INSTITUTUL
DE
MATEMATICA

INSTITUTUL NATIONAL
PENTRU CREATIE
STIINTIFICA SI TEHNICA

ISSN 0250 3638

ON THERMAL EQUATION FOR FLOW IN POROUS MEDIA

by

H.I.ENE and E.SANCHEZ-PALENCIA

PREPRINT SERIES IN MATHEMATICS
No.46/1981

BUCURESTI

Med 17661

ON THERMAL EQUATION FOR FLOW IN POROUS MEDIA

by

H.I.ENE*) and E.Sanchez-PALENCIA**)

May 1981

*) INCREST, Department of Mathematics, Bd.Pacii 222, 79622 Bucharest, Romania

**) Universite Paris VI, Mecanique Theorique, Tour 66 4, Place Jussieu, 75230 Paris, France

ON THERMAL EQUATION FOR FLOW IN POROUS MEDIA

H.I.ENE

INCREST, Department of Mathematics, Bd.Păcii 222
79622 Bucharest, Romania

and

E.SANCHEZ-PALENCIA

Université Paris VI, Mécanique Théorique, Tour 66,
4, Place Jussieu, 75230 Paris, France

Abstract. We establish the form of the energy equation for the flow in porous media. We show that the dissipative term as well as corrective convection terms must be taken into consideration. For the natural convection we prove that the dissipative term desapears.

1. INTRODUCTION

1.1. Generalities

In the general framework of the "homogeneization method" (see Bensoussan, Lions, Papanicolaou [1], or Sanchez-Palencia [2], as general references) we consider the motion of a viscous fluid through a porous medium. The periodic geometric structure of the "pores" is associated with the small parameter ε. It is known that the asymptotic process and the limit equations may have very different structure if several "small parameters" such as the viscosity coefficient are involved in the problem. In a previous paper (Ene and Sanchez-Palencia [3]) we studied some cases where the density, the viscosity or the thermal expansion coefficients are small, but in all these cases the obtained energy equation was the conduction one.

In this paper we consider cases where the energy equation involves the convective and dissipative terms also. The exact physical meaning of the news terms which apears in the energy equation for flow in porous media is given by the non-dimensional numbers.

1.2. General equations

We consider a parallelipipedic period Y of the space of the variables y_i (i=1,2,3) formed by a fluid and a solid part Y_f and Y_s , with smooth boundary Γ . We also denote by Y_f (resp. Y_s) the union of the Y_f (resp. Y_s) parts of all periods, and assume that Y_f (resp. Y_s) is connected. If Ω is the "porous body" in the space of the variables x_i , we introduce the small parameter ε and the fluid domain $\Omega_{\varepsilon f}$ (resp. the solid domain $\Omega_{\varepsilon s}$) defined by

$$\Omega_{\varepsilon f} = \{x;\ x \in \Omega,\ x \in \varepsilon Y_f\},\quad \Omega_{\varepsilon s} = \{x;\ x \in \Omega,\ x \in \varepsilon Y_s\}$$

If ρ , p^ε , T^ε and $\underline{v}^\varepsilon$ denote the density, pressure, temperature and velocity of the incompressible flow, they must satisfy the equations of conservation of momentum, mass and energy

$$\rho_f v_k^\varepsilon \frac{\partial v_i^\varepsilon}{\partial x_k} = - \frac{\partial p^\varepsilon}{\partial x_i} + \frac{\partial \sigma_{ik}^\varepsilon}{\partial x_k} + \rho_f f_i \tag{1.1}$$

$$\frac{\partial v_i^\varepsilon}{\partial x_i} = 0 \tag{1.2}$$

$$\rho_f C_f v_k^{\varepsilon} \frac{\partial T^{\varepsilon}}{\partial x_k} = \tau_{jk}^{\varepsilon} \frac{\partial v_j^{\varepsilon}}{\partial x_k} + \frac{\partial}{\partial x_k}\left(\lambda_f' \frac{\partial T^{\varepsilon}}{\partial x_k}\right) \tag{1.3}$$

in $\Omega_{\varepsilon f}$, and

$$0 = \frac{\partial}{\partial x_k}\left(\lambda_s' \frac{\partial T^{\varepsilon}}{\partial x_k}\right) \tag{1.4}$$

in $\Omega_{\varepsilon s}$, where f_i are the components of the exterior body force by unit mass, τ_{ik}^{ε} are the components of the viscous stress tensor:

$$\tau_{ik}^{\varepsilon} = \mu'\left(\frac{\partial v_i^{\varepsilon}}{\partial x_k} + \frac{\partial v_k^{\varepsilon}}{\partial x_i}\right) \tag{1.5}$$

