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1. Introduction

A basic result in the invariant theory of finite groups acting on local rings is Serre's generalization of Chevalley's classical theorem. Assuming that R is a regular local ring, and G is a finite group of automorphisms of R, which induce the identity map on the residue field, and whose order $|G|$ is invertible in R, Serre's theorem [Se] states that the following are equivalent: (a) G is generated by pseudo-reflections; (b) R is R^G-free ; (c) R^G is regular (cf. also [B_2 , Exercise 7]). Several authors have subsequently investigated the descent of various properties from R to R^G , asuming condition (a) : cf. (3.iii) below. Our purpose in this note is to show, that under reasonable generic restrictions on the action of G , the extension $R^G \subset R$ is free with remarkably well-behaved fibre.

Before going further we fix some notation: $(R,\underline{m},k)$ will always denote a commutative noetherian unitary local ring, and G will be a finite group of automorphisms of R . For an ideal $\underline{a} \subset R$, $G^T(\underline{a}) = \{ g \in G \mid \forall\ x \in R : g(x) - x \in \underline{a} \}$ is the inertia subgroup of $\underline{a}$; we set $H = G^T(\underline{m})$, which is the invariant subgroup of G , equal to $\mathrm{Ker}(G \to \mathrm{Aut}(k))$. The invariant subring $R^G = \{ x \in R \mid \forall\ g \in G : g(x) = x \}$ is a local ring [B_1], with maximal ideal $\underline{m}^G$; when $|H| \in R^{\times}$ (= the group of units of R), R^G is noetherian , and $R^G/\underline{m}^G = k^{G/H}$ (cf. (5) and (6)). An element h of H is called a pseudo-reflection, if $\mathrm{rank}(\varepsilon(h) - \mathrm{id}) = 1$, where $\varepsilon : H \to GL_k(\underline{m}/\underline{m}^2)$ denotes the canonical homomorphism. We denote by $R*G$ the skew group ring, i.e. the free R-module with basis $g \in G$ and (non-commutative when $G \neq 1$) multiplication given by $(x_1g_1)(x_2g_2) = x_1g_1(x_2)\cdot g_1g_2$, and write $R^G[G]$ for the group ring of G over R^G .

We say that *generically the action of* G *is without inertia*, if every associated prime of R has a trivial inertia subgroup. Clearly, it suffices to impose the restriction on the maximal elements of Ass R , and the condition is trivially satisfied when R is a domain.

(1) *Theorem*. *Suppose that* $|H| \in R^{\times}$ *and that generically* G *acts on* R *without inertia*. *If* H *is generated by pseudo-reflections, then*:

(i) R *has a normal basis with respect to* G , *i.e.* $R \simeq R^G[G]$ *as* $R^G[G]$ *-modules* ; in particular:

(i)' R *is a free* R^G*-module of rank* $|G|$;

(ii) *the fibre* $R/\underline{m}^GR$ *is a (artinian) complete intersection. Whose associated graded ring also is a complete intersection.*

Suppose moreover that R contains a primitive $|H|$-th root of unity, and let $h_1,\ldots,h_r$ be the distinct pseudo-reflections, contained in H . In this case [Si, Lemma 6] shows there exist elements $x_1,\ldots,x_r$ in $\underline{m} \setminus \underline{m}^2$, such that $h_i \in G^T(x_i)$, $1 \leq i \leq r$.

(iii) *The Dedekind, Noether, and Kähler differents of* R *over* R^G *are defined, coincide, and when* R *contains a primitive* $|H|$*-th root of unity, they are equal to the principal ideal generated by* $x = \prod_{i=1}^{r} x_i$; *if* $|G| \in R^{\times}$ *the discriminant of* R *over* R^G *is the principal ideal of* R^G , *generated by* $\prod_{g \in G} g(x)$.

We refer to the book of Scheja and Storch [SS_1] for the notions of ramification theory used in this paper.

