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Stefan PAPADIMA

INTRODUCTION

Sullivan's results [13] (see also [1]) showed that the problem of the rational homotopy classification of closed manifolds is of a purely algebraic nature. In more detail, the three steps of the classification may be described as follows.

Starting with the rational cohomology algebra, one must have first a way of classifying Poincaré duality algebras over Q. This step was carried out in [9].

The next step is to detect those Poincaré duality algebras which come from manifolds. Here one may use the following more general algebraic test of "rational surgery", which we shall briefly recall, following [1]. Let X be a 1-connected Q-local space with the property that $H^*(X;Q)$ is a Poincaré duality algebra (P.d.a.) of formal dimension n. Given $\mu \in H_n(X;Q) \setminus \{0\}$ and $p=\sum p_i \in \oplus H^{4i}(X;Q)$, one may always find, if $n \neq 0$ (mod 4) a closed manifold M^n and a map $f:M \longrightarrow X$ with the property:

$$(*) \qquad \begin{array}{c} f \text{ induces a rational homology isomorphism and:} \\ f_*[M]=\mu , \quad f^*(p)=p_M \end{array}$$

If $n=4k$ ($k \neq 1$) then the existence of M and f with the above property is equivalent to the following set of "differentiability conditions":

(D1) the numbers $\{\langle p^{\omega}, \mu \rangle \mid \omega$ a partition of $k\}$ are the Pontrjagin numbers of a closed manifold ;

(D2) the quadratic form $H^{2k}(X;Q) \otimes H^{2k}(X;Q) \longrightarrow Q$ given by the Poincaré duality is a sum of squares over Q ;

(D3) $\sigma(H^{2k}(X;Q),\mu) = \langle L_k(p), \mu \rangle$

(if $\omega = (\omega_1, \ldots, \omega_r)$, p^{ω} stands for the monomial $p_{\omega_1} \ldots p_{\omega_r}$, σ denotes the signature and L_k is the Hirzebruch polynomial).

If H is a P.d.a. one may easily derive necessary and sufficient conditions for H in order to have: $H=H^*(M;Q)$, M a closed manifold, by appropriately reformulating the conditions (D1)-(D3) (see § 1).

At the last step, there is a general theory of the algebraic description of the rational homotopy types (not necessarily manifolds) with a given (arbitrary) cohomology algebra, in terms of minimal models of the Sullivan or Quillen type ([3] and [13], [2] and [11]). As the conditions (D1)-(D3) are involving only the cohomology algebra, every space whose cohomology algebra comes from manifolds is itself of the rational homotopy type of a manifold and this remark completes the solution of the classification problem stated at the beginning.

The aim of this paper is to follow the above program adding some simplifying hypotheses. In order to avoid the complications arising in general at the third step, we shall restrict ourselves to the intrinsic formal cases. A graded algebra H over Q is said to be <u>intrinsically formal</u> (see [3]) if there is exactly one Q-homotopy type X such that $H^*X=H$. For example, it is shown in [5] that a (k-1)-connected P.d.a. of formal dimension less than or equal to 4k-2 is intrinsically formal.

As far as the first step is concerned, there is a certain case when the general classification of [9] takes a particularly neat form, namely the case of <u>homogenously generated</u> algebras H, defined by the property that they are generated as algebras by some homogenous component H^d (see [7] for a geometric interpretation of this condition).

To such a P.d.a. one can associate two more numerical invariants: the rank r defined by: $r=\dim H^d$ and the cuplength c defined by: $\underbrace{H^+ \ldots H^+}_{c} \neq 0$ but $\underbrace{H^+ \ldots H^+}_{c+1}=0$. Denoting by $H^c_{d,reg}(r)$ those degree c homogenous forms q in the variables $x_1,\ldots,x_r$, polynomial for d even and exterior for d odd, with rational coefficients, which have the property that the forms $\frac{\partial q}{\partial x_1},\ldots,\frac{\partial q}{\partial x_r}$ are linearly independent, we constructed for any such q a P.d.a. H_q having d, r and c as invariants and we showed that every homogenously generated P.d.a. arises in this way and, moreover, that the algebras H_{q_1} and H_{q_2} are isomorphic if and only if the forms q_1 and q_2 are linearly equivalent, see [9].

