cylinder was only slightly larger than that of the specimen. The lateral surface of the specimen was lubricated. The piston was compressed either by a standard testing machine or by a dead-weight loading machine. Generally due to very high forces involved, the whole device, though made by high strength steel, will deform. It is expected that many experimental errors may be involved, and not all of these are fully estimated. Therefore the results obtained untill now are considered to be tentative, i.e. indicating qualitatively some properties of the rock only. In all experiments only a single continuous loading followed by a single unloading were done. The constitutive equation proposed is thought to describe the material response in such experiments.

An estimation concerning the stresses and displacement fields involved was done assuming linear elasticity for the rock specimen. We use cylindrical coordinates r, θ, z, the Oz axis coinciding with the specimen symmetry axis. Let us denote by σ_1 the applied axial stress at the upper end $z = \ell$ of the specimen, by τ the frictional stress at the lateral surface $r = R$ of the specimen (which will be assumed to be constant, i.e. independent on z and θ), and by ℓ the height of the specimen. Assuming axial symmetry, that the surface $r = R$ of the steel cylinder is deforming negligible and that at the bottom of the specimen, at $z = 0$ and $r = R$ the displacement is zero, it is easy to show that the mean stress (pressure) can be expressed as

$$\sigma = \frac{1}{3}\,\frac{1+\nu}{1-\nu}\left[\sigma_1 + \frac{2\tau}{R}(\ell - z)\right] \qquad (4.1)$$

where ν is the Poisson's ratio. From this formula it yields that if the lateral friction is small (τ small), if the radius of the specimen is relatively big while its length is relatively small, and if ν is not too far from 0.5 then the mean stress is not too different from σ_1. Formula (4.1) was used only to get a suggestion for the sizes of the specimen to be used. Generally the friction forces are significant since even after unloading a significant force (of the order of several tons) was necessary to push out the specimen from the cylinder.

Thus in this series of experiments a relationship between the axial stress σ_1 and the axial strain ε_1 was established. Both quantities were measured. The other components of the strain were considered to be small in a first approximation, i.e. $\varepsilon_1 = 3\varepsilon$ where ε is the mean strain. Such relationships are anyway suggesting (at least qualitatively) the laws of