Recall that the complete intersection defect, or deviation, of R is defined to be the integer $d(R) = \mu(\underline{a}) - (\dim \tilde{R} - \dim R)$, where $\tilde{R}$ is a regular local ring, given by the Cohen Structure Theorem, such that $\hat{R} = \tilde{R}/\underline{a}$, and μ denotes the minimal number of generators; this is an invariant of R, and $d(R) \geqslant 0$; with equality holding precisely for the complete intersections. When R is Cohen-Macaulay, the type of R is by definition the integer $t(R) = \dim_k \operatorname{Ext}_R^{\dim R}(k,R)$; $t(R) \geqslant 1$, with equality holding precisely for the Gorenstein rings.

(2) Corollary. (i) $d(R^G) = d(R)$; in particular:

(i)' R^G is a complete intersection if and only if this is true for R;

(ii) R^G is Cohen-Macaulay if and only if this is true for R; in this case $t(R^G) = t(R)$, in particular:

(ii)' R^G is Gorenstein if and only if R is Gorenstein;

(iii) if R is a normal domain, then (so is R^G and) $Cl(R^G) \hookrightarrow Cl(R)$; in particular:

(iii)' if R is UFD, then R^G is UFD;

(iv) for every finitely-generated $R*G$-module M, M^G is a finitely-generated R^G-module, and for all $i \geqslant 0$, there is an inequality:

$$\dim_k \operatorname{Tor}_i^R(M,k) \geqslant \dim_{k'} \operatorname{Tor}_i^{R^G}(M^G,k') \quad , \quad k' = R^G/\underline{m}^G .$$

The references are [A, (3.6)] for (i), [HK, (1.24)] for (ii), [F, (6.11)] for (iii), [G_2, (1.1)], (5) and (6) below for (iv).

(3) Remarks. (i) When G acts with generic inertia, the conclusions of the Theorem are not necessarily either true or false. For instance, let A be any local ring, $R = A[X,Y]/(X^2, XY, Y^2)$, σ be the reflection for which $\sigma|_A = id$, $\sigma(X) = X$,

$\sigma(Y) = -Y$. Then $R^{(\sigma)} = A[X]/(X^2)$, and R is not $R^{(\sigma)}$-free (although the fibre is a complete intersection). On the other hand, with $R = A[X,Y]/(X^2,Y^2)$ and the same action of σ as above, R is free over $R^{(\sigma)} = A[X]/(X^2)$.

(ii) It is well-known that the R^G-freeness of R does not in general imply that H is generated by pseudo-reflections, even when $G = H$. A particularly suggestive example is given in [PS, Bemerkung 2] , where $R = k[[U^2,UV,V^2]]$ is a complete intersection normal domain, $\tau(U) = \eta U$, $\tau(V) = V$, with η a primitive sixth root of unity (char(k) $\neq$ 2,3). Then τ is not a pseudo-reflection, but R is free over $R^{(\tau)} = k[[U^6,V^2]]$.

(iii) Assume $G = H$ is generated by pseudo-reflections. When R is a domain, Goto [G_2, Remarks following (1.2)] states that R is R^G-free (with a slightly stronger definition of pseudo-reflections, coinciding with ours when R contains a primitive $|G|$-th root of unity). For analytic normal domains, the same statement has been proved in Storch's Habilitationsschrift (cf. the reference in [PS, Bemerkung 2]). Hochster and Eagon [HE, Proposition 16] have proved the Q^G-freeness of Q , when Q is a graded k-algebra and G is a finite group of degree preserving k-linear automorphisms, generated by elements lying in the inertia subgroups of regular principal homogeneous ideals. We want to note that all these results are contained in statement (i)' of Theorem (1).

(iv) Assume $G = H$ is generated by pseudo-reflections. Watanabe [W_2] has shown, without generic assumptions on the action of G , that the Gorenstein property descends from R to R^G . This has been partially generalized by Goto [G_1], who proved that when R is the quotient of an equicharacteristic local regular ring modulo a G-invariant ideal, the inequalities $d(R^G) \leq d(R)$, and (for R Cohen-Macaulay), $t(R^G) \leq t(R)$ hold. In view of these results and of (2.i), (2.ii) above, it is reasonable to ask the

Question. Assume $|H| \in R^{\times}$ and H is generated by pseudo-reflections. Is it true that $d(R^G) \leqslant d(R)$, and when R is Cohen-Macaulay, that $t(R^G) \leqslant t(R)$?

Of course it is sufficient to handle the case G= H (cf. (4)).

