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C O N T E N T S

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The aim of this paper is the determination of groups of singularities which lies on the degenerate Del Pezzo surfaces. More, we obtain structure theorems for many types of degenerate Del Pezzo surfaces.

We'll work over the field $\mathbb{C}$ of complex numbers. We'll use the standard notations from [6] and [7].

Surface means an integral $\mathbb{C}$-algebraic scheme of dimension 2. We remember that a surface X is called normal iff all its local rings are normal, a surface X is called Gorenstein iff all its local rings are Gorenstein (or, equivalent, the Grothendieck dualising sheaf is invertible, so the surface X has a canonical divisor).

Definition: A rational normal Gorenstein surface with ample anticanonical sheaf is called (degenerate) Del Pezzo surface if it is nonsingular (respectively if it is singular, for "degenerate").

Let be $d:=\omega_X.\omega_X$, where X is a (degenerate) Del Pezzo surface. In [2],[5],[8] is proved that $1\leq d\leq 9$. We want to establis all possible groups of singularities of X for each value of d and to obtain the structure of the surface X.

The method which we'll use, detailed presented in §§1,2, is based on the results from [2], [5], [8], where are obtained structure theorems for normal Gorenstein surfaces with ample anticanonical divisor. Namely, for each $d\in\{1,2,\ldots,8\}$, X is an elliptic cone (i.e. a cone over an elliptic curve) or is rational and is obtained by contracting connected configurations of irreducible curves with self-intersection (-2) that lie on a surface X', and X' is obtained by blowing up (9-d) points in an almost general position in $\mathbb{P}^2$ (see §1 for definitions). A Brieskorn's theorem affirms that the singularities of X - obtained by contracting the above connected configurations, are determined - up to an analytic isomorphism - by configurations. The method used will consist, in the case of rational normal Gorenstein surfaces with ample anticanonical divisor (i.e. (degenerate) Del Pezzo surafaces), in the determination, for each $d\in\{1,2,\ldots,7\}$, of the connected configurations of irreducible curves with self-intersection (-2) that lie on the blown up surface X' in all possible cases (i.e. for each possibility of arrangement of the (9-d) points in almost general positions in $\mathbb{P}^2$).

In this paper are obtained complete results for each $d\in\{2,3,\ldots,9\}$ and partially results for d=1. In §2 is analised the case d=3, in §3 the case d=4, in §4 the case d=2 and in §§5,6 the others cases.

We remark that for d=3 (i.e. the case of cubic surfaces in $\mathbb{P}^3$) all groups of possible singularities are determinde in [4] too, for d=4 (i.e. the case of complete intersections of type (2,2) in $\mathbb{P}^4$) are determined in [9] too and for d=1, d=2 are determined in [11]. But the methods used in these papers are different from ours.

The method used in this paper has the advantage of being applicable to all possible cases and it gives structure theorems for all types of degenerate Del Pezzo surfaces (classified in

§ 1. Some General Results About Algebraic Surfaces

In this § we'll state - without proof - some theorems (with the necessary references) which will be useful further on.

If X is a nonsingular connected surface (over $\mathbb{C}$) we note Pic(X) the Picard's group of X and ω_X the canonical sheaf of X.

If $x \in X$, we note X(x) the surface obtained by blowing up the point $x \in X$. If $x_1 \in X$, $x_2 \in X(x_1)$, ..., $x_r \in X(x_1)(x_2),\ldots,(x_{r-1})$ and $\Sigma := \{x_1,\ldots,x_r\}$ we note $X(\Sigma)$ the surface obtained by blowing up succesively the points $x_1,\ldots,x_r$.

Let be $\Sigma = \{x_1,\ldots,x_r\}$, with $x_1 \in \mathbb{P}^2$, $x_2 \in \mathbb{P}^2(x_1),\ldots,x_r \in \mathbb{P}^2(\Sigma \setminus \{x_r\})$. Then, if $E_1,\ldots,E_r$ are the <u>total</u> transformers of the points $x_1,\ldots,x_r$ in $\mathbb{P}^2(\Sigma)$, we have:

$\mathrm{Pic}(\mathbb{P}^2(\Sigma)) \cong \mathrm{Pic}\,\mathbb{P}^2 \oplus \mathbb{Z}^r \cong \mathbb{Z} \oplus \mathbb{Z}^r$ the isomorphism being

$$\xi \longmapsto (\xi.E_0;\xi.E_1,\ldots,\xi.E_r)$$

where E_0 is the inverse image in $\mathbb{P}^2(\Sigma)$ of a line in $\mathbb{P}^2$.

In $\mathrm{Pic}(\mathbb{P}^2(\Sigma))$ the elements

$$E_0;-E_1;\ldots,-E_r$$

form a $\mathbb{Z}$ - basis and:

1) $E_0^2=1$; $E_i^2=-1$; $E_i.E_j=0$, $(\forall)\ 0<i\neq j$

2) $\omega_{\mathbb{P}^2(\Sigma)} = -3E_0+E_1+\ldots+E_r = (-3;-1,-1,\ldots,-1)$.

3) $\omega_{\mathbb{P}^2(\Sigma)}.E_0=-3$; $\omega_{\mathbb{P}^2(\Sigma)}.E_i=-1$, $(\forall)\ i>0$

$$\omega_{\mathbb{P}^2(\Sigma)}.\omega_{\mathbb{P}^2(\Sigma)}=9-r.$$

<u>Definition</u> ([5]): If $\Sigma = \{x_1,\ldots,x_r\}$ with $x_1 \in \mathbb{P}^2$,

$x_2 \in \mathbb{P}^2(x_1), \ldots, x_r \in \mathbb{P}^2(\Sigma \setminus \{x_r\})$, $r \leq 8$, the points of Σ are in an <u>almost general position</u> iff:

i) any 4 of them are not on a line (if $r \geq 4$);

ii) any 7 of them are not on a conic (if $r \geq 7$);

iii) $(\forall) j$, $1 \leq j \leq r-1$, the point $x_{j+1} \in \mathbb{P}^2(\Sigma_j)$ (with $\Sigma_j = \{x_1, \ldots, x_j\}$) does not lie on any component $\widehat{E}_i$ of E_i $(1 \leq i \leq j)$ such that $\widehat{E}_i^2 = -2$.

<u>Definition</u> (see for example [1]). Let be $f: X \longrightarrow Y$ a desingularisation of the normal singularity of dimension 2, (Y, y). f is called <u>minimal desingularisation</u> iff the reduced fiber $f^{-1}(y)_{red}$ does not contain as components irreducible curves C, with $p_a(C) = 0$ and $C^2 = -1$ (i.e. exceptional curves of the first kind).

It's known that, in the conditions of the definition, there are always minimal desingularisations.

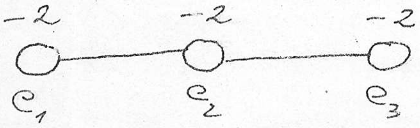
Let be X a (degenerate) Del Pezzo surface and $d := \omega_X \cdot \omega_X$. We'll call d the <u>degree</u> of X. It's known that $1 \leq d \leq 9$ (see [2], [5], [8]).

All is clear if $d \in \{8, 9\}$ (see [2], [5], [8]).

<u>Theorem A</u> ([2], [5], [8]). Let X be a (degenerate) Del Pezzo surface with degree $d \in \{1, 2, \ldots, 7\}$. Then there exist a set of $(9-d)$ points in an almost general position such that $\widetilde{X} \cong \mathbb{P}^2(\Sigma)$ (where $\widetilde{X} \longrightarrow X$ is the minimal desingularisation of X); more, π actions by contracting all the irreducible curves C on X with $p_a(C) = 0$ and $C^2 = -2$ (to singular points of X which are rational double points).

In accordance with theorem A we can try to classify the singularities of degenerate Del Pezzo surfaces X for each value $d \in \{1, 2, \ldots, 7\}$. Using theorem A and the next theorem B, we see that the singularities of X are determined (up to an analytic isomorphism) by the configurations of irreducible curves C with $p_a(C)=0$, $C^2=-2$ which lie on $\tilde{X}$.

If $\tilde{X} \xrightarrow{\pi} X$ is as above and $x \in X$ is a singular point, $\pi^{-1}(x)$ is a configurations of curves C with $p_a(C)=0$, $C^2=-2$ as components. We can associate to $\pi^{-1}(x)$ a graph. For example, if $\pi^{-1}(x)$ has 3 components we can associate the graph



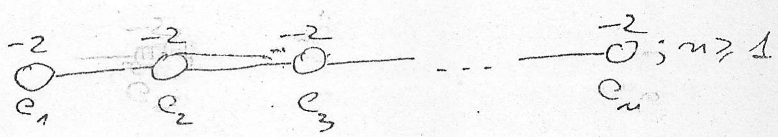
where every circle represents one component, and two circles are joined if the two components which correspond them are cut transversally.

<u>Theorem B</u> (Brieskorn [3a]). Let (X,x) and (X',x') be two singularities which are rational double points. Then (X,x) and (X',x') are analytically isomorphic (i.e. $\hat{\mathcal{O}}_{X,x} = \hat{\mathcal{O}}_{X',x'}$ where the completions are given by the maximal ideals) if and only if the graphs associated to fibers $\pi^{-1}(x)$ and $\pi'^{-1}(x')$ are isomorphic where π and π' are the minimal desingularisations of (X,x) and (X',x') respectively.

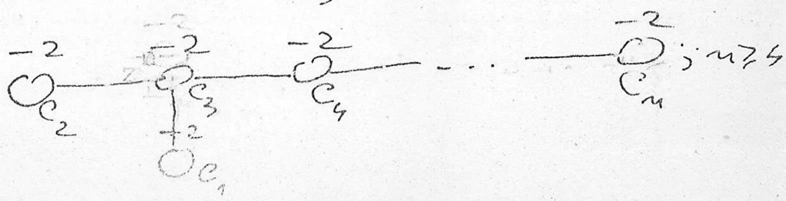
Hence the graph associated to the fiber of the minimal desingularisation in the singular rational double point x, determines (up to an analytic isomorphism) the type of singularity.

We consider the next kinds of rational singularities (to the left it's the standard local equation and to the right there is the graph of the fiber of the minimal desingularisation):

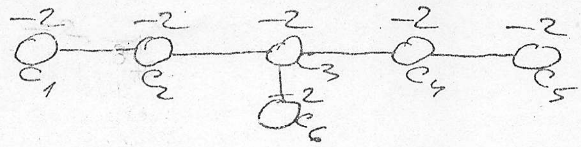
(A_n): $z^{n+1}+z_2^2+z_3^2=0$;



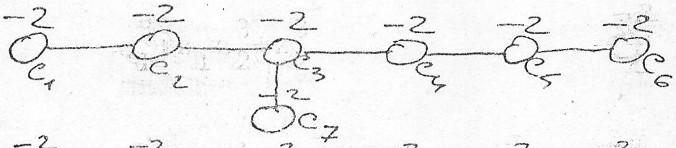
(D_n): $z_1^{n-1}+z_1 z_2^2+z_3^2=0$;



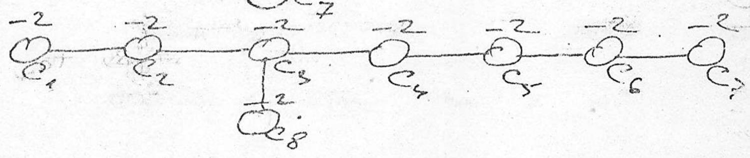
$(E_6):\ z_1^3+z_2^4+z_3^2=0;$



$(E_7):\ z_1^3+z_1z_2^3+z_2^3=0;$



$(E_8):\ z_1^3+z_2^5+z_3^2=0;$



It is known that every singularity which is rational double point is one of the above kinds (see for instance [1], §3).

Hence a (degenerate) Del Pezzo surface has one or more singularities of type $A_n (n\geq 1)$, $D_n (n\geq 4)$, E_6, E_7 or E_8 or it is non-singular.

Let be $(a_1,\dots,a_s)\in \mathbb{N}^{*s}$. We consider the polynomial ring $\mathbb{C}[T_1,\dots,T_s]$ which is graded such that

$$\deg T_i=a_i,\quad i=\overline{1,s}.$$

Then

$$\mathbb{P}(a_1,a_2,\dots,a_s) \stackrel{def}{=} \mathrm{Proj}(\mathbb{C}[T_1,\dots,T_s])$$

is called the <u>weighted projective space</u> of weight $(a_1,\dots,a_s)$ (see [10] too).

Let be the graded ring

$$R(X)=\bigoplus_{m\geq 0} H^0(X,\omega_X^{-m})$$

where $(R(X))_m=H^0(X,\omega_X^{-m})$ and the multiplication is given by the tensor product. Because ω_X^{-1} is ample, there is the canonical isomorphism

---

$$X\cong \mathrm{Proj}(R(X)),\ \text{with}\ \mathcal{O}_X(1)\cong\omega_X^{-1}\quad (\text{see}[6],\ \text{II}).$$

Theorem C ([8]). Let X be a (degenerate) Del Pezzo surface and $d=\omega_X.\omega_X$ ($\in\{1,2,\ldots,9\}$). Then:

i) if $d\geq 4$ then $R(X)\cong\mathbb{C}[T_o,\ldots,T_d]/I$, deg $T_i=1$ ($0\leq i\leq d$) and I is generated by $\frac{d(d-3)}{2}$ quadrics; in particular, for d=4, X is the complete intersection of two quadrics in $\mathbb{P}^4$ (i.e. of type (2,2) in $\mathbb{P}^4$);

ii) if d=3 then $R(X)\cong\mathbb{C}[T_o,T_1,T_2,T_3]/(F)$, deg $T_i=1$ ($0\leq i\leq 3$) and deg F=3; hence X is a cubic surface in $\mathbb{P}^3$;

iii) if d=2 then $R(X)\cong\mathbb{C}[x,y,z,t]/(F)$, deg x=deg y=deg z=1, deg t=2, def F=4, hence X is a hypersurface of degree 4 in the weighted projective space $\mathbb{P}(1,1,1,2)$;

iv) if d=1 then $R(X)\cong\mathbb{C}[x,y,z,t]/(F)$, deg x=deg y=1, deg z=2, deg t=3, deg F=6, hence X is a hypersurface of degree 6 in $\mathbb{P}(1,1,2,3)$

Corollary C1 ([8]):

i) If $d\geq 3$ then ω_X^{-1} is very ample and its global sections give an embedding of X in $\mathbb{P}^d$ as a subvariety of degree d;

ii) if d=2. Then ω_X^{-2} is very ample and its global sections give an embedding of X in $\mathbb{P}^6$ as a subvariety of degree 8;

iii) if d=1 then ω_X^{-3} is very ample and its global sections give an embedding of X in $\mathbb{P}^6$ as a subvariety of degree 9.

Notation: We'll write, for example, $D_4 2A_1 3A_2$, if the surface X has one singularity of type D_4, two singularities of type A_1, three singularities of type A_2 and no others.

§ 2. The Classification of Singularities and Structure Theorems for Degenerate Del Pezzo Surfaces of Degree 3 (i.e. rational normal cubic surfaces in $\mathbb{P}^3$)

In [4] is obtained the classification of singularities of cubic surfaces in $\mathbb{P}^3$, starting from equations; the number of lines contained on each type of surface is obtained as well. In this § we'll obtain these results again with another method and, more in the rational case, we'll determine the ways these surfaces can be obtained starting from $\mathbb{P}^2$ and effectuating blowing ups and contractions (structure theorems for rational normal cubic surfaces, classified after singularities).

Let X be a rational normal cubic surface in $\mathbb{P}^3$. Then $\omega_X^{-1} \cong \mathcal{O}_X(1)$, hence theorem A §1 can be applied; hence X is obtained by blowing up r=9-3=6 points from $\mathbb{P}^2$ in an almost general position and then contracting the integral curves C with $p_a(C)=0$ and $C^2=-2$ (see also theorem C i)).

Lemma 1: Let $\tilde{X} \xrightarrow{\pi} X$ be the minimal desingularisation. Then $\tilde{X} \cong \mathbb{P}^2(\Sigma)$, with $\Sigma=\{P_1,\dots,P_6\}$ in an almost general position. Pic $\tilde{X} \cong \mathbb{Z} \oplus \mathbb{Z}^6$ with $E_o; -E_1,\dots,-E_6$ a $\mathbb{Z}$ - basis (see § 1). Then the integral curves $C \subset \tilde{X}$ with $p_a(C)=0$ and $C^2=-2$ ($\Leftrightarrow \omega_{\tilde{X}}.C=0$) are of one of the following kinds:

(0; -1, 1, 0, 0, 0, 0),

or (1; 1, 1, 1, 0, 0, 0) *

(inverse images of lines which contains 3 points in Σ),

or (2; 1, 1, 1, 1, 1, 1)

(the inverse image of the conic which contains all the points of Σ), as far as they exist.

<u>Proof</u>:

Let be $C=(a;\ b_1,\dots,b_6)\in Pic(\tilde{X})$, with $p_a(C)=0$ and $C^2=-2$.

We have $\omega_{\tilde{X}}=(-3;-1,-1,-1,-1,-1,-1)$ (see § 1). Then:

$$\omega_{\tilde{X}}.\ C = 0 \iff 3a = \sum b_i \qquad (1)$$

$$C^2=-2 \qquad a^2 - \sum b_i^2 = 2 \qquad (2)$$

But

$$(\sum b_i)^2 \leq 6(\sum b_i^2) \quad \text{(Cauchy)}.$$

Replacing from (1) and (2) we obtain:

$$(3a)^2 \leq 6(a^2+2) \iff$$

$$\iff \quad a^2 \leq 4.$$

But $a \geq 0$, hence $a\in\{0,1,2\}$.

Solving in integer numbers the systems

$$\begin{cases} \sum b_i=3\ a \\ \sum b_i^2=a^2+2 \end{cases} \qquad \text{for } a\in\{0,1,2\}$$

we obtain exactly the curves in the lemma.

<u>Lemma 2</u>: The lines on X (i.e. the exceptional divisors of the first kind) are the (isomorphic) images by π of the divisors on X of one of the next kinds:

$(0; -1, 0, 0, 0, 0, 0)$,

or $(1; 1, 1, 0, 0, 0, 0)$

(inverse images of lines which contain 2 points in Σ),

or $(2; 1, 1, 1, 1, 1, 0)$

(inverse images of conics which contain 5 points in Σ), as far as they exist.

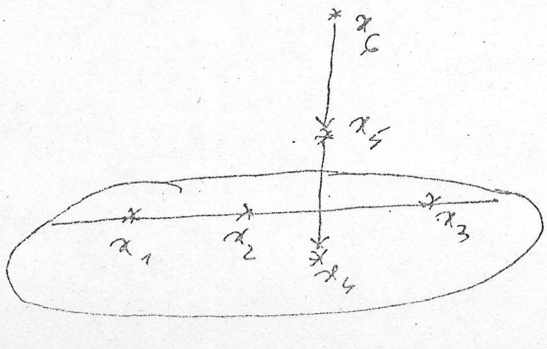
Proof:

We have: $\omega_{\tilde{X}}.\ C=1$ and $C^2=-1$ and we apply the same method as in the proof of lemma 1.

The lemmas 1 and 2 are used for the determination of integral curves C with $p_a(C)=0$ and $C^2=-2$, of the intersection matrix of these, for the determination of lines $D\subset \tilde{X}$ and of the intersection matrix of the lines.

Because every rational normal cubic surface in $\mathbb{P}^3$ is obtained by blowing up successively 6 points in an almost general position in $\mathbb{P}^2$ and by contracting the integral curves C with $p_a(C)=0$ and $C^2=-2$, everything comes back to the analysis of all essentially different possibilities of arrangement of the 6 points in an almost general positions (totally 88 cases, see Appendix 2, §1).

