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ON THERMAL EQUATION FOR THE FLOW OF A VISCOUS COMPRESSIBLE FLUID IN POROUS MEDIA

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Abstract. In the general framework of the homogeneization method it is discused the problem of underground combustion for a viscous compressible fluid. It is proved that the disipative term as well as the compressible and convective terms must be taken into consideration. The non-dimensional parameters gives us the physical meaning of these terms.

1. INTRODUCTION

1.1. Generalities

The problem of motion of a viscous compressible fluid through a porous rigid body, is considered in the general framework of homogeneization method (Bensoussan, Lions, Papanicolaou [1] or Sanchez-Palencia [2], as general references). The periodic geometric structure of the pores is associated with the small parameter $\varepsilon \to 0$. It is known that the asymptotic process and the limit equations may have very different structure if several small parameters are involved in the problem.

In two previous paper (Ene and Sanchez-Palencia [3] [4] was studied some cases where the density, the viscosity or the thermal expansion coefficients are small. In [4] also was studied the problem of underground combustion for a viscous incompressible fluid. In this case the temperature equation contain the convective term and the disipativ one.

In this paper it is studied the problem of the motion of a viscous compressible fluid, but only in the case where it is possible to obtain the Darcy's law. It is known that in other cases [3] the velocity vector may be large and the nonlinear terms of the Navier-Stokes equation may appear in the asymptotic equation ~~to~~ leading a nonlinear Darcy's law.

The physical meaning of these equations is given by the non-dimensional numbers.

1.2. General equations

We consider a fluid domain $\Omega_{\varepsilon f}$ formed by the cavities of a rigid porous solid defined in the following way. We consider a parallelipipedic period Y of the space of the variables y_i (i=1,2,3) formed by a fluid and a solid part Y_f and Y_s, with smooth boundary Γ, and also we denote by Y_f (resp. Y_s) the union of the Y_f (resp. Y_s) parts of all periods. We there assume that Y_f as well as Y_s are connected. If Ω is the porous domain in the space of variables x_i, we define the fluid domain and the solid domain by:

$$\Omega_{\varepsilon f} = \{x \in \Omega;\ x \in \varepsilon Y_f\};\ \Omega_{\varepsilon s} = \{x \in \Omega;\ x \in \varepsilon Y_s\}$$

If ρ^ε, p^ε, T^ε and $\underline{v}^\varepsilon$ denote the density, pressure, temperature and the velocity of the flow, they must satisfy the equations (see for instance Liepman and Roshko [5]):

$$\rho^\varepsilon v_k^\varepsilon \frac{\partial v_i^\varepsilon}{\partial x_k} = -\frac{\partial p^\varepsilon}{\partial x_i} + \frac{\partial \tau_{ik}^\varepsilon}{\partial x_k} + \rho^\varepsilon f_i \tag{1.1}$$

$$\frac{\partial}{\partial x_i}(\rho^\varepsilon v_i^\varepsilon) = 0 \tag{1.2}$$

$$\rho^\varepsilon c_f v_k^\varepsilon \frac{\partial T^\varepsilon}{\partial x_k} - \frac{p^\varepsilon}{\rho^\varepsilon} v_k^\varepsilon \frac{\partial \rho^\varepsilon}{\partial x_k} = \tau_{jk}^\varepsilon \frac{\partial v_j^\varepsilon}{\partial x_k} + \frac{\partial}{\partial x_k}\left(\chi_f' \frac{\partial T^\varepsilon}{\partial x_k}\right) \tag{1.3}$$

in $\Omega_{\varepsilon f}$, and

$$0 = \frac{\partial}{\partial x_k}\left(\chi_s' \frac{\partial T^\varepsilon}{\partial x_k}\right) \tag{1.4}$$

in $\Omega_{\varepsilon s}$, where f_i are the components of exterior body force by unit mass, τ_{ik}^ε are the components of the viscous stress tensor:

$$\tau_{ik}^\varepsilon = \lambda \delta_{ik} \frac{\partial v_j^\varepsilon}{\partial x_j} + \mu \left(\frac{\partial v_i^\varepsilon}{\partial x_k} + \frac{\partial v_k^\varepsilon}{\partial x_i}\right) \tag{1.5}$$

and the state equation is of the form:

$$\rho^{\varepsilon}=\rho_0(1-\alpha T^{\varepsilon}+\beta p^{\varepsilon}) \qquad (1.6)$$

