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TOPOLOGICAL DEGREE FOR SOLVABLE MAPPINGS

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1. INTRODUCTION

In this paper we extend the notion of topological degree introduced by Ship-Fah Wong in such a way that it can be applied to approximation schemes on non-separable topological spaces. For the sake of simplicity we don't build the whole background analogous to that which is used in the study of A-proper mappings, but only survey the results from the theory of mappings defined between the spaces of a dual pair. In this case our degree theory is suited for a class of mappings which includes the fa-continuous closed mappings, and therefore, the already studied, from this point of view, A-proper mappings. Besides the crucial properties, which are the existence theorem for non-zero degree, the additivity on the domain and the invariance under suitable homotopies, some continuity results are obtained. Moreover a few interesting applications are proved.

2. PRELIMINARIES

Let $\mathbb{F}$ be a non-empty directed set with respect to a relation $\leqslant$. For each $E \in \mathbb{F}$ the set

$$\mathbb{F}(E) = \left\{ F \in \mathbb{F} \,\middle|\, E \leqslant F \right\}$$

will be called the section of $\mathbb{F}$ relative to the element E. The set

$S(\mathbb{F})$ of the sections of $\mathbb{F}$, which is a filter base, generates a filter $\mathcal{F}(\mathbb{F})$, called the section filter of the directed set $\mathbb{F}$.

Let $\mathbb{Z}^{\mathbb{F}}$ be the ring of all (generalized) sequences of integers with coordinatewise addition and multiplication. Let the relation $\sim$ on $\mathbb{Z}^{\mathbb{F}}$ be defined in the following way:

$$\{x_F\}_{F\in\mathbb{F}} \sim \{y_F\}_{F\in\mathbb{F}} \quad \text{whenever } \{F\in\mathbb{F} \mid x_F = y_F\} \in \mathcal{F}(\mathbb{F}) .$$

As this equivalence relation is compatible with the ring structure of $\mathbb{Z}^{\mathbb{F}}$, the quotient set ${}^*\mathbb{Z} = \mathbb{Z}^{\mathbb{F}}/_{\sim}$ with the induced operations is a ring. It contains the subring of classes of constant sequences which is isomorphic to $\mathbb{Z}$, which will be, from now on, identified in this way.

A subset $\mathbb{E}$ of $\mathbb{F}$ is said to be a cofinal of $\mathbb{F}$ if $\mathbb{E}\cap\mathbb{F}(E)\neq\emptyset$ for any $E\in\mathbb{F}$. Clearly, $\mathbb{E}$ is also a non-empty directed set with respect to $\leqslant$. The set $\mathcal{C}(\mathbb{F})$ of the cofinals of $\mathbb{F}$ has the following properties.

(i) $\mathcal{F}(\mathbb{F}) \subseteq \mathcal{C}(\mathbb{F})$ and $\mathbb{F}\in\mathcal{C}(\mathbb{F})$

(ii) if $\mathbb{E}_1, \mathbb{E}_2 \in \mathcal{C}(\mathbb{F})$ then $\mathbb{E}_1 \cup \mathbb{E}_2 \in \mathcal{C}(\mathbb{F})$

(iii) if $\mathbb{E}_1 \subseteq \mathbb{E}_2 \subseteq \mathbb{F}$ and $\mathbb{E}_1 \in \mathcal{C}(\mathbb{F})$ then $\mathbb{E}_2 \in \mathcal{C}(\mathbb{F})$

(iv) $\mathbb{E}\in\mathcal{C}(\mathbb{F})$ if and only if $(\mathbb{F}\setminus\mathbb{E}) \notin \mathcal{F}(\mathbb{F})$

3. THE TOPOLOGICAL DEGREE

Let $\langle X,Y\rangle$ be a dual pair of two linear spaces. On X there is a locally convex and separated topology, compatible with the duality, and on Y the weak topology $\sigma(Y,X)$.

Let D be an open bounded subset of X, a mapping $g:\overline{D}\longrightarrow Y$ and a homotopy $G:\overline{D}\times[0,1]\longrightarrow Y$, where $\overline{D}$ is the closure of D and $\dot{D}$ will denote the boundary of D in X.