Let's analyse, for instance, one case of arrangement of the 6 points in an almost general position (fig.1):



5=(4,1)

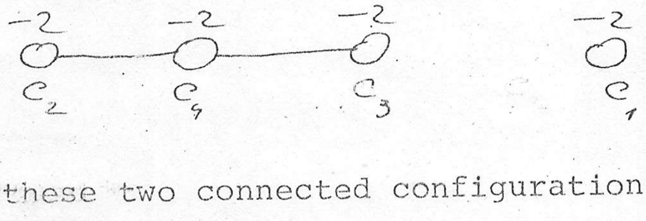
In fig.1 the fact that the points x_1, x_2, x_3, x_4 are encircled means that they are in $\mathbb{P}^2$; the fact that we have joined x_1, x_2, x_3 with a line means that the three points are collinear; the arrow $x_5 \to x_4$ means that x_5 is infinitely near point of x_4, but satisfying the condition of almost general position (see §1); the condition 5=(4,1) means that x_5 corresponds to the direction given by the line which contains x_1 and x_4. Using lemma 1 and the position of points, we obtain that the integral curves C_i on X such that $p_a(C_i)=0$ and $C_i^2=-2$ are (in the basis E_o; $-E_1, \ldots, -E_6$):

$$C_1=(1;\ 1,\ 1,\ 1,\ 0,\ 0,\ 0)$$
$$C_2=(1;\ 1,\ 0,\ 0,\ 1,\ 1,\ 0)$$
$$C_3=(0;\ 0,\ 0,\ 0,\ -1,\ 1,\ 0)$$
$$C_4=(0;\ 0,\ 0,\ 0,\ 0,\ -1,\ 1)$$

The intersection matrix is

$$(C_i.C_j)_{i,j}=\begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 1 \\ 0 & 0 & -2 & 1 \\ 0 & 1 & 1 & -2 \end{bmatrix}$$

Hence, the configurations are:



and, because these two connected configurations are contracting to double rational singular points, and - using theorem B - the configuration on $\tilde{X}$ determines the singularity -up to an analytic isomorphism, it follows that X has A_1A_3 singularities (one singularity of type A_1 and one singularity of type A_3).

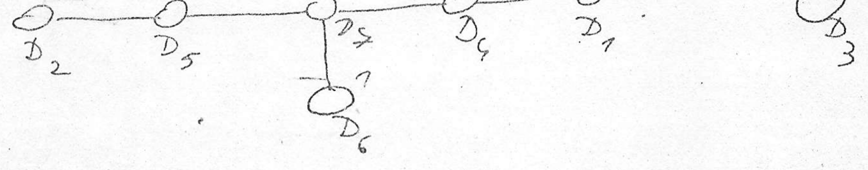
Using lemma 2 we can determine all lines on $\tilde{X}$ (isomorphically transported in X by π), hence all lines on X (how many they are and how they or cut). Using lemma 2 and the position of pointes we obtain that the lines on X are:

$$D_1=(1;\ 0,\ 1,\ 0,\ 1,\ 0,\ 0),$$
$$D_2=(1;\ 0,\ 0,\ 1,\ 1,\ 0,\ 0),$$
$$D_3=(0;-1,\ 0,\ 0,\ 0,\ 0,\ 0),$$
$$D_4=(0;\ 0,\ -1,\ 0,\ 0,\ 0,\ 0),$$
$$D_5=(0;\ 0,\ 0,\ -1,\ 0,\ 0,\ 0),$$
$$D_6=(0;\ 0,\ 0,\ 0,\ 0,\ 0,\ -1),$$
$$D_7=(2;\ 0,\ 1,\ 1,\ 1,\ 1,\ 1).$$

The intersection matrix of lines is

$$(D_i.D_j)_{i,j}=\begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & -1 \end{bmatrix}$$

Hence $\tilde{X}$ (hence X too) has 7 lines (which are the exceptional divisors of the first kind), which are cut as in the next graph:



Analysing in the same way all possible cases we obtain the next theorem (partially established in [4] with another method)

Theorem 1. Let X be a rational normal cubic surface in $\mathbb{P}^3$. Then X has one of the following groups of singularities, all possible: A_1, $2A_1$, A_2, $3A_1$, A_1A_2, A_3, $4A_1$, A_22A_1, A_3A_1, $2A_2$, A_4, D_4, A_32A_1, A_12A_2, A_4A_1, A_5, D_5, $3A_2$, A_5A_1, E_6), in accordance with table 1, Appendix 1, or it is nonsingular.

Observation: In the tables 1-6 of the Appendix 1 on the first column there is the group of singularities, on the second column there is the number of lines and on the third column there are the almost general positions in which can be arranged the points in order to obtain the group of singularities of the first column. In table 3 is not given the number of the exceptional divisors (conics). The notations in the tables are explained in the beginning of the Appendix 1. The same notations are used in Appendix 2 too.

§3. The Classification of Singularities and Structure Theorems for Degenerate Del Pezzo Surfaces of Degree 4 (i.e. rational normal complete intersections of two hyperquadrics in $\mathbb{P}^4$)

In [9] the classification of groups of singularities of these surfaces is obtained too, but using another methods. We'll apply a similar method to that of §2 and we'll obtain again all the possible groups of singularities and more, structure theorems similar to theorems in the table 1 from the Appendix 1 (see table 2 in the Appendix 1).

Let X be a rational normal surface of type (2,2) in $\mathbb{P}^4$ (i.e. the complete intersection of two hyperquadrics in $\mathbb{P}^4$). Becaus ω_X^{-1} is very ample it follows from the theorems A and C i) (§1) that X is obtained by blowing up r=9-4=5 points in an almost general position in $\mathbb{P}^2$, and by contracting the integral curves C with $p_a(C)=0$ and $C^2=-2$.

<u>Lemma 3:</u> Let $\tilde{X} \xrightarrow{\pi} X$ be the minimal desingularisation of X. Then $\tilde{X} \cong \mathbb{P}^2(\Sigma)$ with $\Sigma = \{P_1, \dots, P_5\}$ in an almost general position Pic $\tilde{X} \cong \mathbb{Z} \oplus \mathbb{Z}^5$ with $E_0; -E_1, \dots, -E_5$ as $\mathbb{Z}$ - basis (see §1). Then the integral curves $C \subset \tilde{X}$ with $p_a(C)=0$ and $C^2=-2$ ($\Leftrightarrow \omega_{\tilde{X}}.C=0$) are of one of the next kinds:

$$(0; -1, 1, 0, 0, 0),$$

$$\text{or} \quad (1; 1, 1, 1, 0, 0)$$

(inverse images of lines which contain 3 points in Σ), as far as they exist.

<u>Lemma 4:</u> The lines on X (i.e. the exceptional divisors of the first kind) are the (isomorphic) images by π of the divisors

on $\tilde{X}$ of one of the next kinds:

$$(0;\ -1,\ 0,\ 0,\ 0,\ 0),$$

or $$(1;\ 1,\ 1,\ 0,\ 0,\ 0)$$

(inverse images of lines which contain 2 points in Σ),

or $$(2;\ 1,\ 1,\ 1,\ 1,\ 1)$$

(the inverse image of the conic which contains all the points in Σ), as far as they exist.

The proves of lemmas 3 and 4 are analogously to those of lemmas 1 and 2 respectively (§ 2).

Using the same technics as in § 2 (totally 28 esentially different cases of arrangement of the 5 points in almost general positions; see Appendix 2, § 2), we obtain the next theorem (partially established in [9] too, using another method):

Theorem 2: Let $X \in \mathbb{P}^4$ a rational normal surface of type (2.2). Then X has one of the following groups of singularities, all possible: A_1, $2A_1$, A_2, $3A_1$, A_1A_2, A_3, $4A_1$, A_2, $2A_1$, A_3A_1, A_4, D_4, $A_3 2A_1$, D_5, in accordance with table 2, Appendix 1, or it is non-singular.

§4. The Classification of Singularities and Structure Theorems for Degenerate Del Pezzo Surfaces of Degree 2

In [11] the classification of groups of singularities of these surfaces is obtained too, but using another method. We'll apply a similar method to that of §2 and we'll obtain again all the possible groups of singularities and more, structure theorems similar to theorems in the table 1 from the Appendix 1 (see table 3 in the Appendix 1).

Let X be a degenerate Del Pezzo surface of degree 2. It follows from the theorems A (§1) that X is obtained by blowing up r=9-2=7 points in an almost general position in $\mathbb{P}^2$, and by contracting the integral curves C with $p_a(C)=0$ and $C^2=-2$.

Lemma 5: Let $\tilde{X} \xrightarrow{\pi} X$ be the minimal desingularisation of X. Then $\tilde{X} \cong \mathbb{P}^2(\Sigma)$ with $\Sigma=\{P_1,\ldots,P_7\}$ in an almost general position Pic $\tilde{X}=\mathbb{Z}\oplus\mathbb{Z}^7$ with $E_0; -E_1,\ldots,-E_7$ as $\mathbb{Z}$-basis (see §1). Then the integral curves $C\subset\tilde{X}$ with $p_a(C)=0$ and $C^2=-2$ ($\Leftrightarrow \omega_{\tilde{X}}.C=0$) are of one of the next kinds:

$$(0;\ -1,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0),$$

or $$(1;\ 1,\ 1,\ 1,\ 1,\ 0,\ 0,\ 0),$$

(inverse images of lines which contain 3 points in Σ),

or $$(2;\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 0)$$

(inverse image of conics which contain 6 points in Σ), as far as they exist.

Proof: - similar to the proof of lemma 1. Because the number of the exceptional divisors in this case is not at all stable for a given type of surface ("type" means the group of singularities) we don't insist in this direction.

Using the same technics as in §2 (totally 308 essentially different cases of arrangement of the points in almost general positions) see Appendix 2, §3, we obtain the next theorem (partially established in [11] too using another method):

Theorem 3: Let $X \subset \mathbb{P}^6$ (see corollary C1, §1) a degenerate Del Pezzo surface of degree 2. Then X has one of the following groups of singularities, all possible: A_1, $2A_1$, A_2, $3A_1$, A_1A_2, A_3, $4A_1$, $A_2 2A_1$, A_3A_1, $2A_2$, A_4, D_4, $A_3 2A_1$, $A_1 2A_2$, A_4A_1, A_2A_3, A_1D_4, $5A_1$, $A_2 3A_1$, A_5, D_5, $3A_2$, A_5A_1, $A_3 3A_1$, $6A_1$, $A_1A_2A_3$, $D_4 2A_1$, $2A_3$, A_2A_4, A_1D_5, A_6, D_6, E_6, $D_4 3A_1$, $A_1 2A_3$, A_2A_5, A_1D_6, A_7, E_7; in accordance with table 3, Appendix 1.

§5. The Classification of Singularities and Structure Theorems for Degenerate Del Pezzo Surfaces of Degree $d \in \{5, 6, 7\}$

In [12] the classification of groups of singularities of these types of surfaces is obtained too, using another method.

We'll apply a similar method to that of §2 and we'll obtain again all the possible groups of singularities and more, structure theorems similar to theorems in the table 1 from the Appendix 1 (see tables 4, 5, 6 in the Appendix 1).

Let X be a degenerate Del Pezzo surface of degree d=5,6 or 7. It follows from the theorem A (§1) that X is obtained by blowing up 4, 3 or 2 points in an almost general position in $\mathbb{P}^2$, respectively, and by contracting the integral curves C with $p_a(C)=0$ and $C^2=-2$.

<u>Lemma 6</u>: Let X be a Degenerate Del Pezzo surface of degree 5 and let $\widetilde{X} \xrightarrow{\pi} X$ be the minimal desingularisation of X. Then $\widetilde{X} \cong \mathbb{P}^2(\Sigma)$, with $\Sigma = \{P_1, P_2, P_3, P_4\}$ in an almost general position in $\mathbb{P}^2$; Pic $\widetilde{X} \cong \mathbb{Z} \oplus \mathbb{Z}^4$ with E_0; $-E_1$, $-E_2$, $-E_3$, $-E_4$ as $\mathbb{Z}$- basis (see §1). Then the integral curves $C \subset \widetilde{X}$ with $p_a(C)=0$ and $C^2=-2$ ($\Leftrightarrow \omega_{\widetilde{X}}.C=0$) are of one of the next kinds:

$$(0;\ -1,\ 1,\ 0,\ 0)$$

<u>or</u> $$(1;\ 1,\ 1,\ 1,\ 0)$$

(inverse images of lines which contain 3 points in Σ), as far as they exist.

<u>Lemma 7</u>: Let be X, $\widetilde{X}$, π as in lemma 6. Then the lines of X (i.e. the exceptional divisors of the first kind) are the (isomorphic) images by π of the divisors on $\widetilde{X}$ of one of the next kinds:

$$(0;\ -1,\ 0,\ 0,\ 0),$$

<u>or</u> $$(1;\ 1,\ 1,\ 0,\ 0)$$

(inverse images of lines which contain 2 points in Σ), as far as they exist.

The <u>proves</u> of lemma 6 and 7 are analogously to those of lemma 1 and 2 respectively (§2).

different cases of arrangement of the 4 points in almost general positions in $\mathbb{P}^2$, see Appendix 2, §4) we obtain the next theorem (partially established in [12] too using another method):

Theorem 4: Let $X \subset \mathbb{P}^5$ (see corollary C1, §1) a degenerate Del Pezzo surface of degree 5. Then X has one of the following groups of singularities, all possible: A_1, $2A_1$, A_2, A_1A_2, A_3, A_4 in accordance with table 4, Appendix 1.

Lemma 8: Let X be a degenerate Del Pezzo surface of degree 6 and let $\tilde{X} \xrightarrow{\pi} X$ be the minimal desingularisation of X. Then $\tilde{X} \cong \mathbb{P}^2(\Sigma)$ with $\Sigma = \{P_1, P_2, P_3\}$ in an almost general position in $\mathbb{P}^2$; Pic $\tilde{X} \cong \mathbb{Z} \oplus \mathbb{Z}^3$ with E_0; $-E_1$, $-E_2$, $-E_3$ as $\mathbb{Z}$ - basis (see §1). Then the integral curves $C \subset \tilde{X}$ with $p_a(C)=0$ and $C^2=-2$ ($\Leftrightarrow \omega_{\tilde{X}}.C=0$) are of one of the next kinds:

$$(0; -1, 1, 0)$$

or $$(1; 1, 1, 1)$$

(the inverse image of the line which contains all the points of Σ) as far as they exist.

Lemma 9: Let be X, $\tilde{X}$, π as in lemma 8. Then the lines on X (i.e. the exceptional divisors of the first kind) are the (isomorphic) images by π of the divisors on $\tilde{X}$ of one of the next kinds:

$$(0; -1, 0, 0)$$

or $$(1; 1, 1, 0)$$

(the inverse image of the line which contain all the points of Σ), as far as they exist.

The proves of lemmas 8 and 9 are similar to those of lemmas 1 and 2 respectively (§2).

Using the same technics as in §2 (totally 6 essentially different cases of arrangement of the 3 points in almost general positions in $\mathbb{P}^2$, see Appendix 2, §5) we obtain the next theorem (partially established in [12] too, using another method):

Theorem 5: Let $X \subset \mathbb{P}^6$ (see corollary C1, §1) a degenerate Del Pezzo surface of degree 6. Then X has one of the following groups of singularities, all possible: A_1, $2A_1$, A_2, A_1A_2 in accordance with table 5, Appendix 1.

Lemma 10: Let X be a degenerate Del Pezzo surface of degree 7 and let $\tilde{X} \xrightarrow{\pi} X$ be the minimal desingularisation of X. Then $\tilde{X} \cong \mathbb{P}^2(\Sigma)$ with $\Sigma = \{P_1, P_2\}$, $P_1 \in \mathbb{P}^2$, $P_2 \in \mathbb{P}^2(\{P_1\})$; Pic $\tilde{X} \cong \mathbb{Z} \oplus \mathbb{Z}^2$ with E_0; $-E_1$, $-E_2$ as $\mathbb{Z}$- basis (see §1). Then the integral curves $C \subset \tilde{X}$ with $p_a(C)=0$ and $C^2=-2$ ($\Leftrightarrow \omega_{\tilde{X}} \cdot C = 0$) are of one of the next kind:

$$(0; -1, 1)$$

as far as they exist.

Lemma 11: Let be X, $\tilde{X}$, π as in lemma 10. Then the lines on X (i.e. the exceptional divisors of the first kind) are the (isomorphic) images by π of the divisors on $\tilde{X}$ of one of the next kinds:

$$(0; -1, 0),$$

or $$(1; 1, 1)$$

(the inverse image of the line which contain all the points of Σ), as for as they exist.

The proves of lemmas 10 and 11 are similar to those of lemmas 1 and 2 respectively (§2).

Using the same technics as in §2 (totally 2 essentially different cases, see Appendix 2, §6) we obtain the next theorem (partially established in [12] too, using another method).

Theorem 6: Let $X \subset \mathbb{P}^7$ (see corollary C1, §1) a degenerate Del Pezzo surface of degree 7. Then X has only one singularity of type A1, see table 6, Appendix 1.

§6. On the Classification of Singularities of Degenerate Del Pezzo Surfaces of Degree 1

In [11] the classification of groups of singularities of these surfaces is obtained. In order to apply a similar method to that of § 2 to obtain the possible groups of singularities and structure theorems we see that we have a lot of work, and it's impossible to do it without using a computer. We'll write perhaps another article with the structure theorems (a table similar to tables 1-6 in Appendix 1) obtained using a computer.

In this § we'll obtain only partial results.

Let X be a degenerate Del Pezzo surface of degree 1. It follows from the theorem A (§1) that X is obtained by blowing up r=9-1=8 points in an almost general position in $\mathbb{P}^2$ and by contracting the integral curves C with $p_a(C)=0$ and $C^2=-2$.

Lemma 12: Let $\tilde{X}\xrightarrow{\pi}X$ be the minimal desingularisation of X. Then $\tilde{X}\cong \mathbb{P}^2(\Sigma)$, with $\Sigma=\{P_1,\dots,P_8\}$ in an almost general position in $\mathbb{P}^2$, Pic $\tilde{X}\cong\mathbb{Z}\oplus\mathbb{Z}^8$ with E_o; $-E_1,\dots,-E_8$ as $\mathbb{Z}$-basis (see § 1) Then the integral curves $C\subset\tilde{X}$ with $p_a(C)=0$ and $C^2=-2$ ($\Leftrightarrow \omega_{\tilde{X}}.C=0$) are of one of the next kinds:

(0; -1, 1, 0, 0, 0, 0, 0, 0)

or (1; 1, 1, 1, 0, 0, 0, 0, 0)

(inverse images of lines which contain 3 points in Σ),

or (2; 1, 1, 1, 1, 1, 1, 0, 0)

(inverse images of conics which contain 6 points in Σ),

<u>or</u> $(3;2,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1)$

(inverse images of cubics which contain all the points of Σ and having a double point in one point of Σ),
as far as they exist.

<u>Proof:</u> Similar to the proof of lemma 1 (§1).

<u>Proposition 1:</u> For every group of singularities in the <u>list (1)</u>, there is a degenerate Del Pezzo surface of degree one, $X \subset \mathbb{P}^6$ (see corollary C1 iii), §1), having this group of singularities.

<u>List (1)</u>

A_1, A_2, $2A_1$, A_3, A_1A_2, $3A_1$, A_4, D_4, A_1A_3, A_2 $2A_1$, $2A_2$, $4A_1$, A_5, D_5, A_4A_1, D_4A_1, A_3 $2A_1$, A_2 $3A_1$, A_1 $2A_2$, $5A_1$, A_3A_2, A_6, D_6, E_6, A_5A_1, D_5 A_1, A_4 $2A_1$, D_4 $2A_1$, A_3 $3A_1$, A_2 $4A_1$, $2A_1$ $2A_2$, $6A_1$, A_4A_2, D_4 A_2, D_4 $2A_1$, $A_1A_2A_3$, $3A_2$, $2A_3$, E_7, A_7, D_7, A_1A_6, A_1 D_6, A_1 E_6, A_5 $2A_1$, D_5 $2A_1$, D_4 $3A_1$, A_3 $4A_1$, $A_1A_2A_4$, A_2A_3 $2A_1$, A_1 $3A_2$, A_1 $2A_3$, A_5A_2, A_3A_4, A_3 D_4, E_8, A_8, D_8, D_5 A_2, E_7 A_1, A_1A_7, D_6 $2A_1$, $A_1A_2A_5$, $2A_1$ $2A_3$, E_6 A_2, A_3 D_5, $2A_4$, $4A_2$, $2D_4$.