The boundary conditions on Γ are:

$$v_i^{\varepsilon} = 0 \tag{1.6}$$

$$T^{\varepsilon}\Big|_f = T^{\varepsilon}\Big|_s \tag{1.7}$$

$$\lambda_f' \frac{\partial T^{\varepsilon}}{\partial n}\Big|_f = \lambda_s' \frac{\partial T^{\varepsilon}}{\partial n}\Big|_s \tag{1.8}$$

In order to study the asymptotic process $\varepsilon \to 0$ we consider the classical expansions:

$$v_i^{\varepsilon}(x) = \varepsilon^n v_i^o(x,y) + \varepsilon^{n+1} v_i^1(x,y) + \ldots \tag{1.9}$$

$$p^{\varepsilon}(x) = p^o(x,y) + \varepsilon p^1(x,y) + \ldots \tag{1.10}$$

$$T^{\varepsilon}(x) = T^o(x,y) + \varepsilon T^1(x,y) + \ldots \tag{1.11}$$

where $y = \frac{x}{\varepsilon}$ and all functions are considered to be Y periodic with respectic to the variable y and n is a positive parameter to be defined later (depending on the data). The two-scale asymptotic expansion is obtained by considering that the dependence in x is obtained directly and through the variable y. The derivatives must be considered as:

$$\frac{d}{dx_i} \rightarrow \frac{\partial}{\partial x_i} + \frac{1}{\varepsilon}\frac{\partial}{\partial y_i}$$

1.3. Darcy's law

If we suppose that the viscosity is of the form $\mu' = \mu\,\varepsilon^m$ where μ is constant (independent of ε), it is well known (see Sanchez-Palencia [2], Ene and Sanchez-Palencia [4]) that, for n+m=2, the asymptotic proces lead to the Darcy's law:

$$\tilde{v}_i^o = -\frac{K_{ij}}{\mu}\left(\frac{\partial p^o}{\partial x_j} - f_j\right) \qquad (1.12)$$

$$\mathrm{div}_x \tilde{\underline{v}}^o = 0 \ ; \quad \mathrm{div}_y \underline{v}^o = 0 \qquad (1.13)$$

where $p^o = p^o(x)$ and $\sim$ is the mean operator:

$$\tilde{\cdot} = \frac{1}{|Y|}\int_Y \cdot\, dy \qquad (1.14)$$

The matrix K_{ij}, named "permeability tensor", is defined by:

$$K_{ij} = \frac{1}{|Y|}\int_Y w_i^j\, dy \qquad (1.15)$$

$$\underline{v}^o = \left(f_i - \frac{\partial p^o}{\partial x_i}\right)\underline{w}^i \qquad (1.16)$$

where $\underline{w}^i$ denotes the Y-periodic flow corresponding to a mean pressure gradient equal to the unit vector in the direction of x_i (see Ene and Sanchez [4] for details) and depend on the geometric structure of the period.

2. ENERGY EQUATION

In order to obtain the "macroscopic equation" for the energy, we consider the case where $\lambda' = \lambda \varepsilon^p$ with constant λ in the two phases.