Given the algebra H_q, one may construct a Q-homotopy type X_q, uniquely determined by the property that X_q is formal and $H^*X_q=H_q$ ([13]). When speaking of <u>the rational homotopy type</u> of X we shall always assume that X is <u>homologically 1-connected</u> i.e. $\bar{H}_i(X;Q)=0$ for $i\leq 1$. We shall also assume from now on that all graded algebras H are <u>1-connected</u> i.e. $H^0=Q$ and $H^1=0$ (when H is homogenously generated this is equivalent to $d\geq 2$). With these specifications, it is worth noting that, starting with q, X_q may be explicitely described, either by constructing its Sullivan model (as in [3]) or its Quillen model (as in [6]).

In the homogenously generated case it follows immediately from the result quoted from [5] that any P.d.a. having $c\leq 3$ is intrinsically formal. The example $H=H^*(((P^1C \times P^1C)\#P^2C)\times P^2C)$ examined in [8], shows that this is no longer true for c=4. The first section of the present paper is devoted to the proof of the following:

THEOREM 1. Let M be a compact, boundaryless, orientable, homologically 1-connected differentiable manifold. Suppose that $H^*(M;Q)$ is homogenously generated, with $c \leqslant 3$.

I) c=1 : M has the Q-homotopy type of a sphere S^d.

II) c=2 : if d=2k+1 then M has the Q-homotopy type of a connected sum of p copies of $S^{2k+1} \times S^{2k+1}$;

If d=2k then M has the Q-homotopy type of a complex of the form: $(\underbrace{S^d \vee \ldots \vee S^d}_{r_+} \vee \underbrace{S^d \vee \ldots \vee S^d}_{r_-}) \cup_f e^{2d}$, where $[f] \in \pi_{2d-1}(S^d \vee \ldots \vee S^d)$ is given by:

$$[f] = \sum_{i=1}^{r_+} [S_i^d, S_i^d] - \sum_{j=1}^{r_-} [S_j^d, S_j^d], \text{ with } r_+ \geqslant r_-.$$

The invariants are: the dimension n=4k, the rank $r=r_+ + r_-$ and the signature $\sigma = r_+ - r_-$, subject to a single restriction: σ is a multiple of a certain number σ_k, depending only on k.

III) c=3 : every M has the Q-homotopy type of some X_q, where $q \in H^3_{d,reg}(r)$; X_{q_1} and X_{q_2} have the same Q-type if and only if q_1 and q_2 are linearly equivalent; if d is odd we must have r=3 or $r > 4$.

Remark. The numbers σ_k are computable. Proposition 2 from § 1 and the remark following it provide more concrete information.

Using again [5] it comes out that any 1-connected P.d.a. H of formal dimension $n \leqslant 6$ is intrinsically formal. This fails to be true for n=7, as shown by $H=H^*((S^2 \times S^5) \# (S^2 \times S^5))$ (see § 2). The second section contains the proof of the following:

THEOREM 2. Let M^n be a homologically 1-connected closed manifold, of dimension $n \leqslant 6$.

If n=2 or 3 then M has the Q-type of S^2 or S^3.

If $n=4$ then M has the $\mathbb{Q}$-type of S^4 or of a connected sum of copies of P^2C, r_+ of them with the standard orientation and r_- with the opposite one.

If $n=5$ then M has the $\mathbb{Q}$-type of S^5 or of a connected sum of copies of $S^2 \times S^3$.