<u>Proof:</u> We find in every case, an almost general position in $\mathbb{P}^2$ of the 8 points $P_1, \ldots, P_8$, such that, using the lemma 12 and the same technics as in § 2 to find on $\tilde{X} \cong \mathbb{P}^2(\Sigma)$ $(\Sigma = \{P_1, \ldots, P_8\})$ exactly the desired configurations of integral curves C with $p_a(C)=0$ and $C^2=-2$ (see Appendix 2, § 7).

Finally, let's observe that we can improve the contain of proposition 1. Indeed, from [2] (1 Theorem, (E)) it follows that the number of curves $C \subset \tilde{X}$ with $p_a(C)=0$ and $C^2=-2$ is $\beta \leq 8$. To be more precise, $\beta=9-b_2(X)$, where b_2 is the second Betti's number. Writing all the possible groups of singularities (such that the union of graphs has not more then 8 picks) we note that they are all obtained for the graphs Γ (not necessary connected) with ≤ 6 vertex. When Γ has 7 vertex it might appear the following groups of singularities (corresponding to the graphs Γ with 7 vertex which are not mentioned in the proposition 1): A_4 $3A_1$, A_2 $5A_1$, $3A_1$ $2A_2$,

$A_1 A_2$ D_4, A_3 $2A_2$ (we used the fact that it couldn't be more than 6 singularities, too). For the graphs Γ with 8 vertex we are in the case $\beta=8$, hence $b_2(X)=1$.

Using [2] (the remark after theorem 1) in this case X has the rational cohomology type of $\mathbb{P}^2$ (i.e. X and $\mathbb{P}^2$ have the same Betti's numbers, see [3]). Using now [3], theorem 7, f) for n=8, it follows that the graphs Γ with 8 vertex which appear are exactly those in proposition 1.

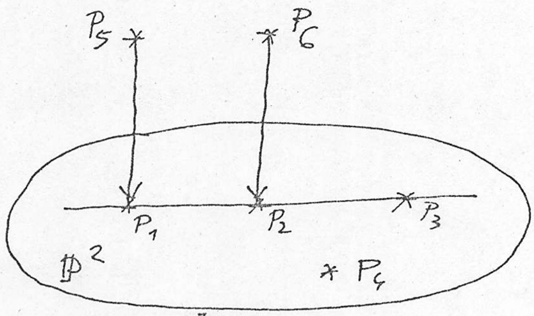
Concluding now, the only exceeding groups of singularities which can appear on degenerate Del Pezzo surfaces of degree except those in proposition 1 are those for the graphs Γ with 7 vertex, namely A_4 $3A_1$, A_2 $5A_1$, $3A_1$ $2A_2$, A_1 A_2 D_4, A_3 $2A_2$ (which in fact don't appear, [11]) about which the method of this § don't give any answer if they really appear or not.

APPENDIX 1

a) Preliminary notations

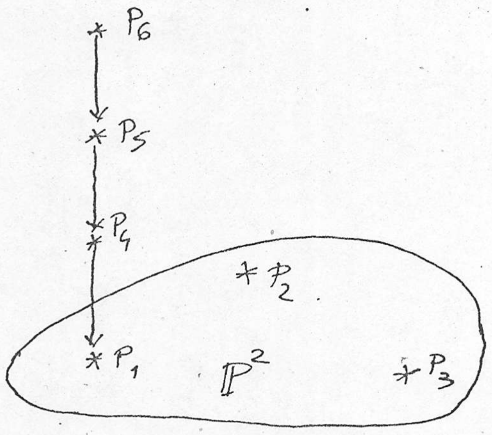
Let be $\Sigma = \{P_1, P_2, P_3, P_4, P_5, P_6\}$ a set of points in an almost general position in $\mathbb{P}^2$ (see § 1).

Let's consider the next three figures:



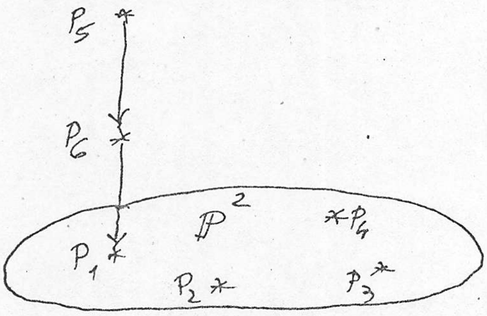
6=(2,4)

figure 1



$5=(4,1)^e$

figure 2

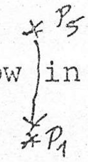


(1,2,3,4,5,6)
on a conic

figure 3

Explanation of the notations in fig.1 - 3:

- The fact that the points P_1, P_2, P_3, P_4 in fig.1, the points P_1, P_2, P_3 in fig.2, the points P_1, P_2, P_3, P_4 in fig.3 are encircled means that these points are in $\mathbb{P}^2$.

- The fact that there is an arrow 

in fig.1 means that the point P_5 is on the total transformation E_1 of the point P_1 after the blowing up of P_1 and satisfying the condition of almost general position (see § 1). Analogously in fig.2,3.

- The fact that there is a line containing the points P_1, P_2, P_3 in fig.1 means that this three points are collinear.

- The notation $6=(2,4)$ in fig.1 means that the line containing P_2 and P_4 satisfies the tangent condition given by P_6.

- The notation $5=(4,1)^e$ in fig.2 means that the line containing P_1 and satisfying the tangent condition given by P_4, satisfies the second order tangent condition given by P_5 too.

- The notation (1, 2, 3, 4, 5, 6) on a conic (fig.3) means that there is an irreducible conic containing the points P_1, P_2, P_3, P_4 and satisfying the tangent condition given by P_6 and the second order condition given by P_5.

b) Final notations

In the tables of Appendix 1 we'll use different notations than in a) for specifying the positions of the points as follows:

- Instead of fig.1 we'll write:

((1, 2, 3, 4))

5/1, 6/2

(1,2,3), (6,2,4)

- Instead of fig.2 we'll write:

((1, 2, 3))

6/5/4/1

(5, 4, 1)

- Instead of fig.3 we'll write:

((1, 2, 3, 4))

5/6/1

(1,2,3,4,5,6)

Observations:

1. In the next tables in the first column there is the group of singularities of the surface, on the second column there is the number of lines of the surface having the group of singularities of the first column and on the third column there are the positions in which can be arranged the points in order to obtain the group of singularities of the first column.

2. If we don't write (1,2,3) we understand that the points P_1, P_2, P_3 are not collinear; analogously for the conic.

3. A set of points $\Sigma = \{P_1, -, P_r\}$, $(r \leq 8)$ is said to be in a general position if:

i) $\Sigma \subset \mathbb{P}^2$;

ii) no 3 points in Σ are collinear (if $r \geq 3$)

iii) no 6 points in Σ are on a conic (if $r \geq 6$).

§1. Table 1 (pages 28 – 31): (d=3)

1	2	3		
The group of singularities	Number of lines	The positions in which can be arranged the points		
Nonsingular	27	general position		
A_1	21	((1,2,3,4,5,6)) (1,2,3)	((1,2,3,4,5,6)) (1,2,3,4,5,6)	((1,2,3,4,5)) 6/1
$2A_1$	16	((1,2,3,4,5,6)) (1,2,3) (2,4,5)	((1,2,3,4,5)) 6/4 (1,2,3)	((1,2,3,4,5)) 6/1 (6,2,1)
		((1,2,3,4,5)) 6/1 (1,2,3,4,5,6)	((1,2,3,4)) 5/1, 6/4	
A_2	15	((1,2,3,4,5,6)) (1,2,3) (4,5,6)	((1,2,3,4,5)) 6/2 (1,2,3)	((1,2,3,4)) 6/5/1
$3A_1$	12	((1,2,3,4,5,6)) (1,2,3), (1,4,5) (3,6,5)	((1,2,3,4,5)) 6/4 (1,2,3), (6,4,1)	
		((1,2,3,4)) 5/1, 6/2 (1,2,3,4,5,6)	((1,2,3,4)) 5/1, 6/2 (5,1,3)	((1,2,3)) 4/1, 5/2 6/3
A_3	10	((1,2,3,4,5)) (1,2,3) (2,4,5), 6/2	((1,2,3,4)) 6/1, 5/2	((1,2,3,4)) 6/5/1 (5,1,2)
		((1,2,3,4)), 6/5/1 (1,2,3)	((1,2,3)) 6/5/4/2	

1	2	3		
A_1A_2	11	((1,2,3,4,5)) (1,2,3), 6/2 (6,2,4)	((1,2,3,4,5)) (1,2,3), 6/4 (6,4,5)	((1,2,3,4,5)) (1,2,3), 6/3 (2,4,5)
		((1,2,3,4)) 5/1, 6/2 (5,1,2)	(1,2,3,4)) 5/1, 6/4 (1,2,3)	((1,2,3,4)) 6/5/1 (1,2,3,4,5,6)
		((1,2,3,4)) 6/5/1 (6,5,1)	((1,2,3,4)) 6/5/4	((1,2,3)) 5/4/1, 6/2
$4A_1$	9	((1,2,3,4,5,6)) (1,2,3),(1,4,5) (4,2,6),(5,3,6)	((1,2,3,4)) 5/1, (5,1,3) 6/2, (6,2,3)	((1,2,3)) 4/1, 5/2, 6/3 (1,2,3,4,5,6)
A_22A_1	8	((1,2,3)) 5/4/1, 6/2 (1,2,3,4,5,6)	((1,2,3)) 5/4/1, 6/2 (5,4,1)	((1,2,3)) 5/4/1, 6/2 (6,2,3)
		((1,2,3,4)) 5/1, (5,1,2) 6/2, (6,2,3)	(1,2,3,4)) 5/1, (5,1,3) 6/2, (6,2,4)	((1,2,3,4,5)) 6/3, (1,2,3) (2,4,5), (6,3,4)
		((1,2,3,4)), (1,2,3) 5/1, 6/4, (6,4,2)	((1,2,3)), 4/1, 5/2 6/3, (4,1,2)	
A_3A_1	7	((1,2,3,4)) 6/1, 5/2 (6,1,4), (1,2,3)	((1,2,3,4)) (1,2,3), 6/5/1 (6,5,1)	((1,2,3,4)) 5/1, 6/4 (1,2,3), (6,4,1)
		((1,2,3,4)) 6/5/4, (1,2,3) (5,4,1)	((1,2,3)), 5/4/1 6/2, (4,1,3)	((1,2,3)), 5/4/1 6/2, (6,2,1)
		((1,2,3)), 6/5/4/2 (1,2,3,4,5,6)	((1,2)), 6/2 5/4/3/1	

1	2	3		
$2A_2$	7	((1,2,3,4)) 5/1, (1,2,3) 6/4, (5,1,4)	((1,2,3,4)) 6/5/4, (1,2,3) (6,5,4)	((1,2)) 4/3/1 6/5/2
A_4	6	((1,2,3,4)) 6/5/1, (1,2,3) (5,1,4)	((1,2,3)), 5/4/1 6/2, (4,1,2)	((1,2,3)), 5/4/1 6/2, (1,2,3)
		((1,2,3)),6/5/4/2 (5,4,2)	((1,2,3)), (1,2,3) 6/5/4/2	((1,6)) 5/4/3/2/1
D_4	6	((1,2,3)), (1,2,3) 4/1, 5/2, 6/3	((1,2,3)), 6/5/4/2 (4,2,1)	
$A_3 2A_1$	5	((1,2,3,4)), 6/1, 5/2 (1,2,3), (6,1,4), (5,2,4)	((1,2,3)), 5/4/1, 6/2 (6,2,1), (5,4,1)	
		((1,2,3)), 5/4/1, 6/2 (6,2,3), (4,1,3)	((1,2)), 5/4/3/1, 6/2 (1,2,3,4,5,6)	
$A_1 2A_2$	5	((1,2,3,4)), 5/1, 6/4 (1,2,3), (6,4,2), (5,1,4)	((1,2,3)), 5/4/1, 6/2 (6,2,3), (5,4,1)	
		((1,2,3)), 4/1, 5/2 6/3, (4,1,2) (5,2,3)	((1,2)), 4/3/1 6/5/2 (1,2,3,4,5,6)	((1,2)), 4/3/1 6/5/2 (6,5,2)
$A_4 A_1$	4	((1,2,3)), 5/4/1 6/2, (6,2,1) (4,1,3)	((1,2,3)), 6/2 5/4/1, (6,2,3) (4,1,2)	((1,2)), 5/4/3/1 6/2, (4,3,1)
		((1,2,3)), (1,2,3) 5/4/1, 6/2 (5,4,1)	((1,2)), 6/2 5/4/3/1 (6,1,2)	((1,6)) 5/4/3/2/1 (1,2,3,4,5,6)
A_5	3	((1,2)), 4/3/1 6/5/2, (3,1,2)	((1,2,3)), (1,2,3) 6/5/4/2, (5,4,2)	((1)) 6/5/4/3/2/1
D_5	3	((1,2)), 5/4/3/1 [illegible]	((1,6)), (3,2,1) 5/4/3/2/1	((1,6)), (2,1,6) 5/4/3/2/1

1	2	3		
$3A_2$	3	((1,2,3)), 4/1, 5/2, 6/3 (4,1,2), (6,3,1), (5,2,3)	((1,2)), 5/3/1, 6/4/2 (5,3,1), (6, 4, 2)	
A_5A_1	2	((1,2)), 4/3/1 6/5/2, (3,1,2) (6,5,2)	((1,2)), 5/4/3/1 6/2, (6,1,2) (4,3,1)	((1)), 6/5/4/3/2/1 (1,2,3,4,5,6)
E_6	1	((1)), 6/5/4/3/2/1, (3,2,1)		

§2. Table 2 (d=4)

1	2	3		
Non-singular	16	general position		
A_1	12	((1,2,3,4,5)), (1,2,3)	((1,2,3,4)), 5/1	
$2A_1$	8 or 9	((1,2,3,4,5)),(1,2,3),(2,4,5)	((1,2,3,4)), 5/1, 5/1/2	
		((1,2,3,4)), (1,2,3), 5/4	((1,2,3)), 4/1, 5/2	
A_2	8	((1,2,3,4)), 5/1, (1,2,3)	((1,2,3)), 5/4/1	
$3A_1$	6	((1,2,3,4)), 5/4, (1,2,3) (5,4,1)	((1,2,3)), 4/1, 5/2 (4,1,3)	
A_1A_2	6	((1,2,3,4)),(1,2,3), 5/1 (5,1,4)	((1,2,3)), (4,1,2) 4/1, 5/2	
		((1,2)), 4/3/1, 5/2	((1,2,3)), 5/4/1, (5,4,1)	
A_3	5	((1,2,3)), 4/1 5/2, (1,2,3)	((1,2,3)), 5/4/1 (4,1,2)	((1,2)), 5/4/3/1
$4A_1$	4	((1,2,3)), 4/1, 5/2, (4,1,3), (5,2,3)		
A_22A_1	4	((1,2,3)), 4/1, 5/2 (4,1,2), (5,2,3)	((1,2)), 4/3/1, 5/2 (4,3,1)	
A_3A_1	3	((1,2)), 4/3/1, 5/2, (5,2,1)		
A_4	3	((1,2)), 4/3/1 5/2, (3,1,2)	((1,2)), 5/4/3/1 (4,3,1)	((1)) 5/4/3/2/1
D_4	2	((1,2)), 5/4/3/1, (3,1,2)		
A_32A_1	2	((1,2)), 4/3/1, 5/2, (5,2,1), (4,3,1)		
D_5	1	((1)), 5/4/3/2/1, (3,2,1)		

§3. Table 3 (pages 33-41) (d=2)

1	2		
The group of singularities	The position in which can be arranged the points		
Nonsingular	General position		
A_1	((1,2,3,4,5,6,7)) (1,2,3,4,5,6)	((1,2,3,4,5,6,7)) (1,2,3)	((1,2,3,4,5,6)) 7/6
$2A_1$	((1,2,3,4,5,6,7)) (1,2,3) (2,3,4,5,6,7)	((1,2,3,4,5,6,7)) (1,2,3) (1,4,5)	((1,2,3,4,5,6)) 7/6, (7,6,1)
	((1,2,3,4,5,6)) 7/6 (1,2,3,4,6,7)	((1,2,3,4,5,6)) 7/4 (1,2,3)	((1,2,3,4,5)) 6/1, 7/2
A_2	((1,2,3,4,5,6,7)) (1,2,3), (4,5,6)	((1,2,3,4,5,6)), 7/6 (1,2,3,4,5,6)	
	((1,2,3,4,5,6)), 7/1 (1,2,3)	((1,2,3,4,5)), 7/6/1	
$3A_1$	((1,2,3,4,5,6,7)) (1,2,3), (1,4,5) (2,3,4,5,6,7)	((1,2,3,4,5,6,7)) (1,2,3), (1,6,7) (1,4,5)	((1,2,3,4,5,6,7)) (1,4,5), (1,2,3) (3,6,5)
	((1,2,3,4,5,6)) 7/4, (1,2,3) (2,3,4,5,6,7)	((1,2,3,4,5,6)) 7/4, (1,2,3) (7,4,1)	((1,2,3,4,5,6)) 7/6, (1,2,3) (3,4,5)
	((1,2,3,4,5)), 6/1, 7/2 (1,6,2,7,3,4)	((1,2,3,4,5)), 6/1, 7/2 (6,1,5)	
	((1,2,3,4,5)), 6/4, 7/5 (1,2,3)	((1,2,3,4)), 5/1, 6/3 7/2	

1	2		
A_1A_2	((1,2,3,4,5,6,7)) (1,2,3), (4,5,6) (2,5,7)	((1,2,3,4,5,6)) 7/6, (7,6,1) (1,2,3,4,5,6)	((1,2,3,4,5,6)), 7/1 (1,2,4,5,6,7) (1,2,3)
	((1,2,3,4,5,6)) 7/1, (1,2,3) (7,1,4)	((1,2,3,4,5,6)) 7/4, (7, 4, 5) (1,2,3)	((1,2,3,4,5,6,7)) 7/1, (1,2,3) (3,4,5)
	((1,2,3,4,5)) 6/1, 7/2 (1,6,2,3,4,5)	((1,2,3,4,5)) 6/1, 7/2 (6,1,2)	((1,2,3,4,5)) 6/1, 7/4, (1,2,3)
	((1,2,3,4,5)), 7/6/1 (1,6,7,2,3,5)	((1,2,3,4,5)), 7/6/1 (7,6,1)	
	((1,2,3,4,5)), 7/6/4, (1,2,3)	((1,2,3,4)), 6/5/1, 7/2	
A_3	((1,2,3,4,5,6)) 7/5, (1,2,3) (4,5,6)	((1,2,3,4,5,6)) 7/3, (1,2,3) (3,4,5)	((1,2,3,4,5)), 6/1 7/2, (1,2,3)
	((1,2,3,4,5)), 7/6/1 (1,6,2,3,4,5)	((1,2,3,4,5)), 7/6/1, (6,1,2)	
	((1,2,3,4,7)), 6/5/1, (1,2,3)	((1,2,3,4)), 7/6/5/1	
$4A_1$	((1,2,3,4,5,6,7)) (1,2,3), (1,6,7) (1,4,5) (2,3,4,5,6,7)	((1,2,3,4,5,6,7)) (1,2,3), (1,7,6) (1,4,5), (3,6,5)	((1,2,3,4,5,6,7)) (1,2,3), (1,4,5) (4,2,6), (6,3,5)
	((1,2,3,4,5,6)) 7/4, (7,4,1) (1,2,3) (2,3,4,5,6,7)	((1,2,3,4,5,6)) 7/6, (1,2,3) (3,4,5),(7,6,3)	((1,2,3,4,5,6)) 7/6, (1,2,3) (3,4,5) (1,2,4,5,6,7)
	((1,2,3,4,5)), 6/1 7/2, (6,1,5) (7,2,5)	((1,2,3,4,5)), 6/4 7/5, (1,2,3) (1,4,5,6,7,3)	((1,2,3,4,5)), 6/4 7/5, (1,2,3) (6,4,1)
	((1,2,3,4)), 5/1, 6/3, 7/2 (1,5,3,6,2,7)	((1,2,3,4)), 5/1, 6/3, 7/2 (5,1,4)	