(v) Statement (iii)' of Corollary (2) shows that in Singh's Theorem 1 [Si] the assumptions that R contains a primitive $|H|$-th root of unity and that G = H can be deleted.

2. Preliminaries

Most of the results of this section require only part of the hypotheses of (1). Keeping the notation introduced in the previous section, we shall introduce the appropriate assumptions in each case.

(4) R^H is a Galois extension of R^G with Galois group G/H, i.e. $R^H \simeq R^G[G/H]$ as $R^G[G/H]$-modules, $\underline{m}^H = \underline{m}^G R^H$ and the field extension $R^G/\underline{m}^G \hookrightarrow R^H/\underline{m}^H$ is Galois with Galois group G/H .

For a proof cf. [SS_1 , §§ 20, 21] , or [B_1] together with [CHR] .

(5) Assume $|H| \in R^{\times}$. Then:

(i) R^G is a (noetherian) local ring.

(ii) An R-module M is finitely-generated over R (if and) only if it is finitely-generated over R^G .

Proof. (i) The noetherian property descends from R to R^H because of the existence of a Reynolds operator $|H|^{-1}t_H$ (cf. (6) below), and from R^H to R^G by faithful flatness (4).

(ii) It is sufficient to prove the finite generation of R over R^G , which follows from the finite generation of R^H over R^G (4), and that of R over R^H [LP, (3.4)] . Note that (i) is a consequence of (ii) by the Eakin-Nagata Theorem.

(6) Let $t_G = \sum_{g\in G} g \in R*G$ denote the trace element. The following conditions are equivalent:

(a) $|H| \in R^{\times}$;

(b) $(\ )^G$ is an exact functor from the category of left $R*G$-modules to that of R^G-modules, and $M^G = t_G M$;

(c) there exists an element $r \in R$ with $t_G r = 1$.

When these conditions are satisfied, $(\ )^G$ carries finitely-generated R-modules to finitely-generated R^G-modules.

Proof. The equivalence of (b) and (c) is proved in [G_2, (2.2)].

Let $\bar{r}$ denote the image of r in k , let $\bar{g}$ denote the image of $g \in G$ in G/H , and let $g_1, \dots, g_m$ be a complete set of representatives in G for the elements of G/H .

Assume (c). Then in $R^G/\underline{m}^G$ one has:

$$1 = t_G \bar{r} = \sum_{i=1}^{m} g_i\left(\sum_{h\in H} h(\bar{r})\right) = |H| \sum_{\bar{g}\in G/H} \bar{g}(\bar{r}) ,$$

hence $|H| \in (R^G)^{\times} \subset R^{\times}$, which is (a) .

Conversely, suppose (a) is satisfied. Since R^H is a Galois extension of R^G (4) , there exists by [CHR, (1.6)] an $r' \in R^H$ such that $\sum_{\bar{g}\in G/H} \bar{g}(r') = 1$. But since $r' = |H|^{-1} t_H r'$, one has:

$$1 = \sum_{\bar{g}\in G/H} \bar{g}(|H|^{-1} t_H r') = t_G(|H|^{-1} r') ,$$

hence (c) holds .

The last assertion follows from (5) .

(7) Suppose G acts generically without inertia on R . Set $N_G(x) = \prod_{g\in G} g(x)$. Let U denote the set of non-zero divisors of R , and consider the multiplicatively closed sets $V = \{ N_H(x) \mid x \in U \} \subset R^H$, $W = \{ N_G(x) \mid x \in U \} \subset R^G$. Then:

(i) $V^{-1}R = U^{-1}R$ and $(U^{-1}R)^H = V^{-1}(R^H)$;

$W^{-1}R = U^{-1}R$ and $(U^{-1}R)^G = W^{-1}(R^G)$;

(ii) $U^{-1}R$ has a normal basis with respect to H and a normal basis with respect to G .

Proof. (i) holds even without the assumption on the action of G : cf. e.g. [SS_1, p.104] .

