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# TANDEM SYSTEMS I.

## THE TIME TO THE (FIRST) SELF-ACTIVATION OF THE WORLD NUCLEAR WEAPONS SYSTEMS

by

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## INTRODUCTION

In Bereanu [1] was introduced a mathematical model of a nuclear arms race between two protagonists and it was proved that due to unavoidable random failures in large computerized systems for data analysis as well as to reduction in the flight durations to targets of nuclear weapons vehicles, two antagonistic nuclear weapons systems merge in time in a single randomly self-activating system. A critical time $t^*$ of the latter was identified such that any failure (false nuclear alarm) at a time $t \geqq t^*$ leads with probability 1 to self-activation (nuclear war by error).

In Bereanu [2] the model was further developed and measures devised which would minimize the probability of self-activation. A more detailed interpretation of the mathematical model in terms of present day nuclear weapons global systems and corresponding doctrines is contained in Bereanu [3] .

In the present paper it is shown that the results in [1] and [2] are significant for more general interactive complex systems which we call tandem systems and which describe situations often met in practice. Then using the initial interpretation of the model, we determine the probability distribution function of the time to the (first) self-activation of the world nuclear weapons system and analyse the effects of various arms control measures on this distribution.

## 1. TANDEM SYSTEMS

1.1. Notations and definitions. Let $S_1(t)$, $S_2(t)$ be two physical systems at time $t$ , each composed of two subsystems, i.e. $S_i(t) \equiv (O_i(t), W_i(t))$, $i=1,2$. $O_i(t)$ and $W_i(t)$ are respectively the operational and informational (warning) subsystems of $S_i(t)$. Each component $O_i(t)$, $W_i(t)$ and the systems $S_i(t)$ can move from the normal state to an "activated" state. We write $\overline{L}_i(t)$ for activated $L_i(t)$ and the activation is expressed by $(L_i \longrightarrow \overline{L}_i)$. There is a basic difference between the operational and informational parts of $S_i(t)$. Thus the operational subsystem of $S_i(t)$ is activated by a decision maker $D_i$ who may either initiate the activation of $O_i(t)$ independently of the state of $S_j(t)$, or as a result of the tandem rule (3). The informational subsystem of $S_i(t)$, when activated as $\overline{W}_i(t)$ merely indicates the activation of $O_j(t)$, $j \neq i$, i.e. provides the necessary information to the decision maker $D_i$ for implementing (3). Physically $W_i(t)$ contains a system of sensors, which collects information about the state of $S_j(t)$, together with means of treating and interpreting this information (for instance, but not necessarily, a computerized system).

As a result of this treatment of information, $W_i(t)$ can remain in the normal state or at some points in time can move into the active state $\overline{W}_i(t)$ , which indicate correctly or, erroneously, due to a failure of $W_i(t)$, an "activation" of $S_j(t)$. The activation of the informational subsystem $W_i(t)$ may, but not necessarily, lead to the transition of the operational subsystem $O_i(t)$ in an active state .

We express this by the following implication

$$(1) \quad (W_i(t) \longrightarrow \overline{W}_i(t)).\ Q_i(t) \Longrightarrow (O_i(t) \longrightarrow \overline{O}_i(t)).$$

Here $Q_i(t)$ is a test which, with a certain probability, assures that the activation of $W_{it}$ is not due to an error, while "." stands for "and" . Based on this test and (3) the decision maker $D_i$ activate $O_i(t)$.

We assume the following equivalence :

$$(2) \quad (S_i(t) \longrightarrow \overline{S}_i(t)) \Longleftrightarrow (O_i(t) \longrightarrow \overline{O}_i(t)).$$

This points out that $O_i(t)$ is the operational part of $S_i(t)$ and only its activation makes active the system as a whole.

The tandem property of the systems $S_i(t)$, $i=1,2$ is expressed through the following implication

$$(3) \quad (S_i(t) \longrightarrow \overline{S}_i(t)) \Longleftrightarrow (S_j(t) \longrightarrow \overline{S}_j(t)) \text{ , } j \neq i.$$

This property consolidate the two systems $S_i(t)$ into a single system $S(t) \equiv (S_1(t), S_2(t))$ from the point of view of activation. Both the activation of $W_i(t)$ and $O_i(t)$ may take different degrees (see [1] , [2] and section 2 of the present paper). However for simplifying the notations in this section we considered only one active state.

1.2. Self-activation of tandem systems

As we mentioned above, $S(t)$ can be activated by the initiative of one or both of the decision makers $D_i$, $i=1,2$. But since the test

$Q_i(t)$ does not rule out the erroneous activation of $W_{it}$ due to a failure of its sensors or/and of its system of treating the information, $S(t)$ can be activated via (1), (2), (3) due to an error in one of the informational subsystems.

We call <u>self-activation</u> of the random systems $S(t)$ at time $t$, an activation $S(t) \longrightarrow \overline{S}(t)$ via (1), (2), (3), i.e. without the deliberate initiative of one of $D_i$, $i=1,2$. In the process of self-activation (1) may require the nominal participation of $D_j$ (although automation may replace it) but such participation does not initiate the activation of $S(t)$. It merely fulfils the tandem condition (3).