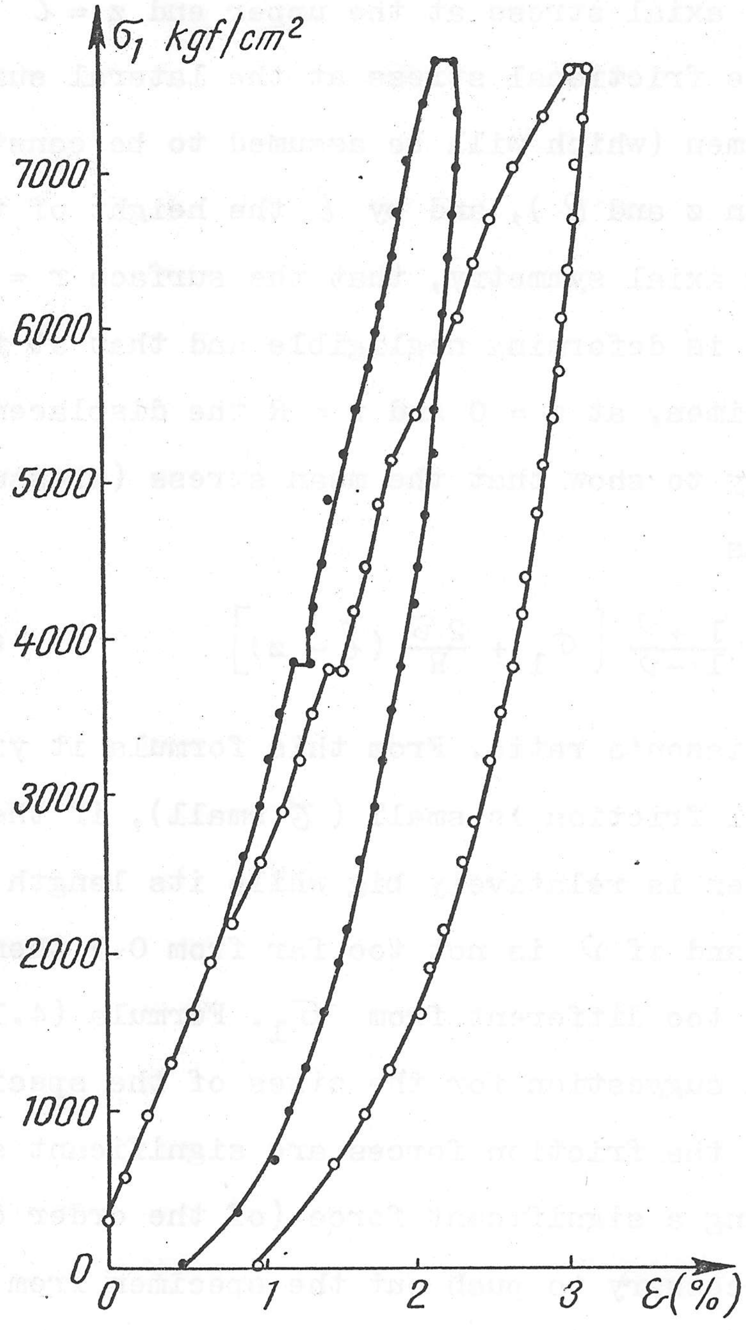

Fig.4.1 Stress-strain curves for volume compressibility for schist for two loading rates

volume compressibility for rocks in confined experiments.

To give an example on fig.4.1 are represented the curves $\sigma_1 \sim \varepsilon_1$ obtained for schist with two loading rates (4 ton/min and 1 ton/min); the unloading was done with the same rate. It was always found that the volume compressibility of various rocks is rate sensitive mainly during loading but also during unloading and that important component of the volume strain is present. The magnitude of the permanent volume strain is also highly dependent on the loading rate: when the loading rate is higher then at the same stress the permanent volume strain is smaller. Both loading and unloading curves are nonlinear with concavity directed towards the σ_1 axis. The permanent compressibility of the volume was checked also by measuring the size of the specimen after the test and also by density measurements. A few preliminary tests have been performed also to check if a creep in volume compressibility is not possible. For this purpose the machine was stopped at various levels of the applied stress and further this stress was kept constant or a dead weight machine was used. It was found that volume strain was increasing though the applied stress was kept constant. The process was a transitory one and after a few minutes or tents of minutes the volume strain became constant and remained so (see the small steps on fig.4.1 as well as the small horizontal plateaux at the top of these curves).

A mathematical model for the volume compressibility can be established following the same procedure as above by making the following assumptions (see fig.4.2). It will be assumed that there is a limit volume strain ε_* which cannot be surpassed no matter how big is the applied pressure (locking