Since for every prime $\underline{p}$ of R, $G^T(\underline{p})$ $G^T(\underline{m}) = H$, H acts on R without generic inertia and it is sufficient to prove (ii) for H. Let $\underline{p}_1, \ldots, \underline{p}_s$ be the maximal elements of Ass R. Since V does not intersect the $\underline{p}_i$'s, we have for $\underline{q}_i = V^{-1}\underline{p}_i = U^{-1}\underline{p}_i$: $H^T(\underline{q}_i) = H^T(\underline{p}_i) = 1$, by [$B_1$, Lemma 3] and our assumption. The $\underline{q}_i$'s being the maximal ideals of the semi-local ring $U^{-1}R$, the extension $(U^{-1}R)^H \subset U^{-1}R$ is Galois according to [CHR, (1.3.f)]. Using the identification $V^{-1}R^H = (U^{-1}R)^H$ given by (i), we see that $V^{-1}R^H$ is semi-local, having a semilocal integral extension $U^{-1}R$. Now (ii) is a consequence of [CHR, (4.2.c)].

Let Q denote the associated graded ring $gr_{\underline{m}}R = \oplus_{i \geqslant 0} \underline{m}^i/\underline{m}^{i+1}$, and let S be the symmetric algebra of $\underline{m}/\underline{m}^2$ over k. Then $\varepsilon : H \longrightarrow GL_k(\underline{m}/\underline{m}^2)$ induces natural degree-preserving actions of H on Q and on S by k-algebra automorphisms. By definition, the following diagram of H-equivariant homomorphisms of graded k-algebras is commutative:

$$\begin{array}{ccc} S^H & \hookrightarrow & S \\ {\scriptstyle p^H}\downarrow & & \downarrow{\scriptstyle p} \\ Q^H & \hookrightarrow & Q \end{array}$$

When $|H| \in R^\times$, both vertical maps are surjective: for p^H this follows from the computation : $p^H(S^H) = p^H(t_H S) = t_H(p(S)) = t_H Q = Q^H$ (cf. (6) and/or [W_2,(3.4)]).

The filtration $\{\underline{m}^i \cap R^H\}_{i \geqslant 0}$ induces, when $|H| \in R^\times$, the $\underline{m}^H$-adic topology on R^H, and $Q^H = \oplus_{i \geqslant 0} \underline{m}^i \cap R^H / \underline{m}^{i+1} \cap R^H$ (cf. [B_2, Exercise 7(c)], or [W_1, Proof of Theorem 4, (2) and (3)]). In particular,

$$Q_i^H = \underline{m}^i \cap R^H / \underline{m}^{i+1} \cap R^H = \underline{m}^i \cap R^H + \underline{m}^{i+1} / \underline{m}^{i+1}$$

$$= (\underline{m}^i \cap R^H)R + \underline{m}^{i+1} / \underline{m}^{i+1} .$$

Set $\overline{R} = R/\underline{m}^H R$, and denote by $\overline{\underline{m}}$ its maximal ideal ; of course, $\overline{R}/\overline{\underline{m}} = k$ (e.g. by (6)).

(8) Assume $|H| \in R^{\times}$. In the notation introduced above, there exist natural H-equivariant surjective homomorphisms of graded- k-algebras:

$$\mathrm{gr}_{\overline{\underline{m}}}\, \overline{R} \longleftarrow Q/Q^H_+ Q \longleftarrow S/S^H_+ S$$

where $(\ )_+$ denotes the irrelevant maximal ideal.

Proof. The right-hand map is simply $p \otimes_{S^H} k$, where $k = S^H/S^H_+$, taking into account that $Q \otimes_{S^H} k = Q/S^H_+ Q = Q/Q^H_+ Q$ by the surjectivity of p^H.

On the other hand note that $I = \mathrm{Ker}(\, Q = \mathrm{gr}_{\underline{m}} R \rightarrow \mathrm{gr}_{\overline{\underline{m}}} \overline{R}\,)$ is the homogeneous ideal with

$$I_n = \underline{m}^H R \cap \underline{m}^n / \underline{m}^H R \cap \underline{m}^{n+1} = \underline{m}^H R \cap \underline{m}^n + \underline{m}^{n+1} / \underline{m}^{n+1} .$$

By the computation made before, we have :

$$(Q^H_+ Q)_n = \sum_{i=1}^{n} \left[(\underline{m}^i \cap R^H)R + \underline{m}^{i+1} / \underline{m}^{i+1}\right] . \left[\underline{m}^{n-i} / \underline{m}^{n-i+1}\right]$$

$$= \sum_{i=1}^{n} (\underline{m}^i \cap R^H)\underline{m}^{n-i} + \underline{m}^{n+1} / \underline{m}^{n+1} .$$

Comparing the two expressions and noticing that $(\underline{m}^i \cap R^H)\underline{m}^{n-i} \subset \underline{m}^H R \cap \underline{m}^n$ for $1 \leqslant i \leqslant n$, one sees that $(Q^H_+ Q)_n \subset I_n$ for all n, and this gives the left-hand epimorphism.