First, ussing (1.11) in (1.2) and (1.3) we have the boundary conditions:

$$T^o\Big|_f = T^o\Big|_s \qquad (2.1)$$

$$T^1\Big|_f = T^1\Big|_s \qquad (2.2)$$

. . . .

$$\lambda_f n_i \frac{\partial T^o}{\partial y_i}\Big|_f = \lambda_s n_i \frac{\partial T^o}{\partial y_i}\Big|_s \qquad (2.3)$$

$$\lambda_f n_i \left(\frac{\partial T^o}{\partial x_i} + \frac{\partial T^1}{\partial y_i}\right)\Big|_f = \lambda_s n_i \left(\frac{\partial T^o}{\partial x_i} + \frac{\partial T^1}{\partial y_i}\right)\Big|_s \qquad (2.4)$$

$$\lambda_f n_i \left(\frac{\partial T^1}{\partial x_i} + \frac{\partial T^2}{\partial y_i}\right)\Big|_f = \lambda_s n_i \left(\frac{\partial T^1}{\partial x_i} + \frac{\partial T^2}{\partial y_i}\right)\Big|_s \qquad (2.5)$$

. . . .

and from (1.3) and (1.4):

$$\rho_f c_f \varepsilon^n \left(\varepsilon^{-1} v_k^o \frac{\partial T^o}{\partial y_k} + v_k^o \frac{\partial T^o}{\partial x_k} + v_k^o \frac{\partial T^1}{\partial y_k} + v_k^1 \frac{\partial T^o}{\partial y_k} + \ldots \right) =$$

$$= \mu \varepsilon^{m+2n} \Big[\varepsilon^{-2} e^o_{jky} \frac{\partial v_j^o}{\partial y_k} + \varepsilon^{-1} \Big(e^o_{jky} \frac{\partial v_j^o}{\partial x_k} + e^o_{jky} \frac{\partial v_j^1}{\partial y_k} +$$

$$+ e^o_{jkx} \frac{\partial v_j^o}{\partial y_k} + e^1_{jky} \frac{\partial v_j^o}{\partial y_k} \Big) + \ldots \Big] + \varepsilon^p \Big\{ \varepsilon^{-2} \operatorname{div}_y (\lambda_f \operatorname{grad}_y T^o) +$$

$$+\varepsilon^{-1}\left[\mathrm{div}_x(\lambda_f \mathrm{grad}_y T^o)+\mathrm{div}_y(\lambda_f \mathrm{grad}_x T^o)+\mathrm{div}_y(\lambda_f \mathrm{grad}_y T^1)\right]+$$

$$+\mathrm{div}_x(\lambda_f \mathrm{grad}_x T^o)+\mathrm{div}_x(\lambda_f \mathrm{grad}_y T^1)+\mathrm{div}_y(\lambda_f \mathrm{grad}_x T^1)+$$

$$+\mathrm{div}_y(\lambda_f \mathrm{grad}_y T^2)+\ldots \Big\} \tag{2.6}$$

$$0=\varepsilon^{p-2}\mathrm{div}_y(\lambda_s \mathrm{grad}_y T^o)+\varepsilon^{p-1}\left[\mathrm{div}_x(\lambda_s \mathrm{grad}_y T^o)+\right.$$

$$\left.+\mathrm{div}_y(\lambda_s \mathrm{grad}_x T^o)+\mathrm{div}_y(\lambda_s \mathrm{grad}_y T^1)\right]+\varepsilon^p\left[\mathrm{div}_x(\lambda_s \mathrm{grad}_x T^o)\right.$$

$$\left.+\mathrm{div}_x(\lambda_s \mathrm{grad}_y T^1)+\mathrm{div}_y(\lambda_s \mathrm{grad}_x T^1)+\mathrm{div}_y(\lambda_s \mathrm{grad}_y T^2)\right]+\ldots \tag{2.7}$$

where:

$$e^{\ell}_{jky}=\frac{\partial v^{\ell}_j}{\partial y_k}+\frac{\partial v^{\ell}_k}{\partial y_j}\ ;\ e^{\ell}_{jkx}=\frac{\partial v^{\ell}_j}{\partial x_k}+\frac{\partial v^{\ell}_k}{\partial x_j}$$

We shall see that, as it usually happens in homogeneization problems, T^o does not depend on y . From (2.6) it is clear that the convective terms are significants if p=n. Moreover from the Darcy's law we have n+m=2, and equations (2.6) and (2.7) give at order ε^{n-2}:

$$\frac{\partial}{\partial y_i}\left(\lambda_{ij}(y)\frac{\partial T^o}{\partial y_j}\right)=0 \tag{2.8}$$

where λ take the values λ_s, λ_f in Y_s and Y_f respectively. Moreover, from (2.1) and (2.3) this equation holds in the hole Y in the sense of distributions and from the Y-periodicity we obtain $T^o = T^o(x)$.