Here $\lambda, \mu, \chi', \alpha$ and β are the two voscosity coefficients, the thermal conductivity, thermal expansion coefficient and compressibility coefficients.

The boundary conditions on Γ are:

$$v_i^{\varepsilon} = 0 \qquad (1.7)$$

$$T^{\varepsilon}\big|_f = T^{\varepsilon}\big|_s \qquad (1.8)$$

$$\chi'_f \frac{\partial T^{\varepsilon}}{\partial n}\Big|_f = \chi'_s \frac{\partial T^{\varepsilon}}{\partial n}\Big|_s \qquad (1.9)$$

In order to study the asymptotic process we suppose the thermal conductivity of the form $\chi'=\varepsilon^2\chi$.

2. UNDERGROUND COMBUSTION EQUATIONS

2.1. Asymptotic expansions

It is well known (see Ene and Sanchez-Palencia [6], or Sanchez-Palencia [2] ch.6) that the Darcy's law must be obtained in the case when the sum of the order of magnitude of the viscosity coefficient with the order of magnitude of the velocity is two. Consequently, we are obliged to search for an asymptotic expansion for the velocity of the form:

$$v_i^{\varepsilon}=\varepsilon^2 v_i^0(x,y)+\varepsilon^3 v_i^1(x,y)+ \dots \qquad (2.1)$$

On the other hand, the expansions for the pressure and temperature, are classical:

$$p^{\varepsilon}=p^0(x,y)+\varepsilon p^1(x,y) + \dots \qquad (2.2)$$

$$T^{\varepsilon}=T^0(x,y)+\varepsilon T^1(x,y)+ \dots \qquad (2.3)$$

In these expansions $y=\frac{x}{\varepsilon}$ and all functions are considered to be Y periodic with respect to the variable y. The two-scale asymptotic expansion is obtained by considering that the dependence in x is obtained directly and through the variable y.

The derivatives must be considered as

$$\frac{d}{dx} \to \frac{\partial}{\partial x} + \frac{1}{\varepsilon}\frac{\partial}{\partial y}$$

Using (2.2) and (2.3) in the state equation (1.6) we obtain the expansion of the density

$$\rho^{\varepsilon} = \rho^{0}(x,y) + \varepsilon \rho^{1}(x,y) + \ldots \qquad (2.4)$$

where

$$\rho^{0}(x,y) = \rho_{0}(1 - \alpha T^{0}(x,y) + \beta p^{0}(x,y)) \qquad (2.5)$$

$$\rho^{1}(x,y) = \rho_{0}(-\alpha T^{1}(x,y) + \beta p^{1}(x,y)) \qquad (2.6)$$

2.2. Continuity equation

Using (2.1) and (2.4) in (1.2) we have at order ε^{2} and ε^{3}:

$$\frac{\partial}{\partial y_i}(\rho^{0} v_i^{0}) = 0 \qquad (2.7)$$

$$\frac{\partial}{\partial x_i}(\rho^{0} v_i^{0}) + \frac{\partial}{\partial y_i}(\rho^{0} v_i^{1} + \rho^{1} v_i^{0}) = 0 \qquad (2.8)$$

As is usually in the homogeneization problems, T^{0} and p^{0} does not depend on y, and consequently ρ^{0} is a function of x only. Then the equation (2.7) takes the form:

$$\mathrm{div}_y\, \underline{v}^{0} = 0 \qquad (2.9)$$

If we apply the mean value operator defined by

$$\tilde{\cdot} = \frac{1}{|Y|}\int_Y \cdot \; dy$$

to the equation (2.8) we obtain the continuity macroscopic equation:

$$\mathrm{div}_x\, (\rho^{0} \underline{\tilde{v}}^{0}) = 0 \qquad (2.10)$$

The mean value of the second term of the equation (2.8) is zero by the Y-periodicity [2].