Let $\mathbb{F}$ be the non-empty directed set (with respect to the inclusion $\subseteq$) of the finite-dimensional sub-spaces F of X. Let denote with j_F the canonical injection of F in X, with j_F^* it's adjoint from Y on F^* (the dual of F), with $g_F=j_F^*\circ g\circ j_F: F\cap\bar{D}\to F$ and with $G_F=j_F^*\circ G(j_F(\cdot),\cdot):(F\cap\bar{D})\times[0,1]\to F$. As $\langle F,F^*\rangle$ is also a dual pair and the only locally convex and separated topology on F is the Euclidean one, we shall identify F^* with F, from now on.

Definition 1. a) The mapping g is called fa-continuous if $\{F\in\mathbb{F} \mid g_F \text{ is continuous}\}\in\mathcal{F}(\mathbb{F})$.

b) Let A be a subset of $\bar{D}$ and $y\in Y$; the mapping g is called solvable for (y,A) if $y\notin g(A)$ implies $\{F\in\mathbb{F} \mid j_F^* y\notin g_F(F\cap A)\}\in\mathcal{F}(\mathbb{F})$.

Remark 1. If $A,B\subseteq\bar{D}$, $y\in Y$ and g is solvable for (y,A) and (y,B) then it is solvable for $(y,A\cup B)$. If $A\subseteq B$, $y\notin g(B)$ and g is solvable for (y,B) then g is solvable for (y,A).

Definition 2. a) The homotopy G is called fa-continuous if $\{F\in\mathbb{F} \mid G_F \text{ is continuous}\}\in\mathcal{F}(\mathbb{F})$.

b) Let A be a subset of $\bar{D}$ and $y\in Y$; the homotopy G is called solvable for (y,A) if $y\notin G(A\times[0,1])$ implies $\{F\in\mathbb{F} \mid j_F^* y\notin G_F((F\cap A)\times[0,1])\}\in\mathcal{F}(\mathbb{F})$.

Remark 2. $\mathcal{F}(\mathbb{F})$ is not an ultrafilter, since $\mathbb{F}\in\mathcal{F}(\mathbb{F})$ but neither the subset of spaces of even dimension nor the subset of spaces of odd dimension belongs to $\mathcal{F}(\mathbb{F})$.

Definition 3. Let $y\in Y\setminus g(\dot{D})$; let g be fa-continuous and solvable for $(y,\dot{D})$. Then $\{F\in\mathbb{F} \mid j_F^* y\notin g_F(F\cap\dot{D}) \text{ and } g_F \text{ continuous}\}\in\mathcal{F}(\mathbb{F})$ and hence the sequence $\{\deg(g_F, F\cap\bar{D}, j_F^* y)\}_{F\in\mathbb{F}}$ (where $\deg(\cdot,\cdot,\cdot)$ denotes the Brouwer degree) determines an element of ${}^*\mathbb{Z}$ which we call the degree of g on D in y, denoted by $\mathrm{Deg}(g,D,y)$.

Remark 3. If $A \subseteq \bar{D}$, $y \in Y$ and g is solvable for (y,A) then $f:\bar{D} \to Y$ defined by $f(x)=g(x)-y$ is solvable for $(0,A)$; if g is fa-continuous then f is fa-continuous; hence if y and g are like in Definition 3 then $Deg(g,D,y)=Deg(f,D,0)$.

Theorem 1. Let $y \in Y \setminus g(\dot{D})$; let g be fa-continuous and solvable for $(y,\dot{D})$ and (y,D). If $Deg(g,D,y) \neq 0$ then $(\exists)$ $x \in D$ such that $g(x)=y$.

Proof. If $Deg(g,D,y) \neq 0$ then

$$\{F \in \mathbb{F} \mid deg(g_F, F \cap D, j_F^* y)=0\} \notin \mathcal{F}(F)$$

and from (iv) § 2 $\{F \in \mathbb{F} \mid deg(g_F, F \cap D, j_F^* y) \neq 0\} \in \mathcal{C}(F)$.

From (iii) § 2 follows $\{F \in \mathbb{F} \mid j_F^* y \in g_F(F \cap D)\} \in \mathcal{C}(F)$. We prove now the assertion by a reductio ad absurdum.

Suppose $y \notin g(D)$; then from the solvability of g for (y,D) and (iv) § 2 we get $\{F \in \mathbb{F} \mid j_F^* y \in g_F(F \cap D)\} \notin \mathcal{C}(F)$; contradiction.