Now, in the same way at order ε^{n-1} we obtain:

$$\frac{\partial}{\partial y_i}\left[\lambda_{ij}(y)\left(\frac{\partial T^o}{\partial x_j} + \frac{\partial T^1}{\partial y_j}\right)\right] = 0 \tag{2.9}$$

or

$$-\frac{\partial}{\partial y_i}\left(\lambda_{ij}(y)\frac{\partial T^1}{\partial y_j}\right) = \frac{\partial T^o}{\partial x_j}\frac{\partial \lambda_{ij}}{\partial y_i} \tag{2.9'}$$

This is the classical equation in homogeneization theory (see Bensoussan, Lions, Papanicolau [1], Sanchez-Palencia [2]) and they give us:

$$\left[\lambda_{ij}(y)\left(\frac{\partial T^o}{\partial x_j} + \frac{\partial T^1}{\partial y_j}\right)\right]^{\sim} = \lambda^h_{ij}\frac{\partial T^o}{\partial x_j} \tag{2.10}$$

$$\lambda^h_{ij} = \left[\lambda_{ij}(y) + \lambda_{ik}(y)\frac{\partial \theta^j}{\partial y_k}\right]^{\sim} \tag{2.11}$$

$$T^1(x,y) = \theta^j(y)\frac{\partial T^o}{\partial x_j} + c(x) \tag{2.12}$$

where θ^j is the solution of the problem:

$$\left\{\begin{array}{l} \text{Find } \theta^j \in H^1_{per}(Y) \text{ with } \tilde{\theta}^j = 0 \text{ satisfying} \\ \displaystyle\int_Y \lambda_{ik}\frac{\partial \theta^j}{\partial y_k}\frac{\partial \varphi}{\partial y_i}dy = -\int_Y \lambda_{ik}\frac{\partial \varphi}{\partial y_k}dy \;,\; (\forall)\, \varphi \in H^1_{per}(Y) \end{array}\right. \tag{2.13}$$

At order ε^n, the equations (2.6) and (2.7) with the boundary conditions (2.4) and (2.5) and the Y-periodicity give:

$$\rho_f c_f \left(v_k^o \frac{\partial T^o}{\partial x_k} + v_k^o \frac{\partial T^1}{\partial y_k}\right) = \mu e^o_{jky} \frac{\partial v_j^o}{\partial y_k} + \tag{2.14}$$

$$+ \frac{\partial}{\partial x_i}\left[\lambda_{ij}(y)\left(\frac{\partial T^o}{\partial x_j} + \frac{\partial T^1}{\partial y_j}\right)\right] + \frac{\partial}{\partial y_i}\left[\lambda_{ij}(y)\left(\frac{\partial T^1}{\partial x_j} + \frac{\partial T^2}{\partial y_j}\right)\right]$$

in Y, where we admit that v_k^o take the value 0 on Y_f. If we take the mean value of the equation (2.14) we have succesively (equations (1.12), (1.13), (1.16), (1.17) and (2.12) are used):

$$\int_Y \frac{\partial}{\partial y_i}\left[\lambda_{ij}(y)\left(\frac{\partial T^1}{\partial x_j} + \frac{\partial T^2}{\partial y_j}\right)\right]dy = \int_{\partial Y} n_i\left[\lambda_{ij}(y)\left(\frac{\partial T^1}{\partial x_j} + \frac{\partial T^2}{\partial y_j}\right)\right]ds = 0$$

$$\left\{\frac{\partial}{\partial x_i}\left[\lambda_{ij}(y)\left(\frac{\partial T^o}{\partial x_j} + \frac{\partial T^1}{\partial y_j}\right)\right]\right\}^{\sim} = \frac{\partial}{\partial x_i}\left(\lambda^h_{ij}\frac{\partial T^o}{\partial x_j}\right)$$