2.3. Darcy's law

The equation (1.1) with (2.1) (2.2) and (2.4) takes the form:

$$\varepsilon^3 \rho^o v_k^o \frac{\partial v_i^o}{\partial y_k} + \dots = -\varepsilon^{-1} \frac{\partial p^o}{\partial y_i} - \frac{\partial p^o}{\partial x_i} - \frac{\partial p^1}{\partial y_i} - \dots + \frac{\partial \tau_{ik}^o}{\partial y_k} + \dots + \rho^o f_i + \dots \tag{2.11}$$

where

$$\tau_{ik}^o = \lambda \delta_{ik} \frac{\partial v_j^o}{\partial y_j} + \mu\left(\frac{\partial v_i^o}{\partial y_k} + \frac{\partial v_k^o}{\partial y_i}\right) \tag{2.12}$$

From (2.11) at order ε^{-1} we have

$$\frac{\partial p^o}{\partial y_i} = 0$$

and consequently $p^o = p^o(x)$.

Also from (2.11) at order ε^o we have

$$0 = -\frac{\partial p^o}{\partial x_i} - \frac{\partial p^1}{\partial y_i} + \mu \Delta_y v_i^o + (\lambda+\mu)\frac{\partial}{\partial y_i}(\mathrm{div}_y \underline{v}^o) + \rho^o f_i$$

or, taking into account (2.9):

$$0 = -\frac{\partial p^o}{\partial x_i} - \frac{\partial p^1}{\partial y_i} + \mu \Delta_y v_i^o + \rho^o f_i \tag{2.13}$$

Now the system (2.9), (2.13) is the classical system (see Ene and Sanchez-Palencia [6] or Sanchez-Palencia [2]) who give us the Darcy's law:

$$\tilde{v}_i^o = -\frac{K_{ij}}{\mu}\left(\frac{\partial p^o}{\partial x_j} - \rho^o f_i\right) \tag{2.14}$$

The matrix K_{ij} named permeability tensor is defined by

$$K_{ij} = \frac{1}{|Y|}\int_Y w_i^j \, dy \tag{2.15}$$

$$\underline{v}^o = \left(\rho^o f_i - \frac{\partial p^o}{\partial x_i}\right)\underline{w}^i \tag{2.16}$$

where $\underline{w}^i$ denote the Y-periodic flow corresponding to a mean pressure gradient equal to the unit vector in the direction of x_i and depend on the geometric structure of the period.

2.4. Energy equation

First, using (2.1)-(2.4) in (1.8), (1.9) we have the boundary condition:

$$T^{0}\Big|_{f} = T^{0}\Big|_{s} \tag{2.17}$$

$$T^{1}\Big|_{f} = T^{1}\Big|_{s} \tag{2.18}$$

....

$$\chi_{f} n_{i} \frac{\partial T^{0}}{\partial y_{i}}\Big|_{f} = \chi_{s} n_{i} \frac{\partial T^{0}}{\partial y_{i}}\Big|_{s} \tag{2.19}$$

$$\chi_{f} n_{i} \left(\frac{\partial T^{0}}{\partial x_{i}} + \frac{\partial T^{1}}{\partial y_{i}}\right)\Big|_{f} = \chi_{s} n_{i} \left(\frac{\partial T^{0}}{\partial x_{i}} + \frac{\partial T^{1}}{\partial y_{i}}\right)\Big|_{s} \tag{2.20}$$