If $n=6$ then M has the $\mathbb{Q}$-type of S^6 or of S_p, the connected sum of of p copies of $S^3 \times S^3$, or of $M_q \# S_p$ where M_q is a closed manifold whose $\mathbb{Q}$-type, denoted by X_q, depends on an arbitrary cubic q and is constructed in Lemma 1 (§2); $M_{q_1} \# S_{p_1}$ and $M_{q_2} \# S_{p_2}$ have the same $\mathbb{Q}$-type if and only if $p_1 = p_2$ and q_1 is linearly equivalent to q_2.

1. Manifolds with homogenously generated cohomology

Let H be a 1-connected P.d.a. of formal dimension n. We begin by describing how to recognize those such algebras which come from manifolds.

PROPOSITION 1. If $n \not\equiv 0 \pmod 4$ then there is a closed manifold M^n such that $H^*(M^n;\mathbb{Q})=H$. If $n=4k$ ($k \geqslant 1$) then the necessary and sufficient condition for H in order to have a representation as above is that there exist: an "orientation" $\mu \in \mathrm{Hom}\,(H^{4k},\mathbb{Q}) \setminus \{0\}$ and a "Pontrjagin class" $p = \sum p_i \in \oplus H^{4i}$ with the properties:

(D*1) the numbers $\{\mu(p^{\omega}) \mid \omega$ a partition of $k\}$ are the Pontrjagin numbers of a closed manifold;

(D*2) the quadratic form $H^{2k} \otimes H^{2k} \longrightarrow \mathbb{Q}$ given by the Poincaré duality is a sum of squares over $\mathbb{Q}$;

(D*3) $\sigma(H^{2k},\mu) = \mu(L_k(p))$

where, if $\omega = (\omega_1,\ldots,\omega_r)$, p^{ω} stands for the monomial $p_{\omega_1} \cdots p_{\omega_r}$, σ denotes the signature and L_k is the Hirzebruch polynomial.

Proof: If $n=4k$ $(k\geq 1)$ then it is plain that the conditions (D^*1) to (D^*3) are necessary. Conversely, if $n\neq 0 \pmod 4$ or if $n=4k$ $(k>1)$, then all we have to do is to consider the formal 1-connected Q-local space X with the property that $H^*(X;Q)=H$ and to check the conditions (D1) to (D3) stated in the introduction. In the remaining case $(n=4)$, if $H^2=0$ then we may take $M=S^4$. If $H^2\neq 0$ we make the remark that the isomorphism type of H is determined by the intersection pairing $H^2\otimes H^2 \longrightarrow Q$ and then, recalling (D^*2), we may take M= the connected sum of r_+ copies of P^2C with standard orientation and of r_- copies of P^2C with the opposite orientation, thus completing the proof.

PROOF OF THEOREM 1 (excepting the divisibility condition on σ):

The case c=1 is rather trivial.

In the other two cases, recall first that the rational homotopy classification coincides with the classification of the cohomology algebras (due to the intrinsic formality property mentioned in the introduction).

In the case c=3 the cohomology algebra of M is of the form $H^*M=Hq$, for some $q\in H^3_{d,reg}(r)$, which means that M has the Q-type of X_q (see Introduction). Xq_1 and Xq_2 have the same Q-type if and only if the algebras Hq_1 and Hq_2 are isomorphic, which in turn is equivalent to q_1 being linearly equivalent to q_2 (see again Introduction). We gave in [9] necessary and sufficient conditions in order to have: $H^c_{d,reg}(r)\neq\emptyset$.

For c=3 they are nonvacuos only if d is odd and in this case they take the form: r=3 or $r\geq 4$. It remains to show that any algebra Hq comes from a closed manifold, which turns to be very easy in this case (using the conditions (D^*1)-(D^*3)) since the only nontrivial possibility is $d=4m$ and in this situation we have $H^{6m}_q=0$.