$$\left(\rho_f c_f v_k^o \frac{\partial T^o}{\partial x_k}\right)^{\sim} = \rho_f c_f \, \delta_{js} \tilde{v}_s^o \frac{\partial T^o}{\partial x_j}$$

$$\left(\rho_f c_f v_k^o \frac{\partial T^1}{\partial y_k}\right)^{\sim} = \rho_f c_f \mu \, \alpha_{ij} (K^{-1})_{si} \tilde{v}_s^o \frac{\partial T^o}{\partial x_j}$$

$$\alpha_{ij} = \left(w_k^i \frac{\partial \theta^j}{\partial y_k}\right)^{\sim} \tag{2.15}$$

$$\left(\mu e^o_{jky} \frac{\partial v_j^o}{\partial y_k}\right)^{\sim} = \frac{\mu}{|Y|}\left[\int_Y \frac{\partial v_j^o}{\partial y_k}\frac{\partial v_j^o}{\partial y_k}dy + \int_Y \frac{\partial v_k^o}{\partial y_j}\frac{\partial v_j^o}{\partial y_k}dy\right] =$$

$$= \frac{\mu}{|Y|}\left[\int_Y \frac{\partial v_j^o}{\partial y_k}\frac{\partial v_j^o}{\partial y_k}dy + \int_Y \frac{\partial}{\partial y_j}\left(v_k^o \frac{\partial v_j^o}{\partial y_k}\right)dy\right] =$$

$$= \frac{\mu}{|Y|} \left[\int_Y \frac{\partial v_j^o}{\partial y_k} \frac{\partial v_j^o}{\partial y_k} \, dy + \int_{\partial Y} n_j v_k^o \frac{\partial v_j^o}{\partial y_k} \, ds \right] =$$

$$= \frac{\mu}{|Y|} \int_Y \frac{\partial v_j^o}{\partial y_k} \frac{\partial v_j^o}{\partial y_k} \, dy = \mu \, (K^{-1})_{ji} \tilde{v}_i^o \tilde{v}_j^o$$

Then, <u>the macroscopic energy equation is:</u>

$$\rho_f c_f \left[\delta_{sj} + \mu \alpha_{ij} (K^{-1})_{si} \right] \tilde{v}_s^o \frac{\partial T^o}{\partial x_j} = \mu \, (K^{-1})_{ji} \tilde{v}_i^o \tilde{v}_j^o + \frac{\partial}{\partial x_i} \left(\lambda^h_{ij} \frac{\partial T^o}{\partial x_j} \right) \qquad (2.16)$$

<u>Remark 2.1</u>. The first term in the right hand side of the equation (2.16) is the viscous dissipation. In the problem of thermal combustion in porous media C.Marle [5] gives a similar term in the energy equation.

<u>Remark 2.2</u>. The corrective term α_{ij} (2.15) in the convective coefficient seems new. This term gives the influence of the difference of thermal conductivity $\lambda_f \neq \lambda_s$. If $\lambda_f = \lambda_s$ the homogeneization of the temperature is trivial and we have $T^1 = 0$, $\theta^j = 0$ and consequently $\alpha_{ij} = 0$.

3. <u>NATURAL CONVECTION</u>

3.1. <u>Darcy's law</u>

In order to obtain the Darcy's law used in the study of natural convection in porous media, we consider the equations of motion of a slightly compressible viscous fluid in the form:

$$\frac{\partial}{\partial x_i}(\rho^\varepsilon v_i^\varepsilon)=0 \tag{3.1}$$

$$\rho^\varepsilon v_k^\varepsilon \frac{\partial v_i^\varepsilon}{\partial x_k} = -\frac{\partial p^\varepsilon}{\partial x_i} + \mu' \frac{\partial^2 v_i^\varepsilon}{\partial x_k \partial x_k} + (\rho^\varepsilon - \rho_o) g \delta_{i3} \tag{3.2}$$

$$\rho^\varepsilon = \rho_o(1-\alpha' T^\varepsilon), \qquad P^\varepsilon = p^\varepsilon + \rho_o g x_3 \tag{3.3}$$