1	2		
$A_2 2A_1$	((1,2,3,4,5,6,7)),(1,2,3), (4,5,6) (4,7,3), (2,7,5)	((1,2,3,4,5,6)),7/1, (1,2,3) (3,4,5), (7,1,4)	
	((1,2,3,4,5,6)),7/1,(1,2,3) (3,4,5), (1,7,2,4,5,6)	((1,2,3,4,5,6)),7/6,(1,2,3) (3,4,5), (7,6,5)	
	((1,2,3,4,5)),6/1 7/2, (6,1,5) ... (7,2,4)	((1,2,3,4,5)),6/1,7/2 (6,1,2), (7,2,3)	((1,2,3,4,5)),6/1,7/2 (6,1,5),(1,5,2,7,3,4)
	((1,2,3,4,5)),6/1 7/4, (1,6,4,7,2,5) (1,2,3)	((1,2,3,4,5)),6/1,7/4 (1,2,3), (6,1,5)	((1,2,3,4)),5/1,6/3 7/2, (1,5,3,6,2,4)
	((1,2,3,4)),5/1, 6/3 7/2, (5,1,2)	((1,2,3,4,5)),7/6/4 (1,2,3),(4,6,7,1,3,5)	((1,2,3,4)),6/5/1, 7/2 (1,5,6,7,2,4)
	((1,2,3,4)),6/5/1 7/2, (7,2,3)	((1,2,3,4)),6/5/1 7/2, (6,5,1)	((1,2,3)),7/4/1 6/2, 7/3
A_3A_1	((1,2,3,4,5,6)), 7/5 (4,5,6),(1,2,3) (7,5,1)	((1,2,3,4,5,6)),7/3 (1,2,3), (3,4,5) (7,3,6)	((1,2,3,4,5)),6/1, 7/2 (1,2,3) (1,6,2,7,4,5)
	((1,2,3,4,5)),6/1 7/2, (1,2,3) (6,1,4)	((1,2,3,4,5)),6/4 7/5, (1,2,3) (6,4,5)	((1,2,3,4,5)),6/1, 7/3 (1,2,3), (2,4,5)
	((1,2,3,4)),5/1, 6/2 7/4,(1,2,3)	((1,2,3,4,5)),7/6/5 (6,1,2),(1,6,7,3,4,5)	((1,2,3,4,5)),7/6/1 (7,6,1),(1,6,2,3,4,5)
	((1,2,3,4,7)), 6/5/1 (1,2,3),(1,5,6,3,4,7)	((1,2,3,4,7)),6/5/1 (1,2,3),(6,5,1)	((1,2,3,4,5)),7/6/4 (1,2,3), (6,4,1)
	((1,2,3,4,5)), 7/6/1 (1,2,3),(2,4,5)	((1,2,3,4)),6/5/1 7/2, (1,5,2,7,3,4)	((1,2,3,4)),6/5/1 7/2, (7,2,1)
	((1,2,3,4)),6/5/1 7/2, (5,1,4)	(1,2,3,4)),6/5/1 7/4, (1,2,3)	((1,2,3,4)),7/6/5/1 (1,5,6,7,2,3)
	((1,2,3,4)),7/6/5/4	((1,2,3)), 6/5/4/1, 7/2	

1	2		
$2A_2$	((1,2,3,4,5,6)),7/1 (1,2,3),(3,4,5), (7,1,6)	((1,2,3,4,5),6/1,7/2 (6,1,2),(1,2,7,3,4,5)	((1,2,3,4,5)),6/1, 7/2 (1,2,3),(6,1,4)
	((1,2,3,4,5)), 6/1, 7/5 (1,2,3),(3,4,5)	((1,2,3,4)),7/6/4 (1,2,3), (7,6,4)	((1,2,3,4)),6/5/1 7/2,(1,5,6,2,3,4)
	((1,2,3,4)),7/1,6/5/4, (1,2,3)	((1,2,3)), 5/4/1, 7/6/2	
A_4	((1,2,3,4,5)), 6/1 7/2 (1,2,3),(2,4,5)	((1,2,3,4,7)),6/5/1 (1,2,3), (5,1,4)	((1,2,3,4)),7/6/4 (1,2,3), (6,4,5)
	((1,2,3,4)),6/5/1 7/2, (5,1,2)	((1,2,3,4)),7/6/5/1 (1,5,6,2,3,4)	((1,2,3,4)),7/6/5/1 (6,5,1)
	((1,2,3,4)),7/6/5/1 (1,2,3)	((1,2,3)),7/6/5/4/1	((1,2,3,4)),6/5/1 7/2, (1,2,3)
D_4	((1,2,3,4)),5/1, 6/2 7/3, (1,2,3)	((1,2,3,4,5)),7/6/2 (1,2,3), (2,4,5)	((1,2,3,4)) 7/6/5/1, (5,1,2)
$A_3 2A_1$	((1,2,3,4,5,6)) 7/1 (1,5,6),(1,2,3) (4,3,6),(5,2,4)	((1,2,3,4,5)),6/1, 7/2 (1,2,3),(6,1,4) (7,2,4)	((1,2,3,4,5)),6/1 7/4, (1,2,3) (6,1,5),(7,4,1)
	((1,2,3,4,5)),6/1, 7/4 (1,2,3),(6,1,5), (7,4,5)	((1,2,3,4,5)),6/4, 7/5 (1,2,3),(6,4,5), (7,5,1)	((1,2,3,4,5)),6/1, 7/3 (1,2,3),(2,4,5) (1,6,3,7,4,5)
	((1,2,3,4,5)),6/1, 7/3 (1,2,3),(2,4,5),(6,1,4)	((1,2,3,4)),5/1,6/3 7/2,(5,1,2),(6,3,4)	((1,2,3,4)),5/1,6/3 7/2(5,1,2),(6,3,2)
	((1,2,3,4)),5/1,6/2 7/4,(1,2,3),(1,5, 2,6,4,7)	((1,2,3,4)),5/1,6/2 7/4,(1,2,3),(7,4,3)	((1,2,3,4,5)),7/6/4 (1,2,3),(6,4,1) (4,6,7,2,3,5)
	((1,2,3,4,5)),7/6/1 (1,2,3),(2,4,5), (1,6,7,3,4,5)	((1,2,3,4)),6/5/1,7/2 (6,5,1), (1,5,2,7,3,4)	((1,2,3,4)),6/5/1, 7/2,(7,2,1), (6,5,1)
	((1,2,3,4)),6/5/1, 7/2,(5,1,4), (1,5,6,2,7,3)	((1,2,3)),6/5/1,7/2 (5,1,4),(7,2,4)	((1,2,3,4)),6/5/1,7/4 (1,2,3), (1,5,6,2,4,7)
	((1,2,3,4)),6/5/1, 7/4, (1,2,3), (6,5,1)	((1,2,3,4)),6/5/1, 7/4, (1,2,3), (7,4,2)	((1,2,3)),5/4/1,6/2, 7/3,(1,4,2,6,3,7)
	((1,2,3)),5/4/1, 6/2,7/3,(6,2,1)	((1,2,3,4)),7/6/5/4,(4,5,6,7,1,2)	
	((1,2,3)),6/5/4/1, 7/2	((1,2,3)),6/5/4/1,7/2 (7,2,3)	

1	2		
$A_1 2A_2$	((1,2,3,4,5)),6/1, 7/4,(1,2,3), (6,1,5),(7,4,2)	((1,2,3,4,5)),6/1,7/4 (1,2,3),(6,1,4),(7,4,2)	((1,2,3,4,5)),6/1,7/5 (1,2,3),(4,2,5), (6,1,4)
	((1,2,3,4,5)),6/1, 7/5,(1,2,3), (4,2,5) (1,6,5,7,3,4)	((1,2,3,4)),5/1,6/3, 7/2, (5,1,2) (1,3,6,2,7,4)	((1,2,3,4)),5/1,6/3 7/2, (5,1,2), (6,3,1)
	((1,2,3,4)),6/5/1, 7/2, (7,2,3), (1,5,6,2,3,4)	((1,2,3,4)),6/5/1,7/2 (7,2,3), (6,5,1)	((1,2,3,4)),6/5/4,7/1 (1,2,3),(4,5,6,1,7,2)
	((1,2,3)),5/4/1,6/2,7/3,(1,5,4,2,6,3)	((1,2,3)),5/4/1, 6/2, 7/3, (6,2,3)	
	((1,2,3)),5/4/1,7/6/2,(1,4,5,2,6,7)	((1,2,3)),5/4/1,7/6/2,(5,4,1)	
A_4A_1	((1,2,3,4,5)),6/1, 7/2,(1,2,3),(2,4,5), (6,1,4)	((1,2,3,4)),5/1,6/2 7/4, (7,4,1,),(1,2,3)	((1,2,3,4,5)),7/6/1 (1,2,3),(2,4,5),(6,1,4)
	((1,2,3,4)),6/5/1, 7/2,(5,1,4), (7,2,3)	((1,2,3,4)),6/5/1,7/2, (5,1,4),(7,2,1)	((1,2,3,4)),6/5/1,7/2 (7,2,3), (5,1,2)
	((1,2,3,4)),6/5/1, 7/2,(1,2,3), (1,5,6,2,7,4)	((1,2,3,4)),6/5/1,7/2, (1,2,3),(6,5,1)	((1,2,3,4)),6/5/1,7/2 (1,2,3),(7,2,4)
	((1,2,3,4)),7/6/5/1 (1,2,3),(1,5,6,7,4,2)	((1,2,3)),6/5/4/1, 7/2, (1,4,5,2,7,3)	((1,2,3)),6/5/4/1, 7/2, (5,4,1)
	((1,2,3)),6/5/4/1,7/2,(7,2,1)	((1,2,3)),7/6/5/4/1,(1,4,5,6,7,2)	
	((1,2)),6/5/4/3/1, 7/2	((1,2,3)),5/4/1,6/2,7/3,(4,1,2)	
A_2A_3	((1,2,3,4,5)),6/1, 7/4 (1,2,3), (6,1,4),(7,4,5)	((1,2,3,4,5)),6/1,7/5, (1,2,3),(2,4,5), (6,1,5)	((1,2,3,4)),5/1,6/2, 7/4,(1,2,3),(5,1,4)
	((1,2,3,4,5)),7/6/1 (1,2,3),(2,4,5), (7,6,1)	((1,2,3,4)),6/5/1,7/2 (7,2,1),(1,5,6,2,3,4)	((1,2,3,4)),6/5/4,7/1 (1,2,3),(6,5,4)
	((1,2,3,4)),6/5/4, 7/1,(1,2,3), (7,1,4)	((1,2,3,4)),6/5/4, 7/1,(1,2,3),(5,4,2)	((1,2,3)),5/4/1,7/6/2 (1,4,5,2,6,3)
	((1,2,3)),5/4/1,7/6/2, (4,1,3)	((1,2,3)),6/5/4/1,7/2,(1,4,5,6,2,3)	
	((1,2)),5/4/3/1,7/6/2	((1,2,3,4,5)),6/1,7/2,(6,1,4) (7,2,5),(1,2,3)	

1	2
A_1D_4	((1,2,3,4)),5/1, 6/2, 7/3, (1,2,3), (5,1,4)
	((1,2,3,4,5)),7/6/2, (1,2,3),(2,4,5), (7,6,2)
	((1,2,3,4)),6/5/1, 7/4, (1,2,3), (7,4,1)
	((1,2,3,4)),7/6/5/1 (5,1,2) (1,5,6,7,3,4)
	((1,2,3,4)),7/6/5/4 (1,2,3),(5,4,1)
	((1,2,3)),6/5/4/1 7/2, (4,1,3)
$5A_1$	((1,2,3,4,5,6,7)) (1,2,3),(3,4,5), (2,7,5),(1,6,5), (3,7,6)
	((1,2,3,4,5,6,7)) (1,4,5),(5,3,6),(1,2,3) (4,2,6), (4,7,3)
	((1,2,3,4,5)),6/1 7/2, (6,1,5) (7,2,5),(1,6,2,7,3,4)
	((1,2,3,4,5)),6/4 7/5,(1,2,3), (6,4,1),(7,5,1)
	((1,2,3,4,5)),6/4,7/5, (1,2,3), (6,4,1) (6,4,2,3,5,7)
	((1,2,3,4)),5/1,6/3, 7/2, (5,1,4) (6,3,4)
	((1,2,3,4,5,6)),7/6, (1,2,3),(3,4,5) (7,6,3),(1,2,4,5,6,7)
	((1,2,3,4,5,6,7)), (1,2,3), (3,4,5), (3,6,7),(1,6,4),(2,6,5)
A_23A_1	((1,2,3,4,5,6,7)) (1,2,3),(4,5,6), (2,7,5),(1,7,6), (4,7,3)
	((1,2,3,4,5)), 6/4 7/5, (1,2,3),(6,4,1) (7,5,2)
	((1,2,3,4)),5/2,6/3, 7/2, (5,1,4), (1,4,3,6,2,7)
	((1,2,3,4)),6/5/1,7/2,(7,2,3) (1,5,6,2,7,4)
	((1,2,3)), 5/4/2, 6/2, 7/3, (5,4,2)
A_5	((1,2,3)),6/5/1, 7/2, (1,2,3), (5,1,4)
	((1,2,3,4)),6/5/1, 7/4, (1,2,3), (5,1,4)
	((1,2,3,4)), 6/5/4, 7/1, (1,2,3),(5,4,1)
	((1,2,3)),5/4/1 7/6/2,(4,1,2)
	((1,2,3,4)),7/6/5/4 (1,2,3),(6,5,4)
	((1,2,3)),5/4/1 7/6/2,(1,2,3)
	((1,2,3,4)),7/6/5/1 (1,2,3), (6,5,1)
	((1,2,3)),6/5/4/1,7/2 (1,2,3)
	((1,2,3)),7/6/5/4/1 (1,4,5,6,2,3)
	((1,2,3)),(1,2,3),7/6/5/4/1
	((1,2)), 7/6/5/4/3/1
D_5	((1,2,3)),(1,2,3),5/4/1,6/2,7/3
	((1,2,3)),(1,2,3),7/6/5/1,(5,1,4)
	((1,2,3)),6/5/4/1,(4,1,2)
	((1,2,3)),7/6/5/4/1,(5,4,1)
	((1,2,3)), 7/6/5/4/1, (4,1,2)

1	2		
$3A_2$	((1,2,3,4,5)),6/1,7/5,(1,2,3) (2,4,5),(6,1,4),(7,5,3)	((1,2,3,4)),5/1, 6/3, 7/2 (5,1,2),(7,2,3),(6,3,1)	
	((1,2,3)),5/4/1, 6/2, 7/3 (6,2,3), (1,4,5,2,3,7)	((1,2,3)), 5/4/1, 7/6/2 (5,4,1), (7,6,2)	
A_5A_1	((1,2,3,4)),6/5/1,7/2 (1,2,3),(5,1,4), (7,2,4)	((1,2,3,4)), 6/5/1,7/4, (1,2,3),(5,1,4),(7,4,2)	((1,2,3)),5/4/1,6/2, 7/3, (4,1,2),(7,3,1)
	((1,2,3)),5/4/1,7/6/2 (4,1,2),(7,6,2)	((1,2,3)),5/4/1, 7/6/2,(1,2,3), (1,4,5,2,6,7)	((1,2,3)),5/4/1, 7/6/2, (1,2,3), (5,4,1)
	((1,2,3)),6/5/4/1, 7/2,(5,4,1), (7,2,3)	((1,2,3),6/5/4/1,7/2, (7,2,1),(5,4,1)	((1,2,3)),6/5/4/1,7/2, (1,2,3), (1,4,5,6,2,7)
	((1,2,3)),7/6/5/4/1,(1,2,3) (1,4,5,6,7,2)	((1,2)), 6/5/4/3/1, 7/2, (1,3,4,5,2,7)	
	((1,2)),6/5/4/3/1,7/2,(7,1,2)	((1,2)), 7/6/5/4/3/1,(1,3,4,5,6,7)	
$A_3 3A_1$	((1,2,3,4,5,6)),7/1 (1,2,3),(1,5,6) (4,2,5),(4,3,6) (7,1,4)	((1,2,3,4,5)),6/1,7/3 (1,2,3),(2,4,5), (6,1,4),(7,3,4)	((1,2,3)),5/1,6/3 7/2, (5,1,2) (6,3,4), (7,2,4)
	((1,2,3,4)),5/1,6/3 7/2, (5,1,2), (6,3,2),(7,2,4)	((1,2,3,4)),5/1,6/2 7/4, (1,2,3),(7,4,3) (1,5,2,6,4,7)	((1,2,3)),5/4/1,6/2 7/3,(1,4,2,6,3,7) (5,4,1)
	((1,2,3)),5/4/1, 6/2, 7/3 (5,4,1),(7,3,1)	((1,2,3)),6/5/4/1, 7/2, (7,2,3) (1,4,5,6,2,7)	
	((1,2,3,4)),6/5/1,7/2,(5,1,4) (7,2,4),(1,5,6,2,7,3)	((1,2,3,4)),6/5/1,7/4,(7,4,2) (1,2,3),(1,5,6,4,7,3)	
$6A_1$	((1,2,3,4,5,6,7)),(1,2,3),(1,4,5) (3,6,5),(1,7,6),(5,7,2) (3,7,4)	((1,2,3,4,5,6,7)),(1,2,3),(1,4,5) (5,3,6),(4,2,6),(4,7,3) (5,7,2)	
	((1,2,3,4)),5/1,6/3, 7/2, (5,1,4), (6,3,4),(7,2,4)	((1,2,3)),5/1,6/3, 7/2, (5,1,4),(6,3,4) (1,5,3,6,2,7)	((1,2,3,4,5)),6/4,7/5, (6,4,1),(7,5,1),(1,2,3) (4,6,5,7,2,3)

1	2		
$A_1A_2A_3$	((1,2,3,4,5)),6/1, 7/3, (1,2,3), (2,4,5),(6,1,4), (7,3,5)	((1,2,3,4,5)),6/1,7/5, (1,2,3),(2,4,5), (6,1,5),(7,5,3)	((1,2,3,4)), 5/1, 6/3,7/2, (5,1,2), (6,3,1), (7,2,4)
	((1,2,3,4)),5/1, 6/2, 7/4,(1,2,3), (5,1,4), (7,4,3)	((1,2,3,4)),6/5/1,7/4, (1,2,3),(7,4,2), (6,5,1)	((1,2,3)),5/4/1,6/2, 7/3, (6,2,3), (7,3,1)
	((1,2,3)),5/4/1, 6/2,7/3,(6,2,1), (1,4,5,2,3,7)	((1,2,3)),5/4/1,6/2, 7/3, (5,4,1),(7,3,2)	((1,2,3)),5/4/1, 7/6/2, (5,4,1), (1,4,2,6,7,3)
	((1,2,3)),5/4/1, 7/6/2 (4,1,3),(1,4,5,2,6,7)	((1,2,3)),6/5/4/1, 7/2,(7,2,3) (1,4,5,6,2,3)	
	((1,2,3,4)),6/5/4,7/1,(1,2,3), (5,4,2),(1,7,4,5,6,3)	((1,2)),5/4/3/1,7/6/2,(7,6,2)	
D_42A_1	((1,2,3)),5/1,6/2, 7/3,(1,2,3),(5,1,4), (6,2,4)	((1,2,3,4)),6/5/1,7/4, (1,2,3),(7,4,1), (6,5,1)	((1,2,3)),5/4/1,6/2, 7/3,(6,2,1),(7,3,1)
	((1,2,3,4)),7/6/5/4 (1,2,3),(5,4,1), (2,3,4,5,6,7)	((1,2,3)),6/5/4/1, 7/2,(4,1,3) (1,4,5,6,2,7)	((1,2,3)),6/5/4/1, 7/2,(4,1,3), (7,2,3)
$2A_3$	((1,2,3,4)),5/1,6/2,7/4, (1,2,3),(5,1,4),(6,2,4)	((1,2,3,4)),6/5/4,7/1,(1,2,3) (7,1,4),(6,5,4)	
	((1,2,3)),5/4/1,7/6/2,(4,1,3), (6,2,3)	((1,2)),5/4/3/1,7/6/2, (1,3,4,5,2,6)	
A_2A_4	((1,2,3,4)),6/5/1, 7/2,(1,2,3),(7,2,4) (6,5,1)	((1,2,3,4)),6/5/4, 7/1,(1,2,3), (5,4,2),(7,1,4)	((1,2,3)),5/4/2, 6/2, 7/3,(4,1,2), (6,2,3)
	((1,2,3)),5/4/1, 7/6/2,(4,1,3), (7,6,2)	((1,2,3)),6/5/4/1,7/2, (7,2,1), (1,4,5,6,2,3)	((1,2)),5/4/3/1, 7/6/2, (1,3,4,2,6,7)
	((1,2)),5/4/3/1, 7/6/2, (4,1,3)	((1,2)),6/5/4/3/1, 7/2, (1,3,4,5,6,2)	((1,2,3,4)),5/1,6/2, 7/4,(1,2,3),(5,1,4), (7,4,2)
A_1D_5	((1,2,3)),5/4/1,6/2,7/3, (1,2,3),(5,4,1)	((1,2,3)),6/5/4/1,7/2,(4,1,3) (7,2,1)	
	((1,2,3)),6/5/4/1,7/2 (4,1,2),(7,2,3)	((1,2,3)),7/6/5/4/1,(4,1,2) (1,4,5,6,7,3)	
	((1,2)),6/5/4/3/1,7/2,(4,3,1)		