Theorem 2. Let $D=D_1 \cup D_2$ where D_1, D_2 are two disjoint open bounded sets in X, $y \in Y \setminus g(\dot{D}_1 \cup \dot{D}_2)$. If g is fa-continuous and solvable for $(y,\dot{D}_1)$ and $(y,\dot{D}_2)$ then $Deg(g,D,y)=Deg(g,D_1,y)+Deg(g,D_2,y)$.

Proof. Clearly $y \notin f(\dot{D}_1)$, $y \notin f(\dot{D}_2)$ and from Remark 1 g is solvable for $(y,\dot{D}_1 \cup \dot{D}_2)$. Then the three degrees are defined and the theorem follow from the sum formula of the finite-dimensional case.

Theorem 3. Let $y \in Y \setminus G(\dot{D} \times [0,1])$; Let G be fa-continuous and solvable for $(y,\dot{D})$. Then $Deg(G(.,t),D,y)$ is independent of t.

Proof. It follows directly from the Definition 2 and the analogous property of the Brouwer degree.

Theorem 4. Let B be another open bounded set of X and $y \in Y$ such that $g^{-1}(y) \subsetneq B \subsetneq D$. Let g be fa-continuous and solvable for $(y, \dot{D})$ and $(y, X \setminus B)$. Then $\mathrm{Deg}(g,B,y)=\mathrm{Deg}(g,D,y)$.

Proof. Clearly $y \notin g(\dot{B})$ and $y \notin g(\dot{D})$; then taking in account the Remark 1 the two degrees are defined. Hence we have only to prove that $\{F \in \mathbb{F} \mid g_F^{-1}(j_F^* y) \subseteq F \cap B\} \in \mathcal{F}(F)$. Supposing that it is not true, we get $\{F \in \mathbb{F} \mid g_F^{-1}(j_F^* y) \not\subseteq F \cap B\} \in \mathcal{C}(F)$, that is $\mathbb{E} = \{F \in \mathbb{F} \mid g_F^{-1}(j_F^* y) \cap \cap (F \setminus B) \neq \emptyset\} \in \mathcal{C}(F)$. Following (iii) § 2 $\{F \in \mathbb{F} \mid j_F^* y \in g_F(F \setminus B)\} \in \mathcal{C}(F)$ as it includes $\mathbb{E}$, and finally $\{F \in \mathbb{F} \mid j_F^* y \notin g_F(F \cap (X \setminus B))\} \notin \mathcal{F}(F)$ which is in contradiction with $y \notin g(X \setminus B)$ as g is solvable for $(y, X \setminus B)$.

Theorem 5. Let $0 \in D$, D symmetric about 0 , $0 \in Y \setminus g(\dot{D})$, where g is fa-continuous and solvable for $(0, \dot{D})$.

If $\{F \in \mathbb{F} \mid g_F$ is odd on $F \cap \dot{D}\} \in \mathcal{F}(F)$ then $\mathrm{Deg}(g,D,0)$ is "odd", that is

$$\{F \in \mathbb{F} \mid \deg(g_F, F \cap \bar{D}, 0) \text{ is odd}\} \in \mathcal{F}(F) ,$$

and in particular $\mathrm{Deg}(g,D,0) \neq 0$ so that $0 \in g(D)$.

Proof. It is a straight consequence of the definitions and of the Borsuk theorem in the finite dimensional case.

Lemma 1. Let $\{g^\delta\}_{\delta \in \Delta}$ (Δ a directed set with $\leqslant$) be a family of fa-continuous mappings from $\bar{D}$ to Y. If $\left\{ \sup_{z\in\bar{D}} |< g^{\delta}(z), x >| \right\}_{\delta\in\Delta}$ is convergent to zero $(\forall) x\in X$, then

$$\left\{ F\in\mathbb{F} \;\middle|\; \sup_{z\in F\cap\bar{D}} |< g_F^{\delta}(z), x >| \xrightarrow[\delta\in\Delta]{} 0 \quad (\forall)\, x\in F \right\} \in \mathcal{F}(\mathbb{F}).$$

Proof. We shall also prove this assertion by a reductio ad absurdum. It follows that $(\exists)\mathbb{E}\in\mathcal{C}(\mathbb{F})$ such that $(\forall) F\in\mathbb{E}$ $(\exists)$ $x_F\in F$ with $\sup_{z\in F\cap\bar{D}} |< g_F^{\delta}(z), x_F >| \not\xrightarrow[\delta\in\Delta]{} 0$