The statement on H-equivariance is clear.

(9) Assume $|H| \in k^{\times}$, and suppose H is generated by pseudo-reflections. Choosing a k-basis $X_1, \ldots, X_n$ of $\underline{m}/\underline{m}^2$, identify the symmetric algebra S with the polynomial ring $k[X_1, \ldots, X_n]$. Then:

(i) S^H is the graded polynomial ring generated by n algebraically independent elements $P_1, \ldots, P_n$ of degree $d_1, \ldots, d_n$ respectively, and S is S^H-free ;

(ii) $\overline{S} = S/S^H_+ S$ is a graded complete intersection, and $\dim_k \overline{S} = d_1 d_2 \ldots d_n = |H|$;

(iii) let $h_1, \dots, h_r$ be the distinct pseudo-reflections contained in H, and let $e_i \in \underline{m}/\underline{m}^2$ be a generator of $\mathrm{Im}(\varepsilon(h_i) - \mathrm{id})$, $1 \leq i \leq r$; then the element $e = \prod_{i=1}^{r} e_i \in S$ is a non-zero scalar multiple of the Jacobian $J = \det\left(\frac{\partial P_\ell}{\partial X_j}\right)$.

References: (i) is part of Chevalley's theorem, as generalized to fields of positive characteristic by Bourbaki [B_2, Theorem 4]; (ii) is an immediate consequence of (i), the statement about $\dim_k \bar{S}$ following from the equalities $\dim_k \bar{S} = \mathrm{rank}_{S^H} S = \dim_{K^H} K = |H|$, K denoting the field of fractions of S; (iii) is contained in [B_2, Proposition 6].

3. Proof of the Theorem

In this section we assume all the hypotheses of (1).

(10) Proof of (1.i)'. Let $\mu_T(M)$ denote the minimal number of generators of the module M over the local ring T.

Since R is a finitely-generated R^H-module by (5), we have from (8) that $\mu_{R^H}(R) \leq |H|$. On the other hand, let $\underline{p}$ be an associated prime of R, and $\underline{p}' = \underline{p} \cap R^H$. Since $\underline{p}'$ does not meet the multiplicatively closed set V, we have by (7) that $R_{\underline{p}'}$ is a free $(R^H)_{\underline{p}'}$-module of rank $|H|$. The inequality $\mu_{R^H}(R) \geqslant \mu_{(R^H)_{\underline{p}'}}(R_{\underline{p}'})$ shows now that R is minimally generated over R^H by $|H|$ elements.

Let $f: F \to R$ be an R^H-epimorphism with F a free R^H-module of rank $|H|$. Localizing at V we see that $V^{-1}f$ is an epimorphism of free modules of the same rank, hence an isomorphism. This implies that V meets all the primes in

Ass_{R^H} (Ker f) $\subset$ Ass R^H . Since by definition V consists of non-zero divisors, this is only possible when Ker f = 0 , i.e. when R is R^H-free of rank $|H|$. Combining this with (4) we see that it also is R^G-free of rank $|G|$.

(11) Proof of (1.ii) . The first part of the argument in (10) shows that as graded k-algebras $\mathrm{gr}_{\bar{\underline{m}}}\,\bar{R}$ and $\bar{S}$ are isomorphic, hence $\mathrm{gr}_{\bar{\underline{m}}}\,\bar{R}$ is a complete intersection by (9.ii) . The statement about the associated graded ring of $R/\underline{m}^G R$ follows, since by (4) we have $R/\underline{m}^G R = R/(\underline{m}^G R^H)R = R/\underline{m}^H R = \bar{R}$. Finally note that if the associated graded ring of a local ring is a complete intersection, then the ring itself is well-known to have the same property (e.g. [W_1, Lemma 10]) .