1	2		
A_6	((1,2,3)),5/4/1, 7/6/2,(4,1,2), (6,2,3)	((1,2,3)),6/5/4/1, 7/2,(1,2,3), (5,4,1)	((1,2)),5/4/3/1, 7/6/2, (6,2,1)
	((1,7)),6/5/4/3/2/1,(1,2,3,4,5,7)	((1)),7/6/5/4/3/2/1	
D_6	((1,2)),5/4/3/1, 7/6/2,(3,1,2)	((1,2,3)),7/6/5/4/1 (1,2,3), (5,4,1)	((1,2)),7/6/5/4/3/1 (3,1,2)
E_6	((1,2)),6/5/4/3/1,7/2,(3,1,2)	((1,2)),7/6/5/4/3/1,(4,1,3)	
$D_4 3A_1$	((1,2,3)),5/1,6/2, 7/3,(1,2,3),(5,1,4) (6,2,4),(7,3,4)	((1,2,3)),5/4/1, 6/2, 7/3,(5,4,1),(6,2,1) (7,3,1)	((1,2,3)),6/5/4/1,7/2, (4,1,3),(7,2,3), (1,4,5,6,2,7)
$A_1 2A_3$	((1,2,3,4)),5/1,6/2,7/4, (1,2,3),(6,2,4),(5,1,4), (7,4,3)	((1,2,3)),5/4/1,6/2,7/3,(5,4,1), (6,2,3),(7,3,1)	
	((1,2,3)),5/4/1,7/6/2, (4,1,3),(6,2,3), (1,4,5,2,6,7)	((1,2)),5/4/3/1, 7/6/2, (7,6,2),(1,3,4,5,2,6)	
A_2A_5	((1,2)),6/5/4/3/1, 7/2, (7,2,1),(1,3,4,5,6,2)	((1,2,3)),5/4/1,6/2,7/3,(4,1,2), (6,2,3), (7,3,1)	
	((1,2,3)),5/4/1,7/6/2,(1,2,3) (5,4,1),(7,6,2)	((1,2)),5/4/3/1, 7/6/2, (4,3,1), (7,6,2)	
A_1D_6	((1,2)),5/4/3/1, 7/6/2, (3,1,2), (7,6,2)	((1,2)),6/5/4/3/1, 7/2,(7,1,2), (4,3,1)	((1,2)),7/6/5/4/3/1, (3,1,2),(1,3,4,5,6,7)
A_7	((1,2)),5/4/3/1,7/6/2, (6,2,1), (4,3,1)	((1)), 7/6/5/4/3/2/1, (1,2,3,4,5,6)	
E_7	((1)), 7/6/5/4/3/2/1, (3,2,1)		

§ 4, Table 4: (d=5)

1	2	3	
Non-singular	10	general position	
A_1	7	((1,2,3,4)), ((1,2,3))	((1,2,3)), 4/1
$2A_1$	5	((1,2,3)), 4/1, (4,1,2)	((1,2)), 3/1, 4/2
A_2	4	((1,2)), 4/3/1	((1,2,3)), 4/1, (1,2,3)
A_1A_2	3	((1,2)), 3/1, 4/2, (3,1,2)	((1,2)), 4/3/1, (4,3,1)
A_3	2	((1,2)), 4/3/1, (3,1,2)	((1)), 4/3/2/1
A_4	1	((1)), 4/3/2/1, (3,2,1)	

§5 Table 5: (d=6)

1	2	3	
Non-singular	6	general position	
A_1	3 or 4	((1,2,3)), (1,2,3)	((1,2)), 3/1
$2A_1$	2	((1,2)), 3/1, (3,1,2)	
A_2	2	((1)), 3/2/1	
A_1A_2	1	((1)), 3/2/1, (3,2,1)	

§6: Table 6: (d=7)

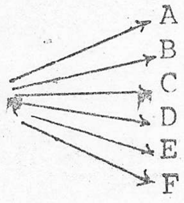
1	2	3
Non-singular	3	general position (i.e. ((1,2)))
A_1	2	((1)), 2/1

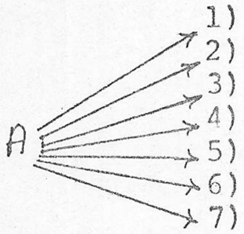
Appendix 2

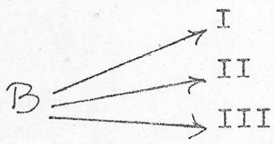
We'll use the same notations as in Appendix 1.

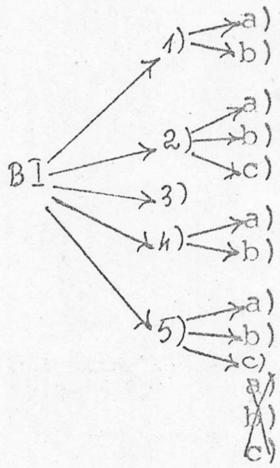
§ 1. The tree of cases for d=3 (see § 2 in the text)

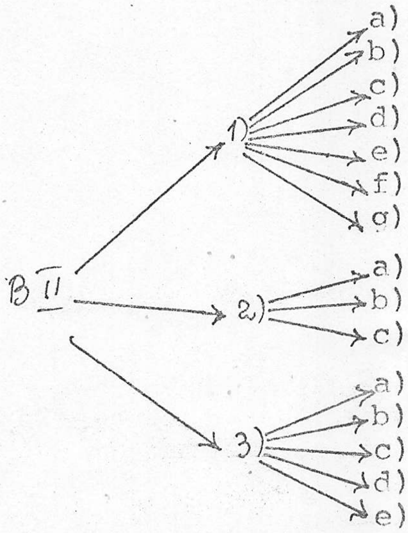
Groups of cases:

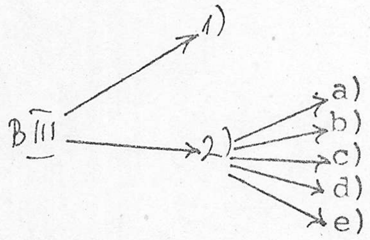


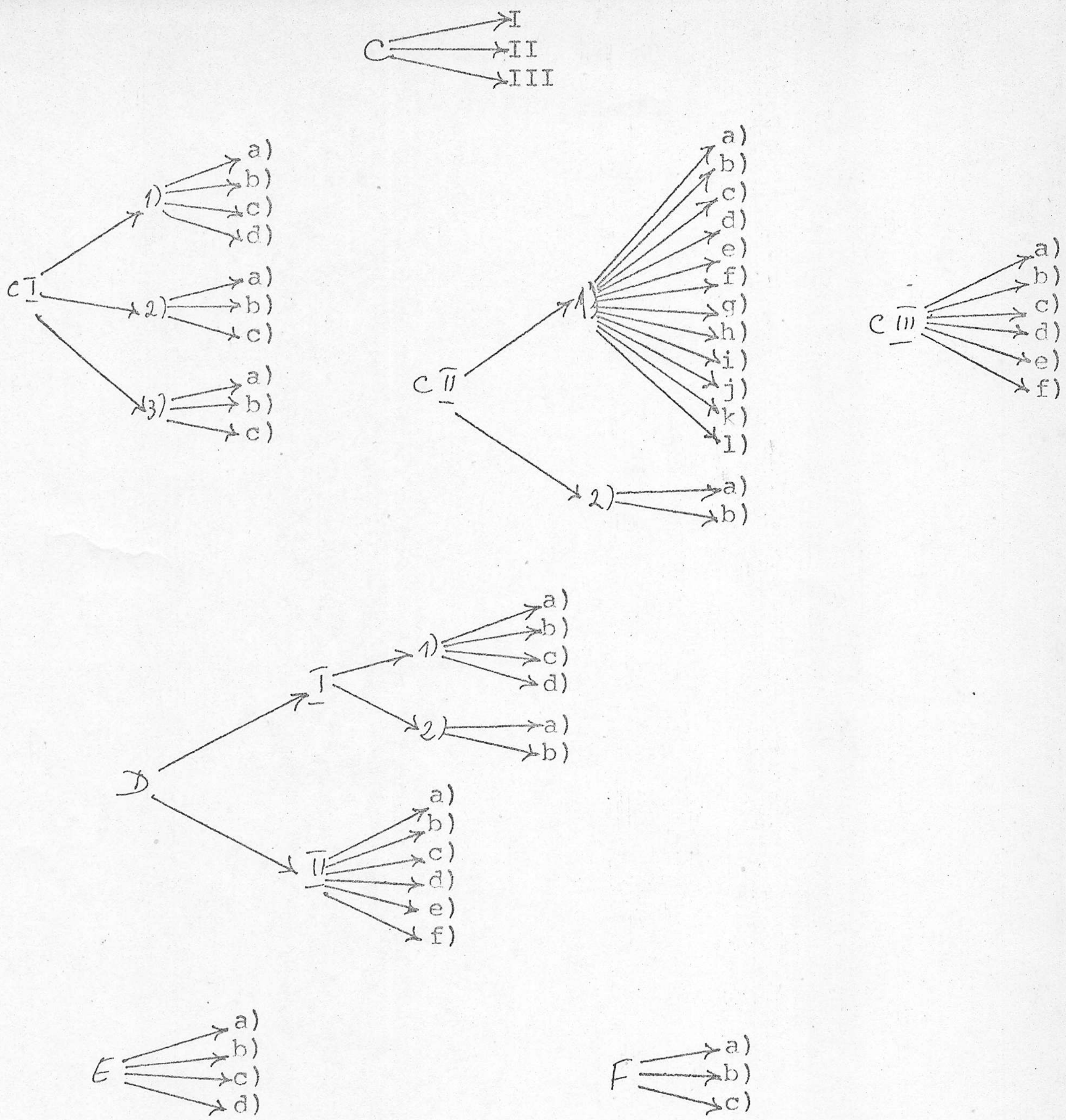












<u>The explanation of the graph:</u>

A; 1): ((1,2,3,4,5,6)), (1,2,3), (4,5,6)

2): ((1,2,3,4,5,6)), (1,2,3)

3): ((1,2,3,4,5,6)), (1,2,3), (3,4,5)

4): ((1,2,3,4,5,6)), (1,2,3), (1,4,5), (3,6,5).

5): ((1,2,3,4,5,6)), (1,2,3), (1,4,5), (4,2,6), (5,3,6)

6): ((1,2,3,4,5,6)), (1,2,3,4,5,6)

7) general position.

B I 1) a) ((1,2,3,4,5)), 6/2, (1,2,3)

b) ((1,2,3,4,5)), 6/2, (1,2,3), (6,2,4)

2) a) ((1,2,4,5)), 6/4, (1,2,3)

b) ((1,2,3,4,5)), 6/4, (1,2,3), (6,4,1)

c) ((1,2,3,4,5)), 6/4, (1,2,3), (6,4,5)

3) ((1,2,3,4,5)), 6/2, (1,2,3), (2,4,5)

4) a) ((1,2,3,4,5)), 6/3, (1,2,3), (2,4,5)

b) ((1,2,3,4,5)), 6/3, (1,2,3), (2,4,5), (6,3,4)

5) a) ((1,2,3,4,5)), 6/1

b) ((1,2,3,4,5)), 6/1, (6,1,2)

c) ((1,2,3,4,5)), 6/1, (1,2,3,4,5,6)

II 1)a)((1,2,3,4)), 5/1, 6/2, (5,1,2)

b)((1,2,3,4)), 5/1, 6/2, (5,1,2), (6,2,3)

c)((1,2,3,4)), 5/1, 6/2,(5,1,3), (6,2,4)

d)((1,2,3,4)), 5/1, 6/2, (5,1,3), (6,2,3)

e)((1,2,3,4)), 5/1, 6/2, (5,1,3)

f)((1,2,3,4)), 5/1, 6/2, (1,2,3,4,5,6)

g)((1,2,3,4)), 5/1, 6/2.

2) a)((1,2,3,4)), 6/1, 5/2, (1,2,3)

b)((1,2,3,4)), 6/1, 5/2, (1,2,3), (6,1,4)

c)((1,2,3,4)), 6/1, 5/2, (1,2,3), (6,1,4), (5,2,4)

3) a)((1,2,3,4)), 5/1, 6/4, (1,2,3)

b)((1,2,3,4)), 5/1, 6/4, (1,2,3), (6,4,2), (5,1,4)

c)((1,2,3,4)), 5/1, 6/4, (1,2,3), (6,4,2)

d)((1,2,3,4)), 5/1, 6/4, (1,2,3), (5,1,4)

e)((1,2,3,4)), 5/1, 6/4, (1,2,3), (6,1,4)

III 1)((1,2,3)), 4/1, 5/2, 6/3 (1,2,3)

2)a) ((1,2,3)), 4/1, 5/2, 6/3, (4,1,2)

b) ((1,2,3)), 4/1, 5/2, 6/3, (4,1,2), (5,2,3)

c) ((1,2,3)), 4/1, 5/2, 6/3, (4,1,2), (5,2,3), (6,3,1)

d) ((1,2,3)), 4/1, 5/2, 6/3, (1,2,3,4,5,6)

e) ((1,2,3)), 4/1, 5/2, 6/3

C I 1)a)((1,2,3,4)), 6/5/1

b)((1,2,3,4)), 6/5/1, (1,2,3,4,5,6)

c)((1,2,3,4)), 6/5/1, (5,1,2)

d)((1,2,3,4)), 6/5/1, (6,5,1)

2)a)((1,2,3,4)), 6/5/1, (1,2,3)

b)((1,2,3,4)), 6/5/1, (1,2,3), (6,5,1)

c)((1,2,3,4)), 6/5/1, (1,2,3), (5,1,4)

3)a)((1,2,3,4)), 6/5/4, (1,2,3)

b)((1,2,3,4)), 6/5/4, (1,2,3), (6,5,4)

c)((1,2,3,4)), 6/5/4, (1,2,3), (5,4,1)

II 1)a)((1,2,3)), 5/4/1, 6/2

b)((1,2,3)), 5/4/1, 6/2, (1,2,3,4,5,6)

c)((1,2,3)), 5/4/1, 6/2, (5,4,1)

d)((1,2,3)), 5/4/1, 6/2, (4,1,3)

e)((1,2,3)), 5/4/1, 6/2, (4,1,2)

f)((1,2,3)), 5/4/1, 6/2, (6,2,1)

g)((1,2,3)), 5/4/1, 6/2, (6,2,1), (5,4,1)

h)((1,2,3)), 5/4/1, 6/2, (6,2,1), (4,1,3)

i)((1,2,3)), 5/4/1, 6/2, (6,2,3)

j)((1,2,3)), 5/4/1, 6/2, (6,2,3), (5,4,1)

k)((1,2,3)), 5/4/1, 6/2, (6,2,3), (4,1,2)

l)((1,2,3)), 5/4/1, 6/2, (6,2,3), (4,1,3)

II 2) a)((1,2,3)), 5/4/1, 6/2, (1,2,3)

b)((1,2,3)), 5/4/1, 6/2, (1,2,3), (5,4,1)

IIIa)((1,2)), 4/3/1, 6/5/2

b)((1,2)), 4/3/1, 6/5/2, (1,2,3,4,5,6)

c)((1,2)), 4/3/1, 6/5/2, (6,5,2)

d)((1,2)), 4/3/1, 6/5/2, (3,1,2)

e)((1,2)), 4/3/1, 6/5/2, (3,1,2), (6,5,2)

f)((1,2)), 4/3/1, 6/5/2, (4,3,1), (6,5,2)

D I 1)a)((1,2,3)), 6/5/4/2

b)((1,2,3)), 6/5/4/2, (1,2,3,4,5,6)

c)((1,2,3)),6/5/4/2, (5,4,2)

d)((1,2,3)), 6/5/4/2, (4,2,1)

2)a)((1,2,3)), 6/5/4/2, (1,2,3)

b)((1,2,3)), 6/5/4/2, (1,2,3), (5,4,2)

II a)((1,2)), 5/4/3/1, 6/2

b)((1,2)), 5/4/3/1, 6/2, (1,2,3,4,5,6)

c)((1,2),5/4/3/1, 6/2, (4,3,1)

d)((1,2)), 5/4/3/1, 6/2, (6,2,1)

e)((1,2)), 5/4/3/1, 6/2, (6,2,1), (4,3,1)

f)((1,2)), 5/4/3/1, 6/2, (3,1,2)

E a)((1,6)), 5/4/3/2/1

b)((1,6)), 5/4/3/2/1, (1,2,3,4,5,6)

c)((1,6)), 5/4/3/2/1, (3,2,1)

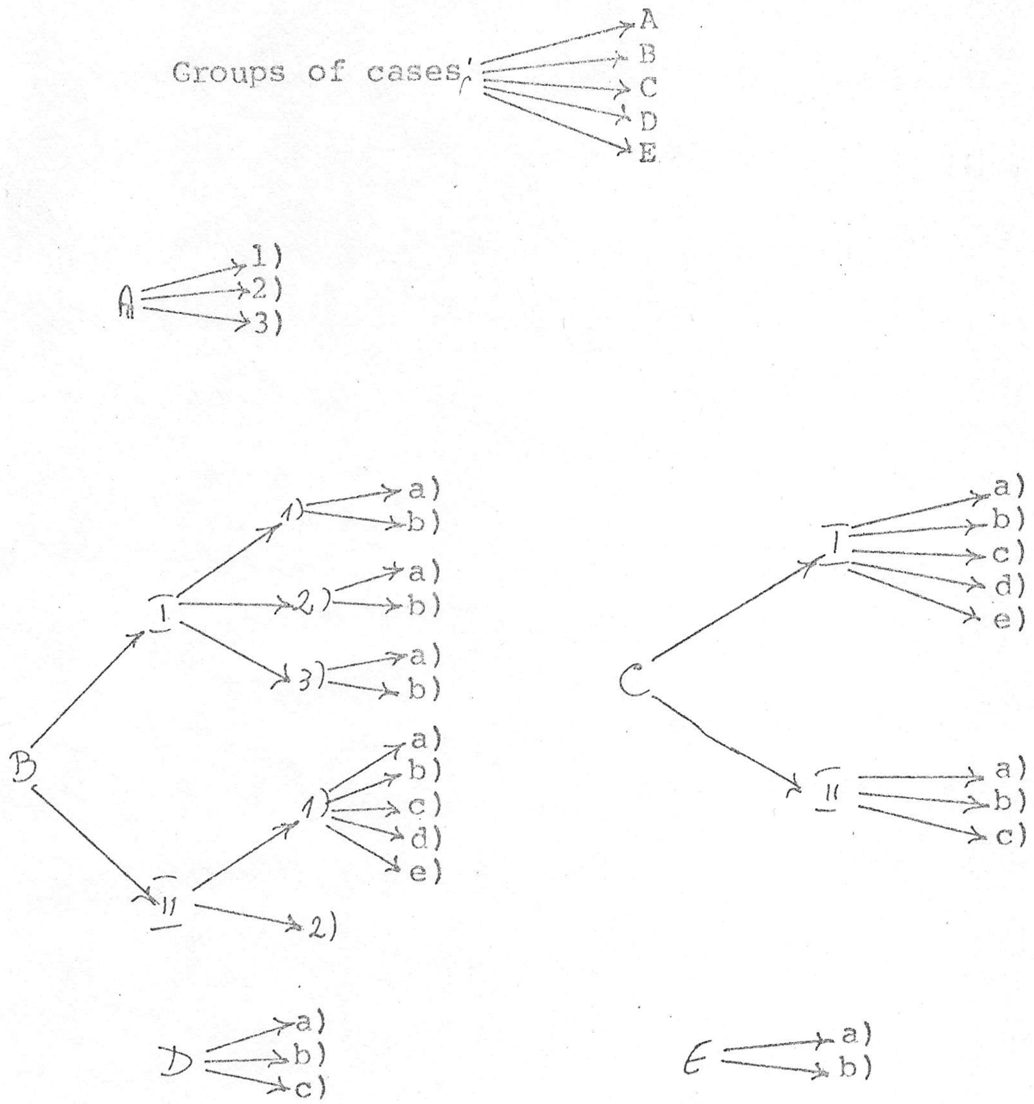
d)((1,6)), 5/4/3/2/1, (2,1,6)

F a)((1)), 6/5/4/3/2/1

b)((1)), 6/5/4/3/2/1, (1,2,3,4,5,6)

c)((1)), 6/5/4/3/2/1, (3,2,1).