The self-activation can occur in a wide variety of tandem systems resulting from a variety of occurances such as a false nuclear alarm generated by the Early Warning System of one of two antagonistic nuclear weapons systems, to a misinterpretation of somebody's smile by annother person.

## 2. <u>TANDEM NUCLEAR WEAPONS SYSTEMS</u>

2.1. <u>Interpretation of the elements of the tandem systems.</u> $S_{it}$, $O_{it}$, $W_{it}$, $i=1,2$ are respectively two antagonistic global nuclear weapons systems, their subsystems of nuclear weapons with their delivery means, and corresponding Early Warning Systems (EWS). The tandem property translates the doctrine "if under nuclear attack, retaliate". See Bereanu [2], [3] for details on launching doctrines and the composition of $O_{it}$ and $W_{it}$ in this interpretation.

According to the notations of [1] let $T_{in}$, $n=1,\ldots$ be the sequence of random times at which the EWS, $W_{it}$, moves into one of the $k(i)$ active states $\overline{W}_{iqt}$ with the positive probabilities (of sum 1) $p_{iq}$, $q=1,\ldots,k(i)$, $i=1,2$. The test $Q_i(t)$ of (1) is defined here via a family of piecewise constant functions $d_{iq}(t)$ - The Efficient Checking Time (ECT)- corresponding to $\overline{W}_{iqt}$ (nuclear alarm of degree $q$ in $S_{it}$ at time $t$). The ECT represents the maximum ammount of time at the disposal of the decision maker $D_i$ for verifying if the nuclear alarm is not due to a failure of the system (a discussion of these types of failures based on an USA Congressional Report of 1981 is contained in [1] ).

The ECT diq(t) is defined by

$$(4)\quad d_{iq}(t) = \min_{k \in K_q} \left\{ \Delta_{jk}(t) \right\} - \delta_i,\ j \neq i,$$

where $\Delta_{jk}(t)$, $k=1,\ldots,r$ is the average delivery time (time to reach its targets) of the fastest launcher of type $k$ in $O_{jt}$, $K_q$ is the subset of $r$ types of launchers of $O_{jt}$, the activation of which is implied by $\overline{W}_{iqt}$, and $\delta_i$ is the time required for activating $O_{it}$ in retaliation.

Let $\{\xi_{in}\}$, $i=1,2$; $n=1,2,\ldots,$ be two sequences each one composed of identically distributed, stochastically independent random variables. The r.v. $\xi_{in}$ represents the time which would be required at $\tau = T_{in}$ to discover if the nuclear alarm indicated by $\overline{W}_{iq}$ is false. In case of a real activation of $S_{jt}$ the realization $x_{in}$ of the r.v. $\xi_{in}$ is infinite and as seen from (5) $O_{i\tau}$ will be activated.

In the interpretation of the tandem systems $S_{it}$, $i=1,2$ of this section $Q_i(\tau)$ is given by $((x_{in} - d_{iq}(\tau)) > 0)$ and the rule of activation of $O_{it}$ is (supposing that at time $T_{in}=\tau$, $W_{i\tau}$ was activated):

$$(5)\quad (W_{i\tau} \longrightarrow \overline{W}^q_{i\tau}).((x_{in} - d_{iq}(\tau)) > 0) \Longrightarrow (O_{i\tau} \longrightarrow \overline{O}_{i\tau}).$$

The condition (5) under which the activation of $W_{it}$ leads to the activation of $O_{it}$ and which corresponds to (1), in the nuclear weapons systems interpretation, means the following: "if under nuclear attack, check the validity of the alarm the maximum available time before retaliating". This is a very cautions rule of activation (see [1], [2] for launching rules advocated at present and is in line with the whole approach of the model, to estimate from below the probability of self-activation.

## 2.2. The random elements of $S_{it}$ and qualitative effects of the arms race

The approach adopted in [1] which assures meaningful results may be summarized as follows :

a) We take into consideration only two main sources of randomness inherent in $W_{it}$ and which according to official published information produce false nuclear alarms: errors in the software of the computer systems of $W_{it}$, failures of components of $W_{it}$.

b) Since there are not enough published data on these systems, we made "optimistic" hypotheses, i.e. such hypotheses which reduce probabilities of errors or failures. Hence we obtain a lower bound on the probability of self-activation. Then we show that because $d_{iq}(t)$, $i=1,2$; $q=1,\ldots,k(i)$ are decreasing functions of $t$, as a result of the arms race (see [1] and [2] for details), there is a critical time $t^*$ such that any false nuclear alarm in $S_t$ at a time $t \geqq t^*$ leads to self-activation with probability one .

It is also shown that because b) the random sequences $T_{iu}$ , i=1,2 can be approximated by Poisson arrival processes [4] and hence the false nuclear alarms in $S_t$ follow again a Poisson process ; see for instance [5] ).

2.3. REMARK. The same or similar approaches can be used for other tandem systems, taking into consideration their specific behaviour and randomnesses.