If M has c=2 then H^*M is determined by the intersection form $H^dM\otimes H^dM \longrightarrow Q$. If $d=2k+1$, then we are dealing with a skew-symmetric nondegenerate form over Q, whose rank must be of the form 2p and whose normal form is then the one coming from the cohomology algebra of a connected sum of p copies of $S^{2k+1} \times S^{2k+1}$. If d=2k then, by (D^*2), the intersection form of M may be written as: $\sum_{i=1}^{r_+} x_i^2 - \sum_{j=1}^{r_-} y_j^2$, a complete set of invariants being given by the dimension, the rank and the signature. Denote by H the P.d.a. of formal dimension 4k having $x_1,\dots,x_{r_+}$, $y_1,\dots,y_{r_-}$, ω as an additive basis for H^+, with: deg x_i=deg y_j=2k and deg ω =4k, whose multiplication is given by the quadratic form written above. We have to show two more things in order to complete the proof of Theorem 1. The first is that, denoting by X the complex described in the statement of the theorem, one has $H^*X=H$ and the second is that H is the cohomology algebra of a closed manifold if and only if the signature of H is divisible by σ_k. We are going to give the proof of the first assertion, postponing the other one until the next Proposition. Corresponding to the given cell decomposition of X one may immediately write down the Quillen minimal model L_X ([11], [4]): as a free graded Lie algebra, L_X is generated by $a_1,\dots,a_{r_+}$, $b_1,\dots,b_{r_-}$, μ, with: deg a_i=deg b_j=2k-1 and deg μ =4k-1, and the differential D is given by:

$$Da_i=Db_j=0 \quad \text{and} \quad D\mu=\sum_{i=1}^{r_+}[a_i,a_i]-\sum_{j=1}^{r_-}[b_j,b_j]$$

Knowing that, in general, the free Lie algebra generators of L_X correspond (with a dimension shift) to an additive basis of $\bar{H}_*(X;Q)$ and that the quadratic part of the differential gives the coalgebra structure (see [2], [11]), the assertion follows.

<u>Remark</u>. Using the same arguments as in the above proof (the case c=3) one may deduce the following slightly more general result: any 1-connected homogenously generated P.d.a. with odd cup-length is the cohomology algebra of a closed manifold.

Define, for any $m \geq 1$, two positive numbers s_m and s'_m by :

(1) <u>s_m generates the group $\{\sigma(M^{4m}) \mid p^{\alpha}[M]=0$ excepting perhaps $p_m[M]\}$</u>

(2) <u>s'_m generates the group $\{\sigma(M^{8m}) \mid p^{\beta}[M]=0$ excepting perhaps $p_{2m}[M]$ and $p_m^2[M]\}$</u>

(where M is a closed manifold, α (β) is a partition of m(2m) and $p^{\alpha}[M^{4m}]$, $p^{\beta}[M^{8m}]$ denote the corresponding Pontrjagin numbers).

For any $k \geq 1$, define a number σ_k by:

(3)
$$\sigma_k = s_k \quad , \quad \text{for } k \text{ odd}$$
$$\sigma_{2m} = s'_m \quad , \quad \text{for } k=2m$$

PROPOSITION 2. i) <u>The invariants n=4k, r and σ in the statement of Theorem 1 (the case c=2, d=2k) may appear if and only if σ is a multiple of σ_k.</u>

ii) <u>$s_k = 2^{2k-1-v_2[(2k)!]}(2^{2k-1}-1)$.numerator (B_k/k), for any k, where $v_2(y)$ denotes the greatest power of 2 dividing y and B_k stands for the k-th Bernoulli number.</u>

iii) <u>σ_{2m} divides both s_{2m} and s_m^2, for any m.</u>

<u>Proof:</u> ii) This was computed in [1]. In particular s_k is nonzero for any k.

i) This is meant to complete the proof of Theorem 1. Recalling the first part of the proof, the assertion is to be reformulated as follows: the P.d.a. H constructed starting from k, r_+ and r_- is the cohomology algebra of a closed manifold if and only if its signature is a multiple of σ_k. If $H=H^*(M^{4k})$ then the divisibility condition on $\sigma(H)$ is an immediate consequence of the definitions (1)-(3), for any k.