$$\rho^\varepsilon C_f v_k^\varepsilon \frac{\partial T^\varepsilon}{\partial x_k} - \frac{P}{\rho^\varepsilon} v_k^\varepsilon \frac{\partial \rho^\varepsilon}{\partial x_k} = \tau_{jk}^\varepsilon \frac{\partial v_j}{\partial x_k} + \frac{\partial}{\partial x_k}\left(\lambda'_f \frac{\partial T^\varepsilon}{\partial x_k}\right) \tag{3.4}$$

where P^ε is the pressure and p^ε is the difference between P^ε and the Archimede's pressure for the reference temperature. Moreover, the coefficients are $\mu' = \mu\varepsilon^m$, $\lambda' = \lambda\varepsilon^p$ and $\alpha' = \alpha\varepsilon^r$ $(0<r<1)$.

Remark 3.1. The state equation $(3.3)_1$ shows that temperature, but not pressure, is taken into account for density. In addition, the compressibility is small. Equations (3.2), (3.3) and $0<r<1$ amounts to the Boussinesq's approximation.

We now consider expansions (1.9), (1.11) for the velocity and the temperature; oppositely, according to the Boussinesq's approximation, we take for pressure (instead of (1.10)):

$$p^\varepsilon(x) = \varepsilon^r p_o^o(x,y) + \varepsilon^{r+1} p^1(x,y) + \ldots \tag{3.5}$$

From (3.3) we have:

$$\rho^\varepsilon = \rho_o(1 - \alpha\varepsilon^r T^o - \alpha\varepsilon^{r+1} T^1 \ \ldots) \tag{3.6}$$

As in the section 1.3, we are obliged to take n+m-2=r and from (3.1) (3.2) we have the Darcy's law:

$$\tilde{v}_i^o = -\frac{K_{ij}}{\mu}\left(\frac{\partial p^o}{\partial x_j} + \alpha \rho_o T^o g \delta_{i3}\right) \tag{3.7}$$

$$\mathrm{div}_x \tilde{\underline{v}}^o = 0 \tag{3.8}$$

3.2. The energy equation

In the equation (3.4) the untrivial convective terms are obtained for n=p. Then at order ε^{n-2} and ε^{n-1} we obtain $T^o = T^o(x)$ and (2.12) with the homogeneized coefficient (2.11). But the terms of viscosity and compressibility are of order ε^{n+r} and are negligible with respect to convective terms (order ε^n). Consequenctly, instead of (2.16), we obtain:

$$\rho_f C_f \left[\delta_{sj} + \mu \alpha_{ij} (K^{-1})_{si}\right] \tilde{v}_s^o \frac{\partial T^o}{\partial x_j} = \frac{\partial}{\partial x_i}\left(\lambda_{ij}^h \frac{\partial T^o}{\partial x_j}\right) \tag{3.9}$$

where the coefficients α_{ij} are given by (2.15).

Remark 3.2. The system (3.7)-(3.9) is the classical system of equations for natural convection in porous media (see Ene and Gogonea [6]).

4. NON-DIMENSIONAL NUMBERS

4.1. The general energy equation

In order to obtain the physical meaning of the news terms which apears in equation (2.16) we take a characteristic lenght ℓ of the pores, a characteristic length L of the domain Ω and a characteristic velocity Q of the filtration velocity $\underset{\sim}{v}^{o}$. Now, the small parameter is $\varepsilon = \frac{\ell}{L}$.

If we introduce the Reynolds number R_{ε}, the Prandtl number P and a new non-dimensional number S_{ε} defined by:

$$R_{\varepsilon} = \frac{Q \rho \ell}{\mu} \tag{4.1}$$

$$P = \frac{\mu c}{\lambda} \tag{4.2}$$

$$S_{\varepsilon} = \frac{\mu Q^2}{\lambda T} \tag{4.3}$$

where T is the difference between the temperature and the reference temperature, equation (2.16) makes sense for:

$$P R_{\varepsilon} \sim \varepsilon^{-1} S_{\varepsilon} \tag{4.4}$$

If P $\sim$ 0(1) (like for usual fluids), (4.4) show that:

$$\frac{\lambda \rho \ell^2 T}{\mu^2 L Q} \sim 0(1) \tag{4.5}$$

This is the condition for taking into acount the dissipative term in (2.16). On the other hand, the corrective term in α_{ij} is always of the some order than the classical convective term (see, nevertheless, Remark 2.2).