§ 2. <u>The tree of cases for d=4</u> (see 3 in the text)



<u>The explanation of the graph:</u>

A 1)((1,2,3,4,5)), (1,2,3)

2)((1,2,3,4,5)), (1,2,3), (2,4,5)

3) general position

B I 1) a)((1,2,3,4)), 5/1

b)((1,2,3,4)), 5/1, (5,1,2)

2) a)((1,2,3,4)), 5/1, (1,2,3)

b)((1,2,3,4)), 5/1, (1,2,3), (5,1,4)

3) a)((1,2,3,4)), 5/4, (1,2,3)

b)((1,2,3,4)), 5/4, (1,2,3), (5,4,1)

II 1) a)((1,2,3)), 4/1, 5/2

b)((1,2,3)), 4/1, 5/2, (4,1,3)

c)((1,2,3)), 4/1, 5/2, (4,1,3), (5,2,3)

d)((1,2,3)), 4/1, 5/2, (4,1,2)

e)((1,2,3)), 4/1, 5/2, (4,1,2), (5,2,3)

2)((1,2,3)), 4/1, 5/2, (1,2,3)

C I a)((1,2)), 4/3/1, 5/2

b)((1,2)), 4/3/1, 5/2, (5,2,1)

c)((1,2)), 4/3/1, 5/2, (5,2,1), (4,3,1)

d)((1,2)), 4/3/1, 5/2, (4,3,1)

e)((1,2)), 4/3/1, 5/2, (3,1,2)

C II a)((1,2,3)), 5/4/1

b)((1,2,3)), 5/4/1, (5,4,1)

c)((1,2,3)), 5/4/1, (4,1,2)

D a)((1,2)), 5/4/3/1

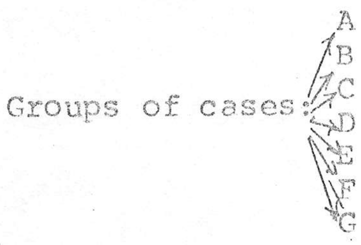
b)((1,2)), 5/4/3/1, (4,3,1)

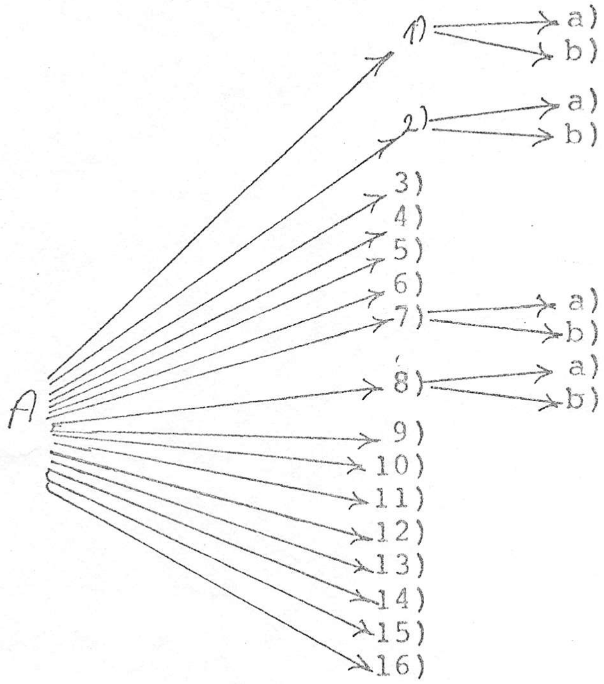
c)((1,2)), 5/4/3/1, (3,1,2)

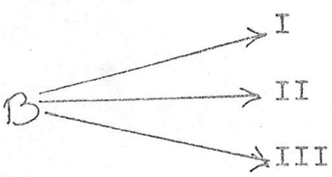
E a)((1)), 5/4/3/2/1

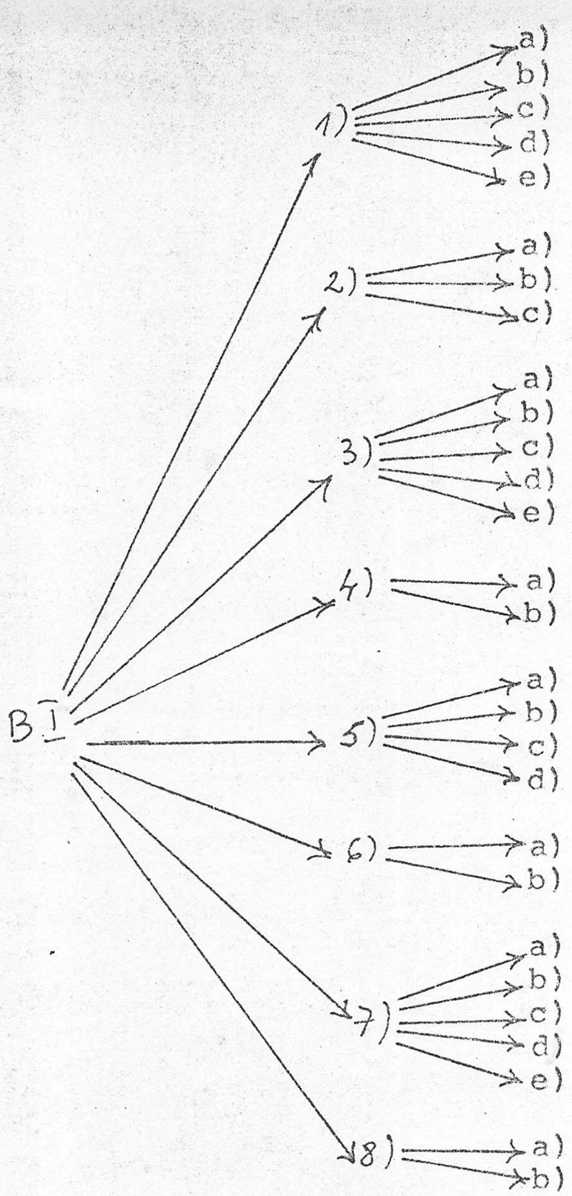
b)((1)), 5/4/3/2/1, (3,2,1)

§3. The tree of cases for d=2 (see §4 in the text)








B I
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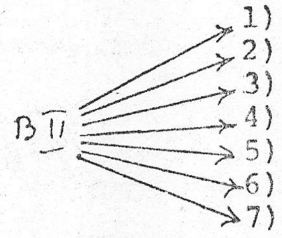
a)
b)
c)
d)
e)
2)
a)
b)
c)
3)
a)
b)
c)
d)
e)
4)
a)
b)
5)
a)
b)
c)
d)
6)
a)
b)
7)
a)
b)
c)
d)
e)
8)
a)
b)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2)
3)
4)
5)
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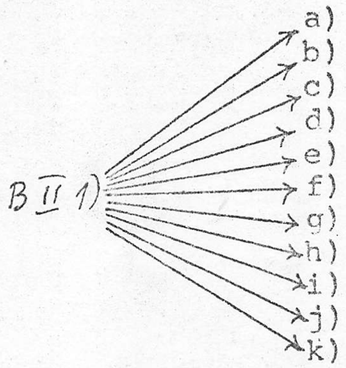
7)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b)
c)
d)
e)
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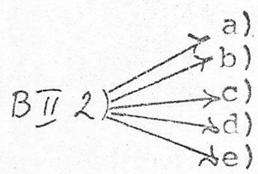
g)
h)
i)
j)
k)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c)
d)
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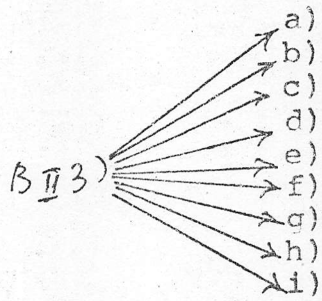
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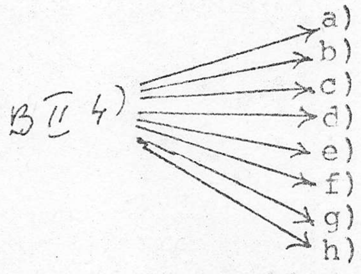
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b)
c)
d)
e)
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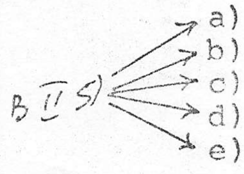
g)
h)


B II 5)
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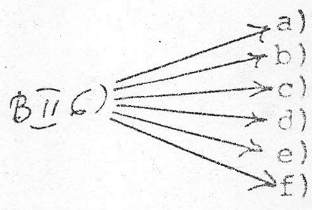
b)
c)
d)
e)


B II 6)
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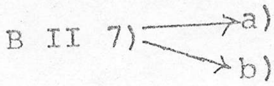
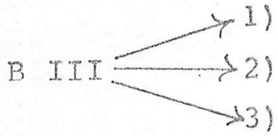
d)
e)
f)


B II 7)
a)
b)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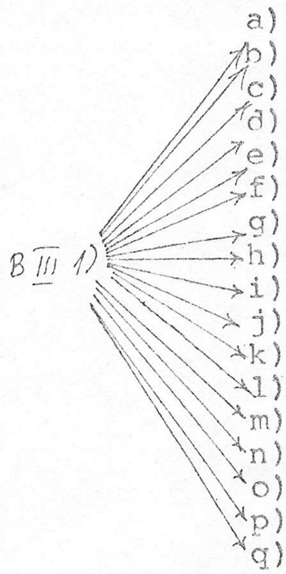
2)
3)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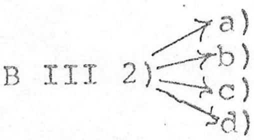
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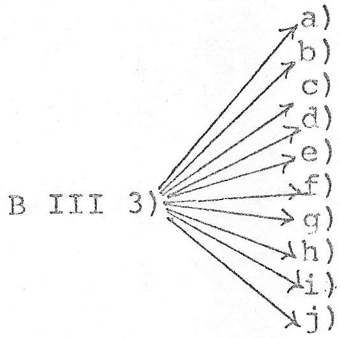
b)
c)
d)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c)
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f)
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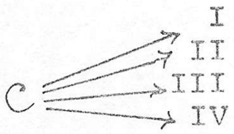
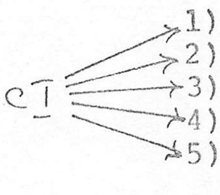
i)
j)


C
I
II
III
IV


C I
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2)
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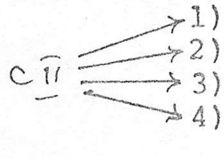
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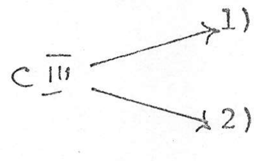
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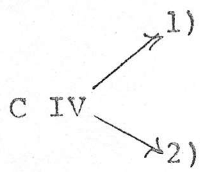
2)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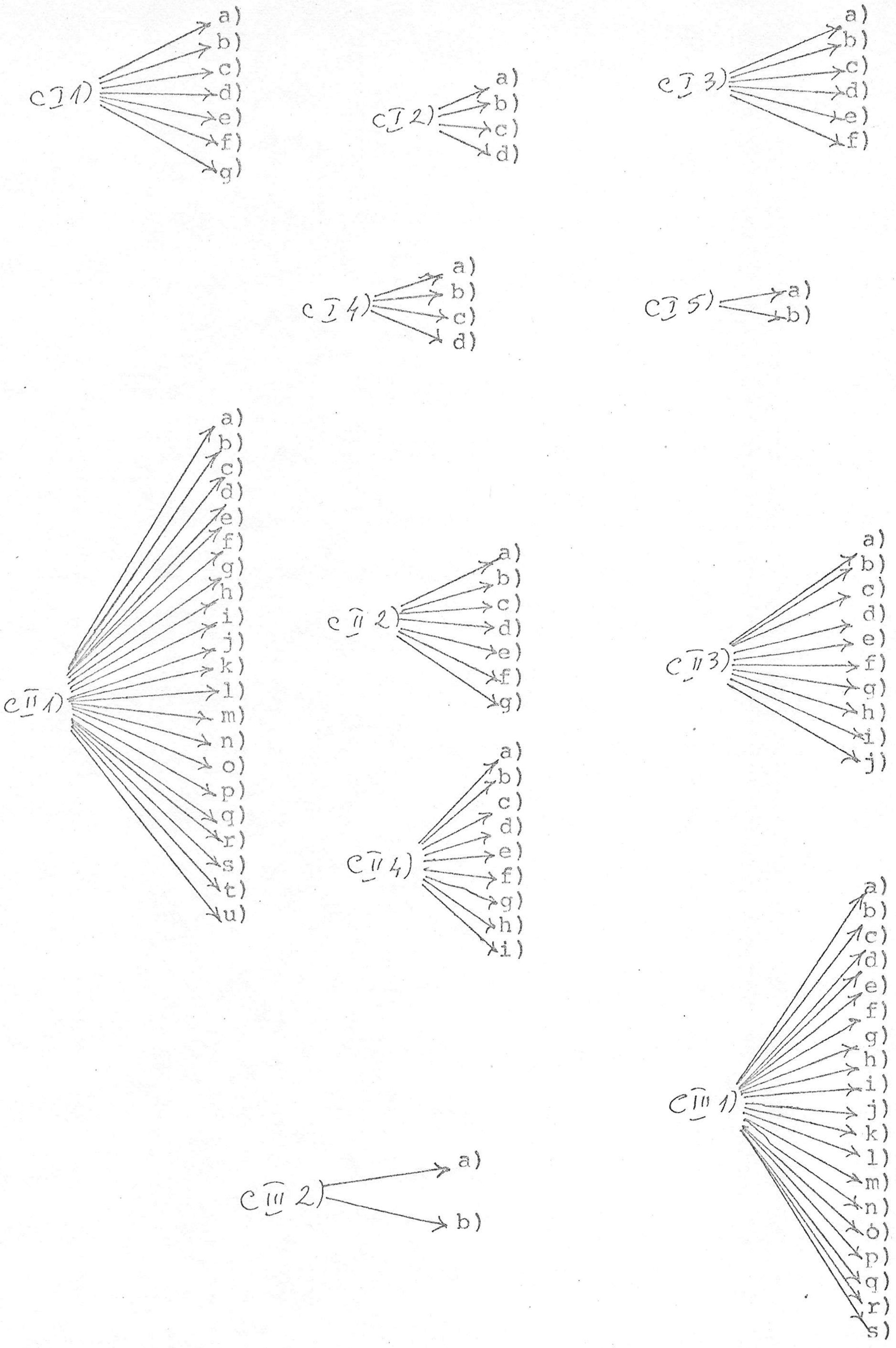
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c I 2) a) b) c) d)
c I 3) a) b) c) d) e) f)
c I 4) a) b) c) d)
c I 5) a) b)
c II 1) a) b) c) d) e) f) g) h) i) j) k) l) m) n) o) p) q) r) s) t) u)
c II 2) a) b) c) d) e) f) g)
c II 3) a) b) c) d) e) f) g) h) i) j)
c II 4) a) b) c) d) e) f) g) h) i)
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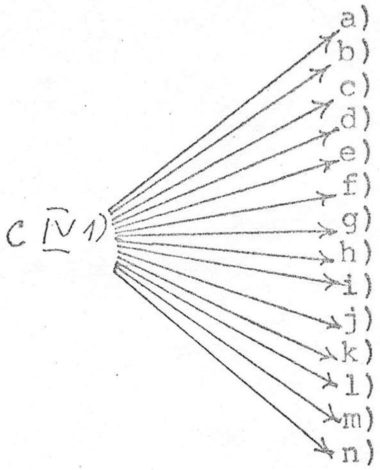
c III 2) a) b)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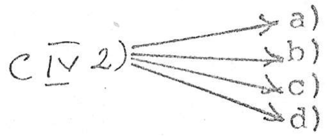
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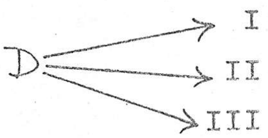
D
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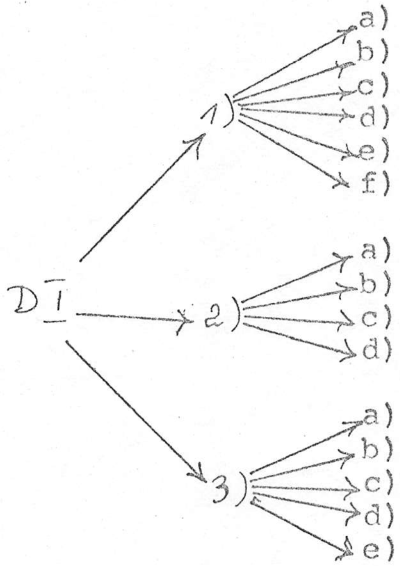
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3)
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b)
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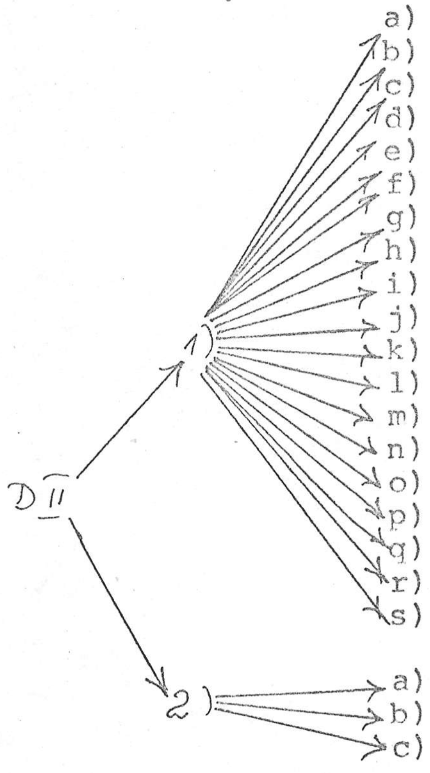
e)


D II
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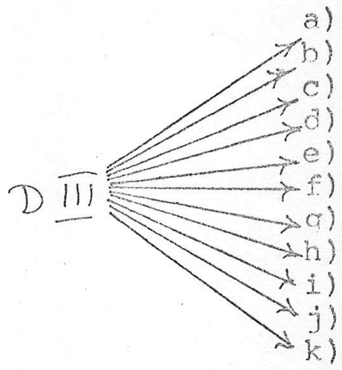
q)
r)
s)
2)
a)
b)
c)


D III
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c)
d)
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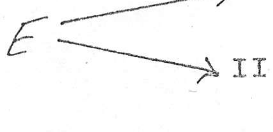
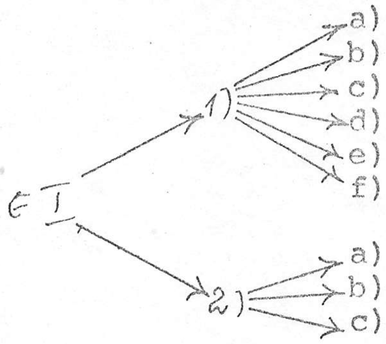
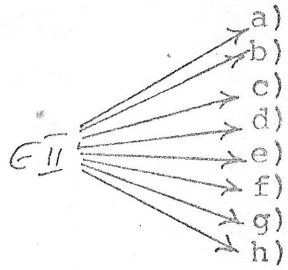
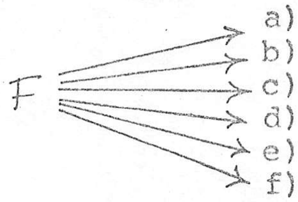
f)
g)
h)
i)
j)
k)


E
I
II







The explanation of the graph:

A 1) a) general position

b) ((1,2,3,4,5,6,7)), (1,2,3,4,5,6)

2) a) ((1,2,3,4,5,6,7)), (1,2,3), (4,5,6,7,2,3)

b) ((1,2,3,4,5,6,7)), (1,2,3)

3) ((11,2,3,4,5,6,7)), (1,2,3), (4,5,6)

4) ((1,2,3,4,5,6,7)), (1,2,3), (4,5,6), (2,5,7)

5) ((1,2,3,4,5,6,7)), (1,2,3), (4,5,6), (2,5,7), (4,7,3)

6) ((1,2,3,4,5,6,7)), (1,2,3), (4,5,6), (2,5,7), (4,7,3), (1,7,6)

7) a) ((1,2,3,4,5,6,7)), (1,2,3), (1,4,5), (2,3,4,5,6,7)

b) ((1,2,3,4,5,6,7)), (1,2,3), (1,4,5).