$$\chi_{f} n_{i} \left(\frac{\partial T^{1}}{\partial x_{i}} + \frac{\partial T^{2}}{\partial y_{i}}\right)\Big|_{f} = \chi_{s} n_{i} \left(\frac{\partial T^{1}}{\partial x_{i}} + \frac{\partial T^{2}}{\partial y_{i}}\right)\Big|_{s} \tag{2.21}$$

and from (1.3) and (1.4):

$$\varepsilon \rho^{0^2} c_f v_k^0 \frac{\partial T^0}{\partial y_k} + \varepsilon^2 c_f \Big(\rho^{0^2} v_k^0 \frac{\partial T^0}{\partial x_k} + \rho^{0^2} v_k^0 \frac{\partial T^1}{\partial y_k} + \rho^{0^2} v_k^1 \frac{\partial T^0}{\partial y_k} +$$

$$+ 2\rho^0 \rho^1 v_k^0 \frac{\partial T^0}{\partial y_k}\Big) + \varepsilon p^0 v_k^0 \frac{\partial \rho^0}{\partial y_k} + \varepsilon^2 \Big(p^0 v_k^0 \frac{\partial \rho^0}{\partial x_k} +$$

$$+ p^0 v_k^0 \frac{\partial \rho^1}{\partial y_k} + p^1 v_k^0 \frac{\partial \rho^0}{\partial y_k} + p^0 v_k^1 \frac{\partial \rho^0}{\partial y_k}\Big) + \ldots =$$

$$= \varepsilon^2 \rho^0 \tau_{jk}^0 \frac{\partial v_j^0}{\partial y_k} + \ldots + \rho^0 \mathrm{div}_y(\chi_f \mathrm{grad}_y T^0) +$$

$$+ \varepsilon \rho^0 \Big[\mathrm{div}_x(\chi_f \mathrm{grad}_y T^0) + \mathrm{div}_y(\chi_f \mathrm{grad}_x T^0) + \mathrm{div}_y(\chi_f \mathrm{grad}_y T^1)\Big] +$$

$$+ \varepsilon \rho^1 \mathrm{div}_y(\chi_f \mathrm{grad}_y T^0) + \varepsilon^2 \rho^0 \Big[\mathrm{div}_x(\chi_f \mathrm{grad}_x T^0) +$$

$$+ \mathrm{div}_x(\chi_f \mathrm{grad}_y T^1) + \mathrm{div}_y(\chi_f \mathrm{grad}_x T^1) + \mathrm{div}_y(\chi_f \mathrm{grad}_y T^2)\Big]$$

$$+ \varepsilon^2 \rho^1 \Big[\mathrm{div}_x(\chi_f \mathrm{grad}_y T^0) + \mathrm{div}_y(\chi_f \mathrm{grad}_x T^0) +$$

$$+ \mathrm{div}_y(\chi_f \mathrm{grad}_y T^1)\Big] + \varepsilon^2 \rho^2 \mathrm{div}_y(\chi_f \mathrm{grad}_y T^0) + \ldots \tag{2.22}$$

$$0 = \mathrm{div}_y(\chi_s \mathrm{grad}_y T^0) + \varepsilon \Big[\mathrm{div}_x(\chi_s \mathrm{grad}_y T^0) +$$

$$+ \mathrm{div}_y(\chi_s \mathrm{grad}_x T^0) + \mathrm{div}_y(\chi_s \mathrm{grad}_y T^1)\Big] + \varepsilon^2 \Big[\mathrm{div}_x(\chi_s \mathrm{grad}_x T^0) +$$

$$+ \mathrm{div}_x(\chi_s \mathrm{grad}_y T^1) + \mathrm{div}_y(\chi_s \mathrm{grad}_x T^1) + \mathrm{div}_y(\chi_s \mathrm{grad}_y T^2)\} + \ldots \quad (2.23)$$

Than equation (2.22) and (2.23) at order ε^0 give:

$$\frac{\partial}{\partial y_i}\left(\chi_{ij}(y)\frac{\partial T^0}{\partial y_j}\right) = 0 \quad (2.24)$$

where χ takes the values χ_s, χ_f in Y_s and Y_f respectively. Moreover, from (2.17) and (2.19) this equation holds in the hole Y in the sens of distributions and from the Y-periodicity we obtain $T^0=T^0(x)$.