Hence $(\exists)\, C_F>0$ and $\Delta_F\in\mathcal{C}(\Delta)$ such that for any $\delta\in\Delta_F$ $\sup_{z\in F\cap\bar{D}} |< g_F^{\delta}(z), x_F >| \geq C_F$. As g_F^{δ} is uniformly continuous on $F\cap\bar{D}$, $(\forall)\,\delta\in\Delta_F$ $(\exists)$ $z_\delta\in F\cap\bar{D}$ for which $|< g_F^{\delta}(z_\delta), x_F >| \geq C_F$; together with

$$< g_F^{\delta}(z_\delta), x_F > = < g^{\delta}(j_F z_\delta), j_F x_F > = < g^{\delta}(z_\delta), x_F >$$

it implies $\sup_{z\in\bar{D}} |< g^{\delta}(z), x_F >| \not\xrightarrow[\delta\in\Delta]{} 0$ which is in contradiction with the hypothesis.

Theorem 6. Let $y\in Y\setminus g(\dot{D})$; Let g be fa-continous and solvable for $(y,\dot{D})$; Let $\{g^{\delta}\}_{\delta\in\Delta}$ be a family of fa-continuous mappings, which is uniformly convergent to g. Then $(\exists)\,\delta_0\in\Delta$ such that $(\forall)\,\delta\geq\delta_0$ g^{δ} is $(y,\dot{D})$-solvable and $\mathrm{Deg}(g,D,y)=\mathrm{Deg}(g^{\delta},D,y)$.

Proof. Let's notice that the family $\{g^{\delta}-g\}_{\delta\in\Delta}$ satisfy the conditions of Lemma 1. Thus

$$\left\{ F\in\mathbb{F} \;\middle|\; \sup_{z\in F\cap\bar{D}} |< g_F^{\delta}(z)-g(z), x >| \xrightarrow[\delta\in\Delta]{} 0 \quad (\forall)\, x\in F \right\} \in \mathcal{F}(\mathbb{F}).$$

It follows from the continuity property of the Brouwer degree $\deg(\cdot, F\cap D, j_F^* y)$ with respect to the uniform topology of the space of continuous mappings $C(F\cap\bar{D},F)$ that $(\exists)\,\delta_0\in\Delta$ such that $(\forall)\,\delta\geq\delta_0$ we have

$$\left\{ F\in\mathbb{F} \;\middle|\; j_F^* y \notin g_F^{\delta}(F\cap\dot{D}) \right\} \in \mathcal{F}(\mathbb{F})$$

(which is in fact more than the $(y,\dot{D})$-solvability of g^{δ})

and $\left\{ F\in\mathbb{F} \;\middle|\; \deg(g_F^{\delta}, F\cap D, j_F^* y) = \deg(g_F, F\cap D, j_F^* y) \right\} \in \mathcal{F}(\mathbb{F})$,

which completes the proof.

4. CLOSED MAPPINGS

In this section we start with some Lemmas which will prove that the general degree theory developped in §3 is suited for the class of fa-continuous closed mappings.

Lemma 2. If $A \subseteq \overline{D}$, $y \in Y \setminus g(A)$ and $\mathbb{E} = \{F \in \mathbb{F} \mid j_F^* y \in g_F(F \cap A)\} \in \mathcal{C}(\mathbb{F})$ then $y \in (g(A))'$.

Proof. Let $x \in X$ and $E \in \mathbb{F}$ with $E \ni x$. Then for any F with $F \in \mathbb{E}$ we have

$$\langle g(x_F) - y, x \rangle = \langle g(x_F) - y, j_F x \rangle = \langle j_F^* \circ g(x_F) - j_F^* y, x \rangle = 0$$

for some special choices of $x_F \in F \cap A$. That is $g(x_F) \longrightarrow y$ and as $g(x_F) \neq y$ the result follows.

Lemma 3. Let g be closed. Then g is solvable for (y, A), $(\forall) y \in Y$ and $(\forall) A$ closed subset of $\overline{D}$.

Proof. Let $y \in Y \setminus g(A)$ and suppose $\{F \in \mathbb{F} \mid j_F^* y \notin g_F(F \cap A)\} \notin \mathcal{F}(\mathbb{F})$. Using (iv)§2 we notice that we are in the conditions of Lemma 2; then $y \in (g(A))'$. As g is closed it results that $y \in g(A)$; contradiction.