8) a) ((1,2,3,4,5,6,7)), (1,2,3), (1,6,7), (1,4,5), (2,3,4,5,6,7)

b) ((1,2,3,4,5,6,7)), (1,2,3), (1,6,7), (1,4,5)

9) ((1,2,3,4,5,6,7)), (1,2,3), (1,7,6), (1,4,5), (3,6,5)

10) ((1,2,3,4,5,6,7)), (1,2,3), (1,7,6), (1,4,5), (3,6,5), (3,7,4)

11) ((1,2,3,4,5,6,7)), (1,2,3), (1,7,6), (1,4,5), (3,6,5), (3,7,4), (5,7,2).

12)((1,2,3,4,5,6,7)), (1,2,3), (1,4,5), (3,6,5)

13)((1,2,3,4,5,6,7)),(1,2,3),(1,4,5),(4,2,6),(4,3,6)

14)((1,2,3,4,5,6,7)),(1,4,5),(1,2,3),(5,3,6),(4,2,6),(4,7,3)

15)((1,2,3,4,5,6,7)),(1,4,5),(1,2,3),(5,3,6),(4,2,6),(4,7,3),
(5,7,2)

16)((1,2,3,4,5,6,7)),(1,2,3),(1,6,4),(2,6,5),(3,4,5),(3,6,7).

B I 1) a)((1,2,3,4,5,6)), 7/6

b)((1,2,3,4,5,6)), 7/6, (1,2,3,4,5,6)

c)((1,2,3,4,5,6)), 7/6, (7,6,1)

d)((1,2,3,4,5,6)), 7/6, (1,2,3,4,5,6), (7,6,1)

e)((1,2,3,4,5,6), 7/6, (1,2,3,4, 6,7)

2)a)((1,2,3,4,5,6)), 7/1, (1,2,3)

b)((1,2,3,4,5,6)), 7/1, (1,2,3), (1,2,4,5,6,7)

c)((1,2,3,4,5,6)), 7/1, (1,2,3), (7,1,4)

3)a)((1,2,3,4,5,6)), 7/4, (1,2,3)

b)((1,2,3,4,5,6)), 7/4, (1,2,3), (2,3,4,5,6,7)

c)((1,2,3,4,5,6)), 7/4, (1,2,3), (7,4,1)

d)((1,2,3,4,5,6)), 7/4, (1,2,3), (7,4,5)

e)((1,2,3,4,5,6)), 7/4, (1,2,3), (7,4,1), (2,3,4,5,6,7)

4)a)((1,2,3,4,5,6)), 7/5, (1,2,3), (4,5,6)

b)((1,2,3,4,5,6)), 7/5, (1,2,3), (4,5,6), (7,5,1)

5)a)((1,2,3,4,5,6)), 7/1, (1,2,3), (3,4,5)

b)((1,2,3,4,5,6)), 7/1, (1,2,3), (3,4,5), (7,1,4)

c)((1,2,3,4,5,6)), 7/1, (1,2,3), (3,4,5), (7,1,6)

d)((1,2,3,4,5,6)), 7/1, (1,2,3), (3,4,5), (1,7,2,4,5,6)

6)a)((1,2,3,4,5,6)), 7/3, (1,2,3), (3,4,5)

b)((1,2,3,4,5,6)), 7/3, (1,2,3), (3,4,5), (7,3,6).

7) a) ((1,2,3,4,5,6)), 7/6, (1,2,3), (3,4,5)

b) ((1,2,3,4,5,6)), 7/6, (1,2,3), (3,4,5), (7,6,5)

c) ((1,2,3,4,5,6)), 7/6, (1,2,3), (3,4,5), (7,6,3)

d) ((1,2,3,4,5,6)), 7/6 ,(1,2,3), (3,4,5), (1,2,4,6,7)

e) ((1,2,3,4,5,6)), 7/6,(1,2,3), (3,4,5), (1,2,4,5,6,7),(7,6,

8) a)((1,2,3,4,5,6)), 7/1, (1,2,3),(1,5,6),(5,2,4), (6,3,4).

b)((1,2,3,4,5,6)), 7/1,(1,2,3), (1,5,6), (5,2,4),(6,3,4), (7,1,4).

II.1) a)((1,2,3,4,5)), 6/1, 7/2

b)((1,2,3,4,5)), 6/1, 7/2, (1,6,2,3,4,5)

c)((1,2,3,4,5)), 6/1, 7/2, (1,6,2,7,3,4)

d)((1,2,3,4,5)), 6/1, 7/2, (6,1,5).

e)((1,2,3,4,5)), 6/1, 7/2, (6,1,5), (7,2,4)

f)((1,2,3,4,5)), 6/1, 7/2, (6,1,5), (7,2,5)

g)((1,2,3,4,5)), 6/1, 7/2, (6,1,2), (7,2,3)

h)((1,2,3,4,5)), 6/1, 7/2, (6,1,2)

i)((1,2,3,4,5)), 6/1, 7/2, (6,1,5), (1,5,2,7,3,4)

j)((1,2,3,4,5)), 6/1, 7/2, (6,1,5), (7,2,5), (1,6,2,7,3,4)

k)((1,2,3,4,5)), 6/1, 7/2, (6,1,2), (1,2,7,3,4,5).

2) a)((1,2,3,4,5)), 6/1, 7/2, (1,2,3)

b)((1,2,3,4,5)), 6/1, 7/2, (1,2,3), (1,6,2,7,4,5)

c)((1,2,3,4,5)), 6/1, 7/2, (1,2,3), (6,1,4)

d)((1,2,3,4,5)), 6/1, 7/2, (1,2,3), (6,1,4), (7,2,5)

e)((1,2,3,4,5)), 6/1, 7/2, (1,2,3), (6,1,4), (7,2,4)

3) a)((1,2,3,4,5)), 6/1, 7/4, (1,2,3).

b)((1,2,3,4,5)), 6/1, 7/4, (1,2,3), (1,6,4,7,2,5)

c)((1,2,3,4,5)), 6/1, 7/4, (1,2,3), (6,1,5), (7,4,2)

d)((1,2,3,4,5)), 6/1, 7/4, (1,2,3), (6,1,5), (7,4,1)

e)((1,2,3,4,5)), 6/1, 7/4, (1,2,3), (6,1,5).

f) ((1,2,3,4,5)), 6/1, 7/4, (1,2,3), (6,1,5), (7,4,5)

g) ((1,2,3,4,5)), 6/1, 7/4, (1,2,3), (6,1,4)

h) ((1,2,3,4,5)), 6/1, 7/4, (1,2,3), (6,1,4), (7,4,2)

i) ((1,2,3,4,5)), 6/1, 7/4, (1,2,3), (6,1,4), (7,4,5)

4) a) ((1,2,3,4,5)), 6/4, 7/5, (1,2,3)

b) ((1,2,3,4,5)), 6/4, 7/5, (1,2,3), (1,4,6,5,7,3)

c) ((1,2,3,4,5)), 6/4, 7/5, (1,2,3), (6,4,1)

d) ((1,2,3,4,5)), 6/4, 7/5, (1,2,3), (6,4,1), (7,5,1)

e) ((1,2,3,4,5)), 6/4, 7/5, (1,2,3), (6,4,1), (7,5,2)

f) ((1,2,3,4,5)), 6/4, 7/5, (1,2,3), (6,4,5)

g) ((1,2,3,4,5)), 6/4, 7/5, (1,2,3), (6,4,5), (7,5,1)

h) ((1,2,3,4,5)), 6/4, 7/5, (1,2,3), (6,4,1), (6,4,2,3,5,7).

5) a) ((1,2,3,4,5)), 6/1, 7/3, (1,2,3), (2,4,5)

b) ((1,2,3,4,5)), 6/1, 7/3, (1,2,3), (2,4,5), (1,6,3,7,4,5)

c) ((1,2,3,4,5)), 6/1, 7/3, (1,2,3), (2,4,5), (6,1,4)

d) ((1,2,3,4,5)), 6/1, 7/3, (1,2,3), (2,4,5), (6,1,4), (7,3,4)

e) ((1,2,3,4,5)), 6/1, 7/3 (1,2,3), (2,4,5), (6,1,4), (7,3,5)

6) a) ((1,2,3,4,5)), 6/1, 7/5 (1,2,3), (2,4,5)

b) ((1,2,3,4,5)), 6/1, 7/5, (1,2,3), (2,4,5), (1,6,5,7,4,3)

c) ((1,2,3,4,5)), 6/1, 7/5, (1,2,3), (2,4,5), (6,1,4)

d) ((1,2,3,4,5)),6/1, 7/5, (1,2,3), (2,4,5), (6,1,4), (7,5,3)

e) ((1,2,3,4,5)), 6/1, 7/5, (1,2,3), (2,4,5), (6,1,5)

f) ((1,2,3,4,5)), 6/1, 7/5, (1,2,3), (2,4,5), (6,1,5),(7,5,3)

7) a) ((1,2,3,4,5)), 6/1, 7/2, (1,2,3), (2,4,5)

b) ((1,2,3,4,5)), 6/1, 7/2, (1,2,3), (2,4,5), (6,1,4).

III. 1) a)((1,2,3,4)), 5/1, 6/3, 7/2

b)((1,2,3,4)), 5/1, 6/3, 7/2, (1,5,3,6,2,4)

c)((1,2,3,4)), 5/1, 6/3, 7/2, (1,5,3,6,2,7)

d)((1,2,3,4)), 5/1, 6/3, 7/2, (5,1,4)

e)((1,2,3,4)), 5/1, 6/3, 7/2, (5,1,4), (1,4,3,6,2,7)

f)((1,2,3,4)), 5/1, 6/3, 7/2, (5,1,4), (6,3,4)

g)((1,2,3,4)), 5/1, 6/3, 7/2, (5,1,4), (6,3,4), (7,2,4)

g')((1,2,3,4)), 5/1, 6/3, 7/2, (5,1,4), (6,3,4), (7,2,4)

(1,5,3,6,2,7) - don't appear,

because it's impossible to have from the point P_4 three tangents to a conic, namely the conic (1,5,3,6,2,7).

h)((1,2,3,4)), 5/1, 6/3, 7/2, (5,1,2)

i)((1,2,3,4)), 5/1, 6/3, 7/2, (5,1,2), (1,3,6,2,7,4)

j)((1,2,3,4)), 5/1, 6/3, 7/2, (5,1,2), (6,3,4)

k)((1,2,3,4)), 5/1, 6/3, 7/2, (5,1,2), (6,3,4), (7,2,4).

l)((1,2,3,4)), 5/1, 6/3, 7/2, (5,1,2), (6,3,1)

m)((1,2,3,4)), 5/1, 6/3, 7/2, (5,1,2), (6,3,1), (7,2,4)

m)((1,2,3,4)), 5/1, 6/3, 7/2, (5,1,2), (6,3,1), (7,2,3).

o)((1,2,3,4)), 5/1, 6/3, 7/2, (5,1,2), (6,3,2)

p)((1,2,3,4)), 5/1, 6/3, 7/2, (5,1,2), (6,3,2), (7,2,4)

q)((1,2,3,4)), 5/1, 6/3, 7/2, (5,1,4),(6,3,4),

(1,5,3,6,2,7)

2) a)((1,2,3,4)), 5/1, 6/2, 7/3, (1,2,3)

b)((1,2,3,4)), 5/1, 6/2, 7/3, (4,2,3), (5,1,4)

c)((1,2,3,4)), 5/1, 6/2, 7/3, (1,2,3), (5,1,4),(6,2,4)

d)((1,2,3,4)), 5/1, 6/2, 7/3, (1,2,3), (5,1,4), (6,2,4), (7,3,4).

3) a)((1,2,3,4)), 5/1, 6/2, 7/4, (1,2,3)

b)((1,2,3,4)), 5/1, 6/2, 7/4, (1,2,3), (1,5,2,6,4,7)

c)((1,2,3,4)), 5/1, 6/2, 7/4, (1,2,3), (5,1,4)

d)((1,2,3,4)), 5/1, 6/2, 7/4, (1,2,3), (5,1,4), (6,2,4)

e)((1,2,3,4)), 5/1, 6/2, 7/4, (1,2,3), (5,1,4), (6,2,4), (7,4,3)

f)((1,2,3,4)), 5/1, 6/2, 7/4, (1,2,3), (7,4,1)

g)((1,2,3,4)), 5/1, 6/2, 7/4, (1,2,3), (7,4,3)

h)((1,2,3,4)), 5/1, 6/2, 7/4, (1,2,3), (5,1,4), (7,4,2)

i)((1,2,3,4)),5/1, 6/2, 7/4, (1,2,3),(5,1,4),(7,4,3)

j)((1,2,3,4)), 5/1, 6/2, 7/4, (1,2,3), (7,4,3), (1,5,2,6,4,7)

C I 1)a)((1,2,3,4,5)), 7/6/1

b)((1,2,3,4,5)), 7/6/1, (1,6,2,3,4,5)

c)((1,2,3,4,5)), 7/6/1, (1,6,7,2,3,5)

d)((1,2,3,4,5)), 7/6/1, (6,1,2)

e)((1,2,3,4,5)), 7/6/1, (6,1,2), (1,6,7,5,4,3)

f)((1,2,3,4,5)), 7/6/1, (7,6,1)

g)((1,2,3,4,5)), 7/6/1, (7,6,1), (1,6,2,3,4,5)

2)a)((1,2,3,4,7)), 6/5/1, (1,2,3)

b)((1,2,3,4,7)), 6/5/1, (1,2,3), (1,5,6,3,4,7)

c)((1,2,3,4,7)), 6/5/1, (1,2,3), (6,5,1)

d)((1,2,3,4,7)), 6/5/1, (1,2,3), (5,1,4)

3)a)((1,2,3,4,5)), 7/6/4, (1,2,3)

b)((1,2,3,4,5)), 7/6/4, (1,2,3), (4,6,7,1,3,5)

c)((1,2,3,4,5)), 7/6/4, (1,2,3), (7,6,4)

d)((1,2,3,4,5)), 7/6/4, (1,2,3), (6,4,1)

e)((1,2,3,4,5)), 7/6/4, (1,2,3), (6,4,5)

f)((1,2,3,4,5)), 7/6/4, (1,2,3), (6,4,1),(4,6,7,2,3,5)

4) a)((1,2,3,4,5)), 7/6/1, (1,2,3), (2,4,5)

b)((1,2,3,4,5)), 7/6/1, (1,2,3), (2,4,5), (1,6,7,3,4,5)

c))((1,2,3,4,5)),7/6/1, (1,2,3), (2,4,5), (7,6,1)

d)((1,2,3,4,5)), 7/6/1, (1,2,3), (2,4,5), (6,1,4)

5) a)((1,2,3,4,5)), 7/6/2, (1,2,3), (2,4,5)

b)((1,2,3,4,5)), 7/6/2, (1,2,3), (2,4,5), (7,6,2)

II1)a)((1,2,3,4), 6/5/1, 7/2

b)((1,2,3,4)), 6/5/1, 7/2, (1,5,6,2,7,4)

c)((1,2,3,4)), 6/5/1, 7/2, (1,5,2,7,3,4)

d)((1,2,3,4)), 6/5/1, 7/2, (1,5,6,2,3,4)

e)((1,2,3,4)), 6/5/1, 7/2, (6,5,1)

f)((1,2,3,4)), 6/5/1, 7/2, (6,5,1), (1,5,2,7,3,4)

g) ((1,2,3,4)), 6/5/1, 7/2, (7,2,3).

h)((1,2,3,4)), 6/5/1, 7/2, (7,2,3), (1,5,6,2,3,4)

i)((1,2,3,4)), 6/5/1, 7/2, (7,2,3), (6,5,1)

j)((1,2,3,4)), 6/5/1, 7/2, (7,2,1)

k)((1,2,3,4)), 6/5/1, 7/2, (7,2,1), (1,5,6,2,3,4)

l)((1,2,3,4)), 6/5/1, 7/2, (7,2,1), (6,5,1)

m)((1,2,3,4)), 6/5/1, 7/2, (5,1,4)

n)((1,2,3,4)), 6/5/1, 7/2, (5,1,4), (1,5,6,2,7,3)

o)((1,2,3,4)), 6/5/1, 7/2, (5,1,4), (7,2,3)

p)((1,2,3,4)), 6/5/1, 7/2, (5,1,4), (7,2,4)

q)((1,2,3,4)), 6/5/1, 7/2, (5,1,4), (7,2,1)

r)((1,2,3,4)), 6/5/1, 7/2, (5,1,2)

s)((1,2,3,4)), 6/5/1, 7/2, (5,1,2), (7,2,3)

t)((1,2,3,4)), 6/5/1, 7/2, (7,2,3), (1,5,6,2,7,4)

u)((1,2,3,4)), 6/5/1, 7/2, (5,1,4), (7,2,4), (1,5,6,2,7,3).

2)a)((1,2,3,4)), 6/5/1, 7/2, (1,2,3)

b)((1,2,3,4)), 6/5/1, 7/2, (1,2,3), (1,5,6,2,7,4)

c)((1,2,3,4)), 6/5/1, 7/2, (1,2,3), (6,5,1)

d)((1,2,3,4)), 6/5/1, 7/2, (1,2,3), (7,2,4)

e)((1,2,3,4)), 6/5/1, 7/2, (1,2,3), (7,2,4), (6,5,1)

f)((1,2,3,4)), 6/5/1, 7/2 (1,2,3), (5,1,4)

g)((1,2,3,4)), 6/5/1, 7/2, (1,2,3), (5,1,4), (7,2,4)

3)a)((1,2,3,4)), 6/5/1, 7/4, (1,2,3)

b)((1,2,3,4)), 6/5/1, 7/4, (1,2,3), (1,5,6,2,4,7)

c)((1,2,3,4)), 6/5/1, 7/4, (1,2,3), (6,5,1)

d)((1,2,3,4)), 6/5/1, 7/4, (1,2,3), (7,4,2)

e)((1,2,3,4)), 6/5/1, 7/4, (1,2,3), (7,4,2), (6,5,1)

f)((1,2,3,4,)), 6/5/1, 7/4, (1,2,3), (7,4,1)

g)((1,2,3,4)), 6/5/1, 7/4, (1,2,3), (7,4,1), (6,5,1)

h)((1,2,3,4)), 6/5/1, 7/4, (1,2,3), (5,1,4)

i)((1,2,3,4)), 6/5/1, 7/4, (1,2,3), (5,1,4), (7,4,2)

j)((1,2,3,4)), 6/5/1, 7/4, (1,2,3), (7,4,2), (1,5,6,4,7,3)

4)a)((1,2,3,4)), 6/5/4, 7/1, (1,2,3)

b)((1,2,3,4)), 6/5/4, 7/1, (1,2,3), (4,5,6,1,7,2)

c)((1,2,3,4)), 6/5/4, 7/1, (1,2,3), (6,5,4)

d)((1,2,3,4)), 6/5/4, 7/1, (1,2,3), (7,1,4)

e)((1,2,3,4)), 6/5/4, 7/1, (1,2,3), (7,1,4), (6,5,4)

f)((1,2,3,4)), 6/5/4, 7/1, (1,2,3), (5,4,1)

g)((1,2,3,4)), 6/5/4, 7/1, (1,2,3), (5,4,2)

h)((1,2,3,4)), 6/5/4, 7/1, (1,2,3), (5,4,2), (7,1,4)

i)((1,2,3,4)), 6/5/4, 7/1, (1,2,3), (5,4,2), (1,7,4,5,6,3).