4.2. Natural convection

In this case it is necessary to take also into acount the Grashof number G_{ε} and the Rayleigh number Ra_{ε}, defined by:

$$G_{\varepsilon} = \frac{g\alpha \rho_o^2 \ell^3 T}{\mu^2} \tag{4.6}$$

$$Ra_{\varepsilon} = G_{\varepsilon} \cdot P \tag{4.7}$$

Then from (3.7) we have:

$$P \cdot R_{\varepsilon} \sim \varepsilon \tag{4.8}$$

and from (3.9):

$$Ra_{\varepsilon} \sim \varepsilon \tag{4.9}$$

Remark 4.1. The Rayleigh number (4.7) is defined at the microscopic level (with ℓ^3). In the study of natural convection in porous media the usual Rayleigh number is defined with two-scales ($\ell^2 L$ instead of ℓ^3):

$$R_a = \frac{\rho \alpha g \ell^2 TL}{\mu \chi} \quad ; \quad \chi = \frac{\lambda}{\rho c} \tag{4.10}$$

From (4.10) and (4.9) it is clear that this number is of order 1. This fact is in good concordance with experimental data.

5. COMPLEMENTS

5.1. Non-steady flow

All considerations concerning the Darcy's law holds for the non-steady case, using a slow scale of time $\tau = \varepsilon^n t$.

In equations (2.16) or (3.9) it apears a new term of the form:

$$(\rho c)^{\sim} \frac{\partial T^{o}}{\partial \tau}$$

or

$$\left[m \rho_f C_f + (1-m) \rho_s C_s \right] \frac{\partial T^{o}}{\partial \tau}$$

where m is the porosity of the medium defined by $m = \frac{|Y_f|}{|Y|}$.

5.2. Diffusion of miscible fluids

It is known that the concentration c of a mixture of miscible fluids satisfies the equation:

$$\frac{\partial c^{\varepsilon}}{\partial t} + v_i^{\varepsilon} \frac{\partial c^{\varepsilon}}{\partial x_i} = \frac{\partial}{\partial x_i}\left(D \frac{\partial c^{\varepsilon}}{\partial x_j}\right) \tag{5.1}$$

where D is the diffusion coefficient. The equation (5.1) coincides with (1.3) if we take $\mu = 0$, $\rho_f C_f = 1$ and $\lambda'_f = D$. It is also clear that if the mixture flows in a solid porous body, this one is impervious and consequently the appropiate boundary condition at Γ is $\frac{\partial c^{\varepsilon}}{\partial n} = 0$. This also amounts to say that the diffusion coefficient in the solid is zero. The homogeneized equation takes a form analogous to (2.16). It is to be noticed that in this case the diffusion coefficient takes necessarly different values in Y_s and Y_f (because $D_s = 0$) and consequently the coefficients analogous to α_{ij} are in general different from zero (see Remark 2.2).

REFERENCES

1 A.BENSOUSSAN, J.L.LIONS and G.PAPANICOLAU, Asymptotic Analysis for Periodic Structures, North-Holland, Amsterdam (1978).

2 E.SANCHEZ-PALENCIA, Topics in non-Homogenous Media and Vibration Theory, Lecture Notes in Physics 127, Springer-Berlin (1980).

3 H.I.ENE and E.SANCHEZ-PALENCIA, Int.J.Enging.Sci., 19, 117 (1981).

4 H.I.ENE and E.SANCHEZ-PALENCIA, J.Mécan.14, 73 (1975).

5 C.MARLE, Int.J.Enging.Sci.(to apear)

6 H.I.ENE and S.GOGONEA, Probleme în teoria filtraţiei, Ed.Academiei, Bucharest (1973).