Lemma 4. Let g be closed. Then g is solvable for (y, A), $(\forall) y \in Y \setminus g(\dot{A})$ and $(\forall)$ A open subset of $\overline{D}$.

Proof. Let $y \in Y \setminus g(A)$; then $y \in Y \setminus g(\overline{A})$ and the result follow using Lemma 3 and Remark 1.

Lemma 5. If the homotopy G has the following properties:

1° $G(\cdot,t)$ is closed $(\forall)$ $t\in[0,1]$

2° $G(x,\cdot)$ is uniformly continuous $(\forall)$ $x\in A$

then G is (y,A)-solvable $(\forall)$ $y\in Y$ and $(\forall) A$ closed in $\overline{D}$.

Proof. Let $y\in Y\setminus G(A\times[0,1])$. Suppose that $(\exists)\, t_o\in[0,1]$ such that $(\forall) n\in\mathbb{N}$ $\quad \mathbb{E}_n=\{F\in\mathbb{F} \mid j_F^* y\in G_F((F\cap A)\times V_n)\}\in\mathscr{C}(\mathbb{F})$, where $V_n=(t_o-\frac{1}{n}, t_o+\frac{1}{n})\cap[0,1]$. That is $(\exists)\, t_o\in[0,1]$ such that $(\forall)$ $n\in\mathbb{N}$ $(\exists)$ $\mathbb{E}_n\in\mathscr{C}(\mathbb{F})$ with the property:
$(\forall)$ $F\in\mathbb{E}_n$ $\quad(\exists)(x_F^n, t_F^n)\in(F\cap A)\times V_n$ $\quad$ for which $j_F^* y=G_F(x_F^n,t_F^n)=j_F^*\circ G(x_F^n,t_F^n)$. From 2° follows that $(\forall) x\in X$ $(\forall) a\in A$ and $(\forall)\,\varepsilon>0$, $(\exists) N(\varepsilon,x)\in\mathbb{N}$ such that for any $n\geq N(\varepsilon,x)$ we have

$$|\langle G(a,t)-G(a,t_o), x\rangle|<\varepsilon \qquad (\forall)\, t\in V_n$$

Particularly $(\forall) n\geq N(\varepsilon,x)$ and $(\forall) F\in\mathbb{E}_n$ we have

$$|\langle G(x_F^n,t_F^n)-G(x_F^n,t_o), x\rangle|<\varepsilon$$

. Let be $E\in\mathbb{F}$ with $E\ni x$; then for $(\forall) F\in\mathbb{E}_n$ with $F\supseteq E$ it follows:

$$|\langle y-G(x_F^n,t_o), x\rangle| = |\langle y-G(x_F^n,t_o), j_F x\rangle| =$$
$$=|\langle j_F^* y-j_F^*\circ G(x_F^n,t_o), x\rangle| = |\langle j_F^*\circ G(x_F^n,t_F^n)-j_F^*\circ G(x_F^n,t_o), x\rangle| =$$
$$=|\langle G(x_F^n,t_F^n)-G(x_F^n,t_o), j_F x\rangle| = |\langle G(x_F^n,t_F^n)-G(x_F^n,t_o), x\rangle|<\varepsilon$$

Then $y\in(G(A,t_o))'$. As A is closed and $G(\cdot,t_o)$ is closed it results $y\in G(A,t_o)$. But this is not true. Hence $(\forall)$ $t\in[0,1]$ $(\exists) V_t$ a neighbourhood of t open in $[0,1]$ such that

$$\mathbb{E}_t=\{F\in\mathbb{F} \mid j_F^* y\notin G_F((F\cap A)\times V_t)\}\in\mathscr{F}(\mathbb{F})$$

As the family $\{V_t\}_{t\in[0,1]}$ is an open covering of $[0,1]$, we can extract a finite subcovering $\{V_{t_i} \mid i=1,2,\dots k\}$. Then

$$\bigcap_{i=1}^{k}\mathbb{E}_{t_i}\subseteq\{F\in\mathbb{F} \mid j_F^* y\in G_F((F\cap A)\times[0,1])\}$$

and as $\mathscr{F}(\mathbb{F})$ is filter it results that G is (y,A)-solvable.

We finish this section with two interesting applications:

Theorem 7. Let y_1, y_2 be two points of the same component (i.e. a maximal linear connected open subset) of the open subset $Y\setminus g(\mathring{D})$, where g is a fa-continuous and closed.