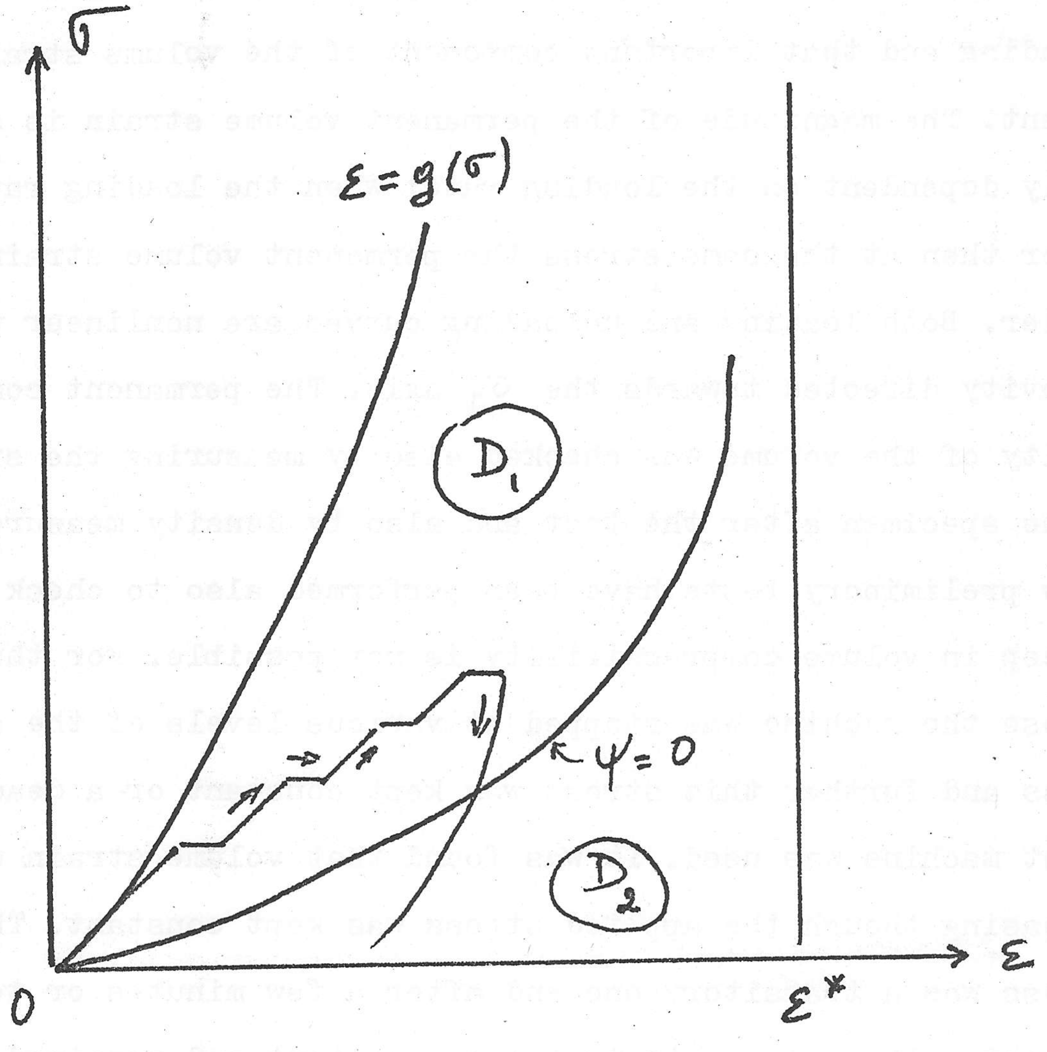

Fig.4.2 Schema of domains and boundaries involved in the constitutive equation for volume compressibility

model). If σ is the pressure and ε the mean strain, then it will be assumed further that by making experiments with the lowest loading rate of interest for the kind of experiments we have in mind, we find a creep-stabilization boundary for the volume compressibility. The equation of such a boundary can be written for instance as

$$\varepsilon = \varepsilon_* \left[1 - \exp(-\frac{\sigma}{\sigma_0})\right] \tag{4.2}$$

where $\sigma_0 > 0$ is a constant of the material.

The previous assumption was made also in conjunction with another kind of experiment in which the same device was used but the loading was applied with a dead-weight machine. This time the load is applied in successive steps. The pressure-volume strain curve which results is step-wise looking. A typical such curve shown on fig.4.2 reminds the Masson-Savart (or Portvin-Le Chatelier) phenomenon. Generally at each additional loading we get an "instantaneous" increase of the pressure and of the volume-strain, followed by a period in which volume-strain continues to increase under constant pressure. The disadvantage of the experiments done with the dead-weight machine is that the maximum pressure which can be reached is smaller than in experiments using a standard testing machine (in our experiments we were unable to surpass about 4 kbar with these machines).

The boundary (4.2) is thought to have distinct properties than the boundary C_c discussed in the previous paragraph: i.e. starting from $\forall(\varepsilon,\sigma) \in D_1$ by any process with constant or decreasing pressure the boundary (4.2) will be reached in an infinite or finite time interval respectively.