In the same way at order ε we obtain:

$$\frac{\partial}{\partial y_i}\left[\chi_{ij}(y)\left(\frac{\partial T^0}{\partial x_j} + \frac{\partial T^1}{\partial y_j}\right)\right] = 0 \quad (2.25)$$

or

$$-\frac{\partial}{\partial y_i}\left(\chi_{ij}(y)\frac{\partial T^1}{\partial y_j}\right) = \frac{\partial T^0}{\partial x_j}\frac{\partial \chi_{ij}}{\partial y_i} \quad (2.26)$$

Note that the convective term of order ε is zero by $T^0=T^0(x)$ and the compressibility term of order ε is also zero by $\rho^0=\rho^0(x)$.

This is the classical equation in homogeneization theory [1][2] and also it apears in the case of underground combustion for the incompressible fluid [4]. Than we have

$$\left[\chi_{ij}(y)\left(\frac{\partial T^0}{\partial x_j} + \frac{\partial T^1}{\partial y_j}\right)\right]^{\sim} = \chi^h_{ij}\frac{\partial T^0}{\partial x_j} \quad (2.27)$$

$$\chi^h_{ij} = \left[\chi_{ij}(y) + \chi_{ik}(y)\frac{\partial \theta^j}{\partial y_k}\right]^{\sim} \quad (2.28)$$

$$T^1(x,y) = \theta^j(y)\frac{\partial T^0}{\partial x_j} + c(x) \quad (2.29)$$

where θ^j is the solution of the problem

Find $\theta^j \in H^1_{per}(Y)$ with $\tilde{\theta}^j=0$ satisfying

$$\int_Y \chi_{ik}\frac{\partial \theta^j}{\partial y_k}\frac{\partial \varphi}{\partial y_i}\,dy = -\int_Y \chi_{ik}\frac{\partial \varphi}{\partial y_k}\,dy \quad (\forall)\varphi \in H^1_{per}(Y) \quad (2.30)$$

At oerder ε^2, taking into account (2.24) and (2.25), with the boundary conditions (2.20) and (2.21) and the Y-periodicity, the equations (2.22) and (2.24) give:

$$\rho^{o^2} c_f v_k^o\left(\frac{\partial T^o}{\partial x_k}+\frac{\partial T^1}{\partial y_k}\right)-p^o v_k^o\left(\frac{\partial \rho^o}{\partial x_k}+\frac{\partial \rho^1}{\partial y_k}\right)=\rho^o \tau^o_{jk}\frac{\partial v_j^o}{\partial y_k}+$$

$$+\rho^o\left\{\frac{\partial}{\partial x_j}\left[\chi_{jk}\left(\frac{\partial T^o}{\partial x_k}+\frac{\partial T^1}{\partial y_k}\right)\right]+\frac{\partial}{\partial y_j}\left[\chi_{jk}\left(\frac{\partial T^1}{\partial x_k}+\frac{\partial T^2}{\partial y_k}\right)\right]\right\} \quad (2.31)$$

in Y, where we admit that v_k^o take the value 0 on Y_s.

The applications of the mean operator to the equation (2.31) give us the macroscopic energy equation. The r.h.s. of this equation is exactly the same as in the case of incompressible flow [4], and then we have:

$$\left(\rho^o \tau^o_{jk}\frac{\partial v_j^o}{\partial y_k}\right)^\sim=\frac{\rho^o\mu}{|Y|}\left[\int_Y \frac{\partial v_j^o}{\partial y_k}\frac{\partial v_j^o}{\partial y_k}dy+\int_Y\frac{\partial v_k^o}{\partial y_j}\frac{\partial v_j^o}{\partial y_k}dy\right]=$$