Then $\mathrm{Deg}(g,D,y_1)=\mathrm{Deg}(g,D,y_2)$.

Proof. We define the homotopy $H(x,t)=g(x)-[ty_1+(1-t)y_2]$ which clearly satisfy the conditions of Lemma 5. Then H is solvable for $(0,\dot{D})$. As $0\in Y\setminus H(\dot{D}\times[0,1])$ and H is fa-continuous it follows from Theorem 3 that $\mathrm{Deg}(H(\ ,t),D,0)$ is independent of t. Particularly, $\mathrm{Deg}(H(\cdot,1),D,0)=\mathrm{Deg}(H(\cdot,0),D,0)$ and the theorem is proved taking in account Remark 3.

Theorem 8 (Schauder). Let X be a Hilbert space, $D=\{x\in X \mid \|x\|<1\}$ If $g:\bar{D}\to\bar{D}$ is a weakly-continuous mapping then g has a fixed point in $\bar{D}$.

Proof. We can suppose that $g(x)\neq x$ $(\forall)x\in\bar{D}$. If we define $H(x,t)=x-tg(x)$ $(\forall)(x,t)\in\bar{D}\times[0,1]$ then for $x\in\dot{D}=\{x\in X \mid \|x\|=1\}$ we have $\|H(x,1)\|=\|x-g(x)\|\neq 0$ and $(\forall)$ $t\in[0,1)$ $\|H(x,t)\|\geq$ $\geq\|x\|-t\|g(x)\|\geq 1-t>0$; hence $0\notin H(\dot{D}\times[0,1])$. Clearly H is fa-continuous, then in order to apply Theorem 3 we have to prove that H is solvable for $(0,\dot{D})$. We shall treat separately the cases $t\in[0,1)$ and $t=1$. Suppose that $(\exists)$ $t_0\in[0,1)$ such that $\{F\in\mathbb{F} \mid j_F^*0\notin H_F(F\cap\dot{D},t_0)\}\notin\mathcal{F}(\mathbb{F})$, that is $(\exists)$ $\mathbb{E}\in\mathcal{C}(\mathbb{F})$ such that $(\forall)$ $F\in\mathbb{E}$ $(\exists)x_F\in F\cap\dot{D}$ with $j_F^*x_F=t_0g_F(x_F)$. It follows

$$0=\langle j_F^*(x_F-t_0g(x_F)),x_F\rangle=\|x_F\|^2-t_0\langle g(x_F),x_F\rangle\geq 1-t_0\quad;$$

contradiction. Now, suppose that $(\exists)$ $\mathbb{E}\in\mathcal{C}(\mathbb{F})$ such that $(\forall)F\in\mathbb{E}$ $(\exists)x_F\in F\cap\dot{D}$ with $j_F^*x_F=g_F(x_F)$. Let $x\in X$ and $E\in\mathbb{F}$ with $E\ni x$. Then $(\forall)F\in\mathbb{E}$ with $F\supseteq E$: $\langle x_F-g(x_F),x\rangle=\langle j_F^*(x_F-g(x_F)),x\rangle=0$ hence $x_F-g(x_F)\xrightarrow[F\in\mathbb{E}]{}0$. As $\{x_F\}_{F\in\mathbb{E}}\subseteq\dot{D}$ and $\bar{D}$ is weakly compact $(\exists)x_0\in\bar{D}$ and $(\exists)$ $\mathbb{E}'\in\mathcal{C}(\mathbb{E})$ such that $x_F\xrightarrow[F\in\mathbb{E}']{}x_0$. Taking in account the weak continuity of g it follows $g(x_0)=x_0$; contradiction. Then from Theorem 3 it results $\mathrm{Deg}((H(\cdot,1),D,0)=\mathrm{Deg}(H(\cdot,0),D,0)$ $=\mathrm{Deg}(I_X,D,0)=1\neq 0$ as $\{F\in\mathbb{F} \mid deg(I_F,F\cap D,0)=1\}\in\mathcal{F}(\mathbb{F})$. Because we can prove that $H(\cdot,1)$ is solvable for $(0,D)$ exactly as for the pair $(0,\check{D})$, from Theorem 1 results that $(\exists)x\in D$ such that $H(x,1)=0$; contradiction.

R E F E R E N C E S

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