Another assumption made is that by making the fastest possible test from the set of experiments we have in mind we get an "instantaneous" response for volume compressibility which may be expressed for instance as

$$\varepsilon = g(\sigma) \tag{4.3}$$

if the experiments start from zero stress and zero strain states. Here $g(\sigma)$ is a non-negative function of class C^1 on $0 < \sigma$, $0 < \varepsilon < \varepsilon_*$. In fig.4.2 D_2 is the domain

$$\varepsilon_*\left[1 - \exp\left(-\frac{\sigma}{\sigma_0}\right)\right] \leqslant \varepsilon < \varepsilon_*, \quad \sigma > 0$$

and domain D_1 is

$$g(\sigma) < \varepsilon < \varepsilon_*\left[1 - \exp\left(-\frac{\sigma}{\sigma_0}\right)\right], \quad \sigma > 0 .$$

Points belonging to D_1 are possible strain-stress states which can be reached in loading processes when stress is increasing. Some points in D_2 can be reached during unloading (decreasing stress) while others as for instance those in the neighbourhood of $\varepsilon = \varepsilon_*$, $\sigma = 0$ cannot be reached by any conceivable experiment. Power functions seem to be suitable for several rocks to be used in (4.3). In particular (4.3) can be a straight line.

In order to describe the volume compressibility a rate type constitutive equation of the form

$$\dot{\varepsilon} = \left[\frac{1}{3\,K(\sigma)} + f(\varepsilon, \sigma, \operatorname{sgn}\dot{\sigma})\right]\dot{\sigma} + \psi(\varepsilon, \sigma) \tag{4.4}$$

is used. The instantaneous properties in loading are defined by

$$f(\varepsilon, \sigma, \operatorname{sign}\dot{\sigma}) = \begin{cases} \varphi(\varepsilon, \sigma) & , \quad \dot{\sigma} \geqslant 0 \\ 0 & , \quad \dot{\sigma} < 0 \end{cases} \tag{4.5}$$

where

$$\psi(\varepsilon,\sigma) = \frac{dg(\sigma)}{d\sigma} - \frac{1}{3\,K(\sigma)} \quad , \quad \forall(\varepsilon,\sigma) \in D_1 \qquad (4.6)$$

and $K(\sigma)$ is the variable modulus in unloading. In a first approximation one can assume K = const.

Function $\psi(\varepsilon,\sigma)$ describing the slow deformation of the volume (creep) is defined by

$$\psi(\varepsilon,\sigma) = \begin{cases} \eta\left\{\varepsilon_*\left[1 - \exp(-\frac{\sigma}{\sigma_0})\right] - \varepsilon\right\}, & \varepsilon < \varepsilon_*\left[1 - \exp(-\frac{\sigma}{\sigma_0})\right] \\ 0 \quad , & \varepsilon > \varepsilon_*\left[1 - \exp(-\frac{\sigma}{\sigma_0})\right] \end{cases} \qquad (4.7)$$

where η is a volume viscosity coefficient, which in a first approximation will be considered to be constant and is given in $\sec^{-1}$ (or $(\text{day})^{-1}$).

Making the rough assumption that σ and σ_1 are of the same order of magnitude the constants involved in (4.4) can be determined by the tests mentioned. Thus for schist approximate values are $\varepsilon_* = 0.0108$, $\sigma_0 = 4000$ kgf cm^{-2} (39228 N cm^{-2}), $\eta = 43$ d^{-1}. For natural chalk which is a soft rock with σ_r = 45 kgf cm^{-2} (441 N cm^{-2}) and initial density $\rho = 1.67$ g cm^{-3} we get $\varepsilon_* = 0.07$, $\sigma_0 = 550$ kgf cm^{-2} (5394 N cm^{-2}) and η = 3.80 ± 1.07 d^{-1}. The value of η was determined from several tents of tests (of horizontal plateaux at various levels of stresses). Generally it was found that η is quite constant. Since a small number of experiments were used to determine these constants, the values given must be considered only as indicative pending further experimental data.

§5 Conclusions

It was shown how with several diagnostic tests one can determine elastic viscoplastic constitutive equations for rocks to be used in one-dimensional compressive loadings and in the compressibility of the volume. A three-dimensional generalization of the model based on these one-dimensional models is not yet possible since experiments which would reveal the relationship between shearing properties and volume variation (dilatancy) are still necessary. Certainly that even experiments of the kind discussed are further necessary to make precise many details in the model (mainly more exact values of the constants involved etc.).

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