## 3. THE TIME TO THE (FIRST) SELF-ACTIVATION

3.1. Preliminaries. Let $\lambda$ be the arrival rate of the Poisson process $M = \{M_t;\ t \geq 0\}$ according to which the false nuclear alarms take place in the system $S_t$ of section 2. The process M is the superposition of two Poisson processes representing the arrival counting processes of false nuclear alarms in $S_{1t}$ and $S_{2t}$ , which in turn are Poisson processes with arrival rates $\lambda_1, \lambda_2$ . Their interarrival times are independent exponential random variables.

We denote by $t_\ell$ , $\ell = 1,\dots,L$ the points in time in a forseeable horizon where at least one of the functions $d_{iq}(t)$, q=1,...,k(i), i=1,2 has a jump (due to the deployment of faster or/and nearer to targets nuclear weapon vehicles. Thus in each of the interval $[t_{\ell-1}, t_\ell)$, $d_{iq}(t)$ are constants, say $d_{iq\ell}$ , decreasing in $\ell$ . The intervals $[t_{\ell-1}, t_\ell)$ are measured in number of years, even decades. We shall therefore in what follows replace $d_{iq}(t)$ by constants $d_{iqs}$ , s indicating a "stability period" of the flight time to reach each other targets by the nuclear weapons vehicles constituting $O_{it}$ in that period of time.

3.2. The time until self-activation. We shall compute now the probability distribution of the time until self-activation take place. This time is the sum of interarrival times of all false nuclear alarms which preceeded the self-activation. The number of such interarrival times is a random variable N with $P\{N = \nu\} = p_\nu$ and $p_\nu = uv^{\nu-1}$ , where u is the probability that a false nuclear alarm leads to self-activation, while v = 1-u is the probability that the alarm is called off after discovering that it is due to an error.

Suppose we consider one false nuclear alarm in the global system $S_t$. From 3.1. and [1] the alarm takes place in $S_{1t}$ with probability $\lambda_1\lambda^{-1}$ and $S_{2t}$ with probability $\lambda_2\lambda^{-1}$ . It is easily seen from (5 that

$$(6)\qquad v = 1/\lambda \sum_{i=1}^{2} \sum_{q=1}^{k(i)} p_{iq}\, \lambda_i\, F_i(d_{iqs}).$$

We recall that $F_i(.)$ is the probability distribution function of the independent identically distributed r.v.s. $\{\xi_{in}\}$, i=1,2.

The time until the (first) self-activation $U_\nu$ is given by the sum with a random number of terms

$$(7)\qquad U_\nu = X_1 + X_2 + \ldots + X_\nu$$

where $X_1, X_2, \ldots$ are mutually independent exponential random variables with parameter $\lambda$ with common density function $f(.)$. Hence the density of $U_\nu$ is the $\nu$-fold convolution of $f$ with itself $f^{\nu *}$. It follows (see Feller [6] that the density function of the duration to the self-activation is given by

$$(8)\qquad w(x) = \sum_{\nu=1}^{\infty} p_\nu f^{\nu *} = u\lambda e^{-\lambda x} \sum_{\nu=1}^{\infty} v^{\nu-1} \frac{(\lambda x)^{\nu-1}}{(\nu-1)!} = u\lambda e^{-u\lambda x},$$

i.e. <u>the time until the (first) self-activation is an exponential r.v. with parameter $u\lambda$</u>, and $u=1-v$, where $v$ is given by (6). Hence its probability distribution function is $1-e^{-u\lambda x}$. The probability that the time until self-activation becomes greater than $T$, $\phi(T)$, is thus given by

$$(9)\qquad \phi(T) = e^{-u\lambda T}$$

We shall call it T-<u>probability of survival</u>.Formulae (6) and (9) allow to analyse the effects of various nuclear arms control or developments of the arms race on the T-probability of survival. For instance maintaining a "stability period" as defined in 3.1, maintain unchanged the T-probability of survival. This happens for instance through the freeze of development and deployment of new nuclear weapons systems with shorter flight times.

The eliminations of nuclear weapons vehicles with the shortest flight times during a given "stability period" increase the probability of survival while drastic reduction of time flight in at least one of the systems $O_{it}$, i=1,2, reduce dramatically $\phi(T)$.

<u>REFERENCES</u>

[1] Bereanu,B.: On the probability of self-activation of the world nuclear weapons stockpile , Methods of Operations Research, 44(1981), 507-520

[2] Bereanu,B.: The self-activation property of the world nuclear weapons stockpile. A minimum risk approach,Rev.Roum.Math.Pures et Appl.,27(1982), 647-662

[3] Bereanu, B.: The self-activation of the world nuclear weapons system, J.Peace Research, 20(1983), 49-57.
[4] Barlow, R.E. and Proschan F.: Mathematical theory of reliability, Wiley, New York, 1965.
[5] Cinlar, E.: Introduction to stochastic processes, Prentice-Hall, Englewood Cliffs, N.J., 1975.
[6] Feller, W.: An introduction to probability theory and its applications, vol.II, Wiley, New York, 1966.