For the converse implication we shall denote by μ_o the orientation of H defined by $\mu_o(\omega)=1$ and we shall first examine the easy case, when k is odd and the Pontrjagin class p must have only one component, namely p_k. If σ_k divides $\sigma(H^{2k},\mu_o)=\sigma$ then, by definitions (1) and (3), $\sigma=\sigma(M^{4k})$, where M^{4k} is as in definition (1). We may then take: $\mu=\mu_o$ and $p=p_k=p_k[M]\cdot\omega$ and with this choice it is immediate to see that we have: $\mu(p^\alpha)=p^\alpha[M]$ for any partition α of k, thus checking condition (D^*1) from Proposition 1. The equalities $\mu(p^\alpha)=p^\alpha[M]$ together with the Hirzebruch formula imply that $\mu(L_k(p))=\sigma(M)=\sigma(H^{2k},\mu)$, i.e. condition (D^*3). Condition (D^*2) being verified by the very construction of H we may conclude by applying Proposition 1.

Suppose now that k is even, say k=2m. If $s'_m=s_{2m}$ then we may argue exactly as before. It is clear, anyway, that s'_m divides s_{2m}, for any m. If $s'_m\neq s_{2m}$ then, choosing N_o^{8m} as in definition (1) with the property that $\sigma(N_o^{8m})=s_{2m}$ and M_o^{8m} as in definition (2) with the property that $\sigma(M_o^{8m})=s'_m$, we deduce that $\sigma(N_o)=a.\sigma(M_o)$ for some nonzero integer a and that $p_m^2[M_o]\neq 0$. The manifold $M=N_o-a.M_o$ fulfils the conditions of definition (2) and has the additional properties: $\sigma(M)=0$ and $p_m^2[M]\neq 0$. We finally deduce the existence of a manifold M_1 as in definition (2) and with the properties: $\sigma(M_1)=0$ and $p_m^2[M_1]>0$.

If σ_{2m} divides σ then we may write $\sigma=\sigma(M^{8m})$, with M^{8m} as in (2). By eventually adding a multiple of the manifold M_1 previously constructed we may suppose in addition that $p_m^2[M]>0$. We shall then take: $\mu=p_m^2[M]\cdot\mu_o$ and $p=p_m+p_{2m}$, where $p_m=x_1$ and $p_{2m}=(p_{2m}[M]/p_m^2[M])\cdot\omega$; condition (D^*1) is checked by showing: $\mu(p^\beta)=p^\beta[M]$, for any partition β of 2m. We deduce as before that: $\mu(L_k(p))=\sigma(M)=\sigma(H^{4m},\mu_o)=\sigma(H^{4m},\mu)$ (the last equality uses $p_m^2[M]>0$). The use of Proposition 1 finishes the proof.

iii) If $s_m=\sigma(N_o^{4m})$, N_o being as in (1), then, by [10], we also have: $s_m=\sigma(N^{4m})$, where N has the property that: $p_i(N)=0$ for $i\neq m$, p_i being the i-th rational Pontrjagin class. We deduce that $s_m^2 = \sigma(N^{4m} \times N^{4m})$ is a multiple of s'_m (the other divisibility assertion in the statement being immediate).

Remark. One might also try to compute the numbers σ_{2m} by using the congruences of [12] which describe the Pontrjagin numbers of closed manifolds together with the Hirzebruch signature formula.

2. Low dimensional homologically 1-connected manifolds

The present classification reduces, similarly to the one in the previous section, to the enummeration of the rational cohomology algebras of the manifolds under discussion, due to the intrinsic formality of the 1-connected P.d.a.'s of formal dimension less than or equal to 6 (see the introduction). Here a great simplification arises from the fact that the "differentiability" conditions of Proposition 1 are nontrivial only for n=4 and in that case they were already worked out in § 1.