$$=\rho^o\mu(K^{-1})_{kj}\tilde{v}_j^o\tilde{v}_k^o$$

$$\left\{\frac{\partial}{\partial x_j}\left[\chi_{jk}\left(\frac{\partial T^o}{\partial x_k}+\frac{\partial T^1}{\partial y_k}\right)\right]\right\}^\sim=\frac{\partial}{\partial x_j}\left(\chi^h_{jk}\frac{\partial T^o}{\partial x_k}\right)$$

$$\left\{\frac{\partial}{\partial y_j}\left[\chi_{jk}\left(\frac{\partial T^1}{\partial x_k}+\frac{\partial T^2}{\partial y_k}\right)\right]\right\}^\sim=0.$$

The mean value of the terms appearing in the l.h.s. of equation (2.3) give:

$$\left(p^o v_k^o\frac{\partial \rho^o}{\partial x_k}\right)^\sim=p^o\tilde{v}_k^o\frac{\partial \rho^o}{\partial x_k}.$$

$$\left(p^o v_k^o\frac{\partial \rho^1}{\partial y_k}\right)^\sim=\frac{p^o}{|Y|}\int_Y v_k^o\frac{\partial \rho^1}{\partial y_k}dy=$$

$$=\frac{\rho^o}{|Y|}\left(\int_Y\frac{\partial}{\partial y_k}(\rho^1 v_k^o)dy-\int_Y \rho^1\frac{\partial v_k^o}{\partial y_k}dy\right)=$$

$$=\frac{\rho^o}{|Y|}\int_{\partial Y}\rho^1 v_k^o n_k dy=0$$

$$\left[\rho^{o^2}c_f v_k^o\left(\frac{\partial T^o}{\partial x_k}+\frac{\partial T^1}{\partial y_k}\right)\right]^\sim=\rho^{o^2}c_f\tilde{v}_k^o\frac{\partial T^o}{\partial x_k}$$

Then, the macroscopic energy equation is

$$\rho^o c_f\tilde{v}_k^o\frac{\partial T^o}{\partial x_k}-\frac{p^o}{\rho^o}\tilde{v}_k^o\frac{\partial \rho^o}{\partial x_k}=\mu(K^{-1})_{kj}\tilde{v}_j^o\tilde{v}_k^o+\frac{\partial}{\partial x_j}\left(\chi^h_{jk}\frac{\partial T^o}{\partial x_k}\right) \quad (2.32)$$

It is necessary to note that the macroscopic state equation is

$$\rho^o=\rho_o(1-\alpha T^o+\beta p^o) \quad (2.33)$$

2.5. Complete system of equations

Then, the complete systems of equations for the underground combustion in the case of a viscous compressible fluid is given by the equations (2.10) (2.14) (2.32) (2.33).

In the vectorial form this system is of the form:

$$\begin{aligned} &\underline{v} = -\frac{K}{\mu}(\text{grad } p - \rho \underline{f}) \\ &\operatorname{div}(\rho \underline{v}) = 0 \\ &\rho = \rho_0 (1-\alpha T + \beta p) \\ &\rho c_f \underline{v} \text{grad } T - \frac{p}{\rho} \underline{v} \text{ grad} \rho = \frac{\mu}{K} \underline{v}^2 + \chi^h \Delta T \end{aligned} \tag{2.34}$$

2.6. Non-steady flow

All consideration concerning the Darcy's law holds for the non-steady case, using a slow scale of time $\tau = \varepsilon^2 t$.

New terms appears in equations (2.10) and (2.32).

The equation (2.8) have also a term of the form $\frac{\partial \rho^0}{\partial \tau}$ and then the mean value of this term is

$$\left(\frac{\partial \rho^0}{\partial \tau}\right)^{\sim} = \frac{\partial}{\partial \tau}\left(\frac{1}{|Y|}\int_{Y_f} \rho^0 \, dy\right) = \frac{\partial}{\partial \tau}(m \rho^0) \tag{2.35}$$

where m is the porosity of the medium: defined by $m = \frac{|Y_f|}{|Y|}$.