(12) Proof of (1.i) . First note that R is a finitely-generated projective $R^G[G]$-module, since it is cyclic projective over R∗G by [G_2, (2.2)] , and the last ring is $R^G[G]$-free of finite rank by (1.i)' . According to [Ba, Chapter XI, (5.1)] in order to establish the isomorphism of $R^G[G]$-modules $R \simeq R^G[G]$, it is sufficient to prove that it holds after localization at a multiplicatively closed subset of R^G , consisting of non-zero divisors. This is given by (7.ii) .

Note. The argument above essentially repeats [G_2 , (5.1) , (3) $\Rightarrow$ (1)] .

(13) Proof of (1.iii) . The ring R being a finitely-generated free R^G-module, all three differents are defined : cf. [SS_1, §§ 15 , 16] . The equality of the Dedekind and Noether differents holds in general for such extensions [SS_1, (16.8)] , while the equality of the Noether and Kähler differents is a consequence of (1.ii) by [SS_2 , (5.6)] . We shall simply refer to the different of R over R^G and denote it by $D(R|R^G) = D$.

We next prove that D is generated by x , assuming that

R contains a primitive $|H|$-th root of unity. Since R^H is unramified over R^G, we can further assume $G = H$. First note that the Jacobian J of (9.iii) has a non-zero image in $\overline{S}$, and generates $(0:\overline{S}_+)$. An easy way to see this is to consider the Koszul complex K on the images of $X_1,\ldots,X_n$ in $\overline{S}$, with generators $T_1,\ldots,T_n$ in degree one. From (9.ii) we see that the d_i's are invertible in k, hence by Euler's formulas the homology classes of $z_i = \sum_{\ell=1}^{n} \frac{\partial P_\ell}{\partial X_j} T_j$ form a basis of $H_1(K)$ $(1 \leq i \leq n)$. The artinian ring $\overline{S}$ being a complete intersection by (9.ii), it follows that $(0:\overline{S}_+)$ is the one-dimensional vector space isomorphic to $(z_1 \wedge z_2 \wedge \ldots \wedge z_n) = (J.T_1 \wedge T_2 \wedge \ldots \wedge T_n)$.

Clearly, the element e_i defined in (9.iii) is, up to a non-zero scalar multiple from k, an initial form of the element x_i appearing in the formulation of the statement to be proved, hence the image $\overline{x}$ of the initial form of x in $gr_{\underline{m}}\overline{R}$ is equal to ae, $a \in k^\times$, which is non-zero by (9.iii), the preceding discussion, and the isomorphism $gr_{\underline{m}}\overline{R} \simeq \overline{S}$ established in (10). In particular, since $\overline{x}$ belongs to the socle of $gr_{\underline{m}}\overline{R}$, it is the image of an element contained in the highest possible non-zero level of the $\underline{m}$-adic filtration of $\overline{R}$, hence can be identified to an element of this ring. Once this is done, $\overline{x}$ is necessarily a generator of the socle of $\overline{R}$, being a non-zero element in the one-dimensional socle of an artinian complete intersection (cf. (1.ii)). Also, with this identification, $\overline{x}$ is simply the image of x in $\overline{R}$ under the canonical projection from R.

Now remark that it follows from (1.ii) and [SS_1, (16.5)] (or equivalently: [SS_2, (15.1)]), that D is principal, generated, say, by $y \in \underline{m}$. Clearly, x is in D, since every prime of R containing some of the x_i's has a non-trivial inertia subgroup, hence is ramified over R^H. By the change of rings properties of the different [SS_1, (15.1)], $D(k|\bar{R})$ is the image of D in $\bar{R}$, hence generated by $\bar{y}$. Since $|H| \in R^{\times}$, we can apply [SS_2, (4.7)] and the remarks following it, to obtain that $\bar{y}$ generates the socle of $\bar{R}$, hence the images in $\bar{R}$ of x and y differ by a multiple from $k^{\times}$. This implies the equality $xR = yR = D$.

The last statement is immediate from [SS_1, (16.11), (21.14)] applied to the expression of the different.

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