LEMMA 1. The rational homotopy classification of homologically 1-connected closed manifolds M of dimension 6, having $H^3M=0$ and $\dim H^2M=r$ coincides with the linear classification of the cubics q in r variables (no restrictions on q!). More precisely, for any such cubic q there is a manifold M_q with all the stated properties, whose Q-type X_q depends only on q, and such that: any M as above has the Q-type of some M_q and M_{q_1} has the Q-type of M_{q_2} if and only if q_1 is linearly equivalent to q_2.

Proof. Following [14] we shall describe a bijection between the set of isomorphism classes of 1-connected P.d.a.'s H of formal dimension 6, having $H^3=0$ and $\dim H^2=r$, and the set of linear isomorphism classes of cubics q in r variables. To H we associate the cubic q_H defined by:

$q_H(x,y,z)=\mu(x.y.z)$, for any $x,y,z\in H^2$ ($\mu\in \mathrm{Hom}(H^6,Q)\setminus\{0\}$ being an orientation).

Conversely, given a cubic q in the variables $x_1,\dots,x_r$, we denote by G the Q-vector space having $x_1,\dots,x_r$ as a basis, we construct H_q additively by: $H_q=Q\oplus G\oplus G^*\oplus Q$ (with degrees 0,2,4 and 6) and then define the multiplication of H_q by: $x.y(z)=q(x,y,z)$, for any $x,y,z\in G$ and $x.y^*=y^*.x=y^*(x)$ for any $x\in G$ and $y^*\in G^*$. It is not difficult to see that these constructions induce a bijection between the appropriate sets of equivalence classes, as asserted.

For any cubic q, let us denote by X_q the unique Q-homotopy type with the property that $H^*X_q=H_q$. Proposition 1 guarantees the existence of a closed manifold M_q in the Q-type of X_q. The other assertions of the Lemma follow at once from the P.d.a classification.

PROOF OF THEOREM 2: The cases n=2,3 are trivial.

For n=4, if $H^2M^4=0$ then obviously M has the Q-type of S^4. If $H^2M^4\neq 0$ then we argue as in the proof of Proposition 1.

For n=5, if $H^2M^5=0$ then M has the Q-type of S^5, whereas if $\dim H^2M^5=p>0$ then H^*M is isomorphic to the cohomology algebra of a connected sum of p copies of S^2xS^3.

For n=6, if H^2M^6 and H^3M^6 are both zero, then M has the Q-type of S^6. If $H^2M^6=0$ but $\dim H^3M^6=2p>0$, then $H^*M=H^*S_p$. Finally if $\dim H^2M^6=r>0$ and $\dim H^3M^6=2p\geq 0$ then H^*M is the connected sum of the subalgebras $H=H^{*\mathrm{even}}M$ and $H'=H^*S_p$. We may apply the Lemma in order to finish the proof.

<u>Example</u>. Writing $H=H^*((S^2xS^5)\#(S^2xS^5))$, the Quillen model L_H of the formal Q-homotopy type corresponding to H may be described as follows, using the algorithm of [6]: L_H is the free graded Lie algebra generated by x,y,a,b,μ, with: deg x=deg y=1, deg a=deg b=4 and deg μ =6, with diffe-

rential given by: $d_H x$, $d_H y$, $d_H a$ and $d_H b$ are zero and $d_H \mu = [a,y] + [b,x]$. One may construct another minimal differential graded Lie algebra (L,d) with the same generators and the same quadratic part of the differential, by defining: $dx=dy=0$, $da= [x, [x,y]]$, $db= [y, [x,y]]$ and $d\mu = [a,y] + [b,x]$. (L,d) will then represent a Q-homotopy type with cohomology algebra H ([2], [11]) and it is not difficult to see that (L_H, d_H) and (L,d) are not isomorphic, showing H not to be intrinsically formal (actually the two minimal models represent the only possible Q-homotopy types having H as cohomology algebra).

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