On the other hand in the equation (2.31) it appears a term of the form $(\rho c)^{\sim} \frac{\partial T^0}{\partial \tau}$, like in the case of incompressible flow [4].

Consequently the system (2.34) in the non-steady case takes the form:

$$\begin{aligned} &\underline{v} = -\frac{K}{\mu}(\text{grad } p - \rho \underline{f}) \\ &\frac{\partial (m\rho)}{\partial t} + \operatorname{div}(\rho \underline{v}) = 0 \\ &\rho = \rho_0 (1-\alpha T + \beta p) \\ &(\rho c)^{\sim} \frac{\partial T}{\partial t} + \rho_f c_f \underline{v} \text{grad } T - \frac{p}{\rho} \underline{v} \text{ grad} \rho = \frac{\mu}{K} \underline{v}^2 + \chi^h \Delta T \end{aligned} \tag{2.36}$$

3. NON-DIMENSIONAL NUMBERS

We take a characteristic length l of the pores, a characteristic length L of the domaine Ω and a characteristic velocity Q of the filtration velocity $\tilde{\underline{v}}^{o}$. Now, the small parameter is well defined $\varepsilon = \ell/L$. It is known [6] that the permeability is of the form $K=K^{*}l^{2}$, where K^{*} is a non-dimensional permeability.

If we introduce the Reynolds number R_ε by

$$R_\varepsilon = \frac{Q \rho_0 \ell}{\mu} \tag{3.1}$$

it is known [6] that the Darcy's law hold for $R_\varepsilon \sim 0(\varepsilon^{-1})$ or in the equivalently form

$$\frac{\ell^{2} \rho_0 Q}{\mu L} \sim 0(1) \tag{3.2}$$

In order to obtain the physical meaning of the terms which appears in the energy equation (2.32), like in the incompressible case, we introduce the non-dimensional number S_ε and the Prandtl number [6]:

$$S_\varepsilon = \frac{\mu Q^{2}}{\chi T}, \quad P = \frac{\mu c}{\chi} \tag{3.3}$$

where T is the difference between the temperature and the reference temperature. Than, with (3.2), the equation (2.32) makes sense for $S_\varepsilon \sim 0(\varepsilon^{2})$ and $P \sim 0(\varepsilon^{2})$, or:

$$\frac{\mu L^{2} Q^{2}}{\ell^{2} \chi T} \sim 0(1), \quad \frac{\mu c L^{2}}{\chi \ell^{2}} \sim 0(1) \tag{3.4}$$

These are the conditions for taking into account all terms in (2.32).

4. COMPLEMENTS

It is interesting to compare the system (2.36) with the classical system of underground combustion for incompressible fluid [4]:

$$\underline{v} = -\frac{K}{\mu}\left(\mathrm{grad} \; - \rho \underline{f}\right)$$

$$\mathrm{div}\, \underline{v} = 0 \tag{4.1}$$

$$(\rho c)^{\sim} \frac{\partial T}{\partial t} + \rho_f c_f \underline{v}\; \mathrm{grad}\; T = \frac{\mu}{K} \underline{v}^{2} + \chi^{1} \Delta T$$

The system (4.1) is uncoupled and consequently the Darcy's law and the continuity equation gives the velocity and the pressure, without the temperature field. The temperature may be determined by the third equation (4.1), taking into account the velocity field.

Contrary to this case, when we take into consideration the compressibility and the viscosity of the fluid, we obtain the system (2.36), who is a coupled system. That means that it is impossible to determin any quantity without the influence of the others. Then it is necessary to integrate the couplet system (2.36) in order to obtain the velocity, the pressure, the temperature and the density.

Finally, it is interesting to note that the incompressible case is a particular case of the compressible one. Of course the compressibility term is zero, and from (2.36) we obtain (4.1).

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