III 1) a)((1,2,3)), 5/4/1, 6/2, 7/3

b)((1,2,3)), 5/4/1, 6/2, 7/3, (1,4,5,2,6,3)

c)((1,2,3)), 5/4/1, 6/2, 7/3, (1,4,2,6,3,7)

d)((1,2,3)), 5/4/1, 6/2, 7/3, (6,2,3)

e)((1,2,3)), 5/4/1, 6/2, 7/3, (6,2,3), (1,4,5,2,3,7)

f)((1,2,3)), 5/4/1, 6/2, 7/3, (6,2,3), (7,3,1)

g)((1,2,3)), 5/4/1, 6/2, 7/3, (6,2,1)

h)((1,2,3)), 5/4/1, 6/2, 7/3, (6,2,1), (1,4,5,2,3,7)

i)((1,2,3)), 5/4/1, 6/2, 7/3, (6,2,1), (7,3,2)

j)((1,2,3)), 5/4/1, 6/2, 7/3, (5,4,1)

k)((1,2,3)), 5/4/1, 6/2, 7/3, (5,4,1), (1,4,2,6,3,7)

l)((1,2,3)), 5/4/1, 6/2, 7/3, (5,4,1), (7,3,1)

m)((1,2,3)), 5/4/1, 6/2, 7/3, (5,4,1), (7,3,2)

n)((1,2,3)), 5/4/1, 6/2, 7/3, (5,4,1), (6,2,3), (7,3,1)

o)((1,2,3)), 5/4/1, 6/2, 7/3, (5,4,1), (6,2,1), (7,3,1)

p)((1,2,3)), 5/4/1, 6/2, 7/3, (4,1,2)

q)((1,2,3)), 5/4/1, 6/2, 7/3, (4,1,2), (7,3,1)

r)((1,2,3)), 5/4/1, 6/2, 7/3, (4,1,2), (6,2,3)

s)((1,2,3)), 5/4/1, 6/2, 7/3, (4,1,2), (6,2,3), (7,3,1)

2) a)((1,2,3)), 5/4/1, 6/2, 7/3, (1,2,3)

b)((1,2,3)), 5/4/1, 6/2, 7/3, (1,2,3), (5,4,1)

IV)1)a)((1,2,3)), 5/4/1, 7/6/2

b)((1,2,3)), 5/4/1, 7/6/2, (1,4,5,2,6,7)

c)((1,2,3)), 5/4/1, 7/6/2, (1,4,5,2,6,3)

d)((1,2,3)), 5/4/1, 7/6/2, (5,4,1)

e)((1,2,3)), 5/4/1, 7/6/2, (5,4,1), (1,4,2,6,7,3)

f)((1,2,3)), 5/4/1, 7/6/2, (5,4,1), (7,6,2).

g)((1,2,3)), 5/4/1, 7/6/2, (4,1,3), (1,4,5,2,6,7)

h)((1,2,3)), 5/4/1, 7/6/2, (4,1,3)

i)((1,2,3)), 5/4/1, 7/6/2, (4,1,3), (7,6,2)

j)((1,2,3)), 5/4/1, 7/6/2, (4,1,3), (6,2,3)

k)((1,2,3)), 5/4/1, 7/6/2, (4,1,3), (6,2,3), (1,4,5,2,6,7)

l)((1,2,3)), 5/4/1, 7/6/2, (4,1,2)

m)((1,2,3)), 5/4/1, 7/6/2, (4,1,2), (7,6,2)

n)((1,2,3)), 5/4/1, 7/6/2, (4,1,2), (6,2,3)

2) a)((1,2,3)), 5/4/1, 7/6/2, (1,2,3)

b)((1,2,3)), 5/4/1, 7/6/2, (1,2,3), (1,4,5,2,6,7)

c)((1,2,3)), 5/4/1, 7/6/2, (1,2,3), (5,4,1)

d)((1,2,3)), 5/4/1, 7/6/2, (1,2,3), (5,4,1), (7,6,2)

DI1) a)((1,2,3,4)), 7/6/5/1

b)((1,2,3,4)), 7/6/5/1, (1,5,6,7,2,3)

c)((1,2,3,4)), 7/6/5/1, (1,5,6,2,3,4)

d)((1,2,3,4)), 7/6/5/1, (6,5,1)

e)((1,2,3,4)), 7/6/5/1, (5,1,2)

f)((1,2,3,4)), 7/6/5/1, (5,1,2), (1,5,6,7,3,4)

2) a)((1,2,3,4)), 7/6/5/1, (1,2,3)

b)((1,2,3,4)), 7/6/5/1, (1,2,3), (1,5,6,7,4,2)

c)((1,2,3,4)), 7/6/5/1, (1,2,3), (6,5,1)

d)((1,2,3,4)), 7/6/5/1, (1,2,3), (5,1,4)

3) a)((1,2,3,4)), 7/6/5/4, (1,2,3)

b)((1,2,3,4)), 7/6/5/4, (1,2,3), (4,5,6,7,1,2)

c)((1,2,3,4)), 7/6/5/4, (1,2,3), (6,5,4)

d)((1,2,3,4)), 7/6/5/4, (1,2,3), (5,4,1)

e)((1,2,3,4)), 7/6/6/4, (1,2,3), (5,4,1), (2,3,4,5,6,7)

II 1)a)((1,2,3)), 6/5/4/1, 7/2

b)((1,2,3)), 6/5/4/1, 7/2, (1,4,5,6,2,7)

c)((1,2,3)), 6/5/4/1, 7/2, (1,4,5,6,2,3)

d)((1,2,3)), 6/5/4/1, 7/2, (1,4,5,2,7,3)

e)((1,2,3)), 6/5/4/1, 7/2, (5,4,1)

f)((1,2,3)), 6/5/4/1, 7/2, (7,2,3)

g)((1,2,3)), 6/5/4/1, 7/2, (7,2,3), (1,4,5,6,2,7)

h)((1,2,3)), 6/5/4/1, 7/2, (7,2,3), (1,4,5,6,2,3)

i)((1,2,3)), 6/5/4/1, 7/2, (5,4,1), (7,2,3)

j)((1,2,3)), 6/5/4/1, 7/2, (7,2,1)

k)((1,2,3)), 6/5/4/1, 7/2, (7,2,1) (1,4,5,6,2,3)

l)((1,2,3)), 6/5/4/1, 7/2, (7,2,1), (5,4,1)

m)((1,2,3)), 6/5/4/1, 7/2, (4,1,3)

n)((1,2,3)), 6/5/4/1, 7/2, (4,1,3), (1,4,5,6,2,7)

o)((1,2,3)), 6/5/4/1, 7/2, (4,1,3), (7,2,3)

p)((1,2,3)) , 6/5/4/1, 7/2, (4,1,3), (7,2,3), (1,4,5,6,2,7)

g)((1,2,3)), 6/5/4/1, 7/2, (4,1,3), (7,2,1)

r)((1,2,3)), 6/5/4/1, 7/2, (4,1,2)

s)((1,2,3)), 6/5/4/1, 7/2, (4,1,2), (4,2,3)

2) a)((1,2,3)), 6/5/4/1, 7/2, (1,2,3)

b)((1,2,3)), 6/5/4/1, 7/2, (1,2,3),(1,4,5,6,2,7)

c)((1,2,3)), 6/5/4/1, 7/2, (1,2,3), (5,4,1)

III a)((1,2)), 5/4/3/1, 7/6/2

b)((1,2)), 5/4/3/1, 7/6/2, (1,3,4,5,2,6)

c)((1,2)), 5/4/3/1, 7/6/2, (1,3,4,2,6,7)

d)((1,2)), 5/4/3/1, 7/6/2, (7,6,2)

e)((1,2)), 5/4/3/1, 7/6/2, (7,6,2), (1,3,4,5,2,6)

f)((1,2)), 5/4/3/1, 7/6/2, (4,3,1).

g)((1,2)), 5/4/3/1, 7/6/2, (4,3,1), (7,6,2)

h)((1,2)), 5/4/3/1, 7/6/2, (6,2,1)

i)((1,2)), 5/4/3/1, 7/6/2, (3,1,2)

j)((1,2)), 5/4/3/1, 7/6/2, (3,1,2), (7,6,2)

k)((1,2)), 5/4/3/1, 7/6/2, (6,2,1), (4,3,1)

EI1)a)((1,2,3)), 7/6/5/4/1

b)((1,2,3)), 7/6/5/4/1, (1,4,5,6,7,2)

c)((1,2,3)), 7/6/5/4/1, (5,4,1)

d)((1,2,3)), 7/6/5/4/1, (4,1,2)

e)((1,2,3)), 7/6/5/4/1, (4,1,2), (1,4,5,6,7,3)

f)((1,2,3)), 7/6/5/4/1, (1,4,5,6,2,3)

2) a)((1,2,3)), 7/6/5/4/1, (1,2,3)

b)((1,2,3)), 7/6/5/4/1, (1,2,3), (1,4,5,6,7,2)

c)((1,2,3)), 7/6/5/4/1, (1,2,3), (5,4,1)

II a)((1,2)), 6/5/4/3/1, 7/2

b)((1,2)), 6/5/4/3/1, 7/2, (1,3,4,5,6,2)

c)((1,2)), 6/5/4/3/1, 7/2, (1,3,4,5,2,7)

d)((1,2)), 6/5/4/3/1, 7/2, (4,3,1)

e)((1,2)), 6/5/4/3/1, 7/2, (3,1,2)

f)((1,2)), 6/5/4/3/1, 7/2, (7,1,2)

g)((1,2)), 6/5/4/3/1, 7/2, (7,1,2), (1,3,4,5,6,2)

h)((1,2)), 6/5/4/3/1, 7/2, (7,2,1), (4,3,1)

F a)((1,2)), 7/6/5/4/3/1

b)((1,2)), 7/6/5/4/3/1, (1,3,4,5,6,7)

c)((1,2)), 7/6/5/4/3/1, (1,3,4,5,6,2)

d)((1,2)), 7/6/5/4/3/1, (4,3,1).

e)((1,2)), 7/6/5/4/3/1, (3,1,2)

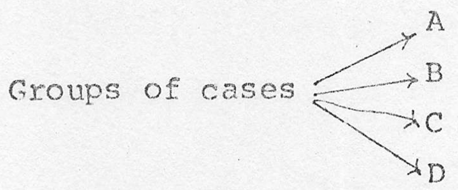
f)((1,2)), 7/6/5/4/3/1, (3,1,2), (1,3,4,5,6,7)

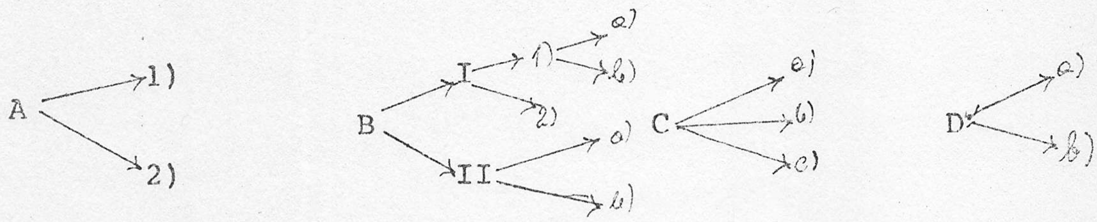
G a)((1)), 7/6/5/4/3/2/1

b)((1)), 7/6/5/4/3/2/1, (1,2,3,4,5,6)

c)((1)), 7/6/5/4/3/2/1, (3,2,1).

§4. The tree of cases for d=5 (see §5 in the text)





The explanation of the graph:

A 1) general position

2) ((1,2,3,4)), (1,2,3)

B I 1) a)((1,2,3)), 4/1, (4,1,2)   b) ((1,2,3)), 4/1,

2) ((1,2,3)), 4/1, (1,2,3)

II a)((1,2)), 3/1, 4/2

b)((1,2)), 3/1, 4/2, (3,1,2)

C a)((1,2)), 4/3/1

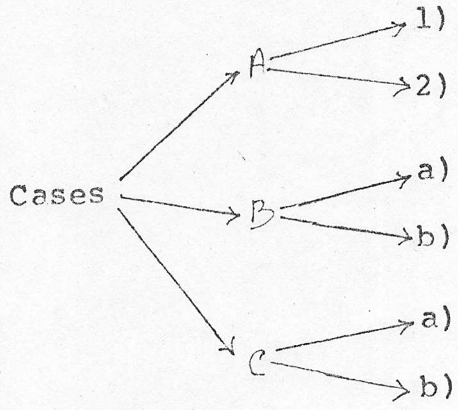
b)((1,2)), 4/3/1, (4,3,1)

c)((1,2)), 4/3/1, (3,1,2)

D a)((1)), 4/3/2/1

b)((1)), 4/3/2/1, (3,2,1)

§5. The tree of cases for d=6 (see §5 in the text)



The explanation of the graph:

A 1) general position

2) ((1,2,3)), (1,2,3)

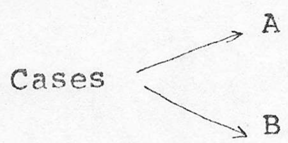
B a) ((1,2)), 3/1

b) ((1,2)), 3/1, (3,1,2)

C a) ((1)), 3/2/1

b) ((1)), 3/2/1, (3,2,1)

§6. The tree of cases for d=7 (see §5 in the text)



The explanation of the graph:

A :((1,2))

B:((1)), 2/1.

§7. The proof of the Proposition 1 (§6 in the text)

In this § we'll present necessary positions of the 8 points in order to obtain the desired groups of singularities on (degenerate) Del Pezzo surfaces of degree 1, presented in the list (1) (§6, page 23).

For A_1: ((1,2,3,4,5,6,7)), 8/1

For $2A_1$: ((1,2,3,4,5,6)), 7/1, 8/2

For A_3: ((1,2,3,4,5,6)), 7/1, 8/2, (1,2,3)

For A_2: ((1,2,3,4,5,6,7)), 8/1, (1,2,3)

For A_1A_2: ((1,2,3,4,5,6)), 7/1, (1,2,3), 8/4

For $3A_1$: ((1,2,3,4,5)), 6/1, 7/2, 8/3

For A_4: ((1,2,3,4)), 8/7/6/5/4

For D_4: ((1,2,3,4,5)), 8/7/6/5, (6,5,1)

For A_1A_3: ((1,2,3,4)), 8/7/6/1, 5/2

For $A_2 2A_1$: ((1,2,3,4)), 8/7/1, 6/2, 5/3

For $2A_2$: ((1,2,3,4)), 8/7/1, 6/5/2

For $4A_1$: ((1,2,3,4)), 5/1, 6/2, 7/3, 8/4

For A_5: ((1,2,3)), 8/7/6/5/4/1

For D_5: ((1,2,3,4)), 8/7/6/5/1, (5,1,2)

For A_4A_1: ((1,2,3)), 8/7/6/5/1, 4/2

For D_4A_1: ((1,2,3,4)), 8/7, 6/2, (6,2,1), 5/3

For A_32A_1: ((1,2,3)), 8/7, 6/1, 5/2, 4/3

For A_23A_1: ((1,2,3,8)), 5/4/2, 6/2, 7/3, (5,4,2)

For A_12A_2: ((1,2,3)), 5/4/1, 7/6/2, 8/3

For $5A_1$: ((1,2,3,4)), 5/1, 6/2, 7/3, 8/4, (5,1,6,2,7,3)

For A_3A_2: ((1,2,3)), 8/7/6/1, 5/4/2

For A_6: ((1,2)), 8/7/6/5/4/3/1

For D_6: ((1,2,8)), 5/4/3/1, 7/6/2, (3,1,2)

For E_6: ((1,2,8)), 6/5/4/3/1, 7/2, (3,1,2)

For A_5A_1: ((1,2)), 8/7/6/5/4/1, 3/2

For D_5A_1: ((1,2,8)), 6/5/4/3/1, 7/2, (4,3,1)

For A_42A_1: ((1,4,6)), 8/7/3/2/1, 5/4, 7/6

For D_42A_1: ((1,2,3,8)), 6/5/4/1, 7/2, (4,1,3), (7,2,3)

For A_33A_1: ((1,2,3,4)), 5/4/1, 6/2, 7/3, (1,4,2,6,3,7), (5,4,1)

For A_24A_1: ((1,4,6,8)), 3/2/1, 5/4, 7/6, (5,4,8), (7,6,8).

For $2A_12A_2$: ((1,2,3,7)), 8/7, 4/1, 5/2, 6/3, (5,2,3), (4,1,2)

For $6A_1$: ((1,2,3,8)), 5/1, 6/3, 7/2, (5,1,4), (6,3,4),
(1,5,3,6,2,7)

For A_4A_2: ((1,2)), 8/7/6/5/1, 4/3/2.

For D_4A_2: ((1,5,6)), 4/3/2/1, 8/7/6, (2,1,5)

For D_42A_1: ((1,5,6,8)), 4/3/2/1, 7/6, (2,1,5), (7,6,5)

For $A_1A_2A_3$: ((1,2,8)), 5/4/3/1, 7/6/2, (7,6,2)

For $3A_2$: ((1,2,3,4)), 8/7/1, 6/5/2, (8,7,1), (6,5,2)

For $2A_3$: ((1,2,8)), 5/4/3/1, 7/6/2, (1,3,4,5,2,6)

For E_7: ((1,8)), 7/6/5/4/3/2/1, (3,2,1)

For A_7: ((1)), 8/7/6/5/4/3/2/1

For D_7: ((1,2,3)), 7/6/5/4/1, 8/2, (1,2,3), (5,4,1)

For A_1A_6: ((1,7)), 6/5/4/3/2/1, 8/7, (8,7,1)

For A_1D_6: ((1,2,8)), 6/5/4/3/1, 7/2, (7,2,1), (4,3,1)

For A_1E_6: ((1,7)), 6/5/4/3/2/1, 8/7, (3,2,1)

For A_52A_1: ((1,7)), 6/5/4/3/2/1, 8/7, (8,7,1), (1,2,3,4,5,6)

For D_52A_1: ((1,5,7)), 4/3/2/1, 6/5, 8/7, (6,5,1), (8,7,1)

For D_43A_1: ((1,2,3,8)), 5/4/1, 6/2, 7/3, (5,4,1), (6,2,1), (7,3,1)

For A_34A_1: ((1,5,7)), 4/3/2/1, 6/5, 8/7, (1,2,3,4,5,6), (1,2,3,4,7,8)

For $A_1A_2A_4$: ((1,6)), 5/4/3/2/1, 8/7/6, (8,7,6)

For $A_2A_32A_1$: ((1,2,3,4)), 8/5/1, 6/3, 7/2, (7,2,4), (6,3,4), (8,5,1)

For $A_1 3A_2$: ((1,2,7)), 4/3/1, 6/5/2, 8/7, (4,3,1), (6,5,2)

For $A_1 2A_3$: ((1,2,3,8)), 5/4/1, 7/6/2, (4,1,3), (6,2,3), (1,4,5,2,6,7

For $A_5 A_2$: ((1,2,3,8)), 5/4/1, 7/6/2, (1,2,3), (5,4,1), (7,6,2)

For $A_3 A_4$: ((1,5)), 4/3/2/1, 8/7/6/5, (3,2,1)

For $A_3 D_4$: ((1,5)), 4/3/2/1, 8/7/6/5, (1,2,5,6,7,8)

For E_8: ((1)), 8/7/6/5/4/3/2/1, (3,2,1)

For A_8: ((1,5,8)), 4/3/2/1, 7/6/5, (1,5,8), (7,6,5), (3,2,1)

For D_8: ((1,6)), 5/4/3/2/1, 8/7/6, (7,6,1), (3,2,1)

For $D_5 A_2$: ((1,6)), 5/4/3/2/1, 8/7/6, (3,2,1)

For $E_7 A_1$: ((1,6)), 5/4/2/1, 8/7/6, (8,7,6), (2,1,6)

For $A_1 A_7$: ((1,5,8)), 4/3/2/1, 7/6/5, (1,5,8), (7,6,5), (1,2,3,4,5,6)

For $D_6 2A_1$:((1,2,3,8)), 7/4/1, 5/2, 6/3, (1,2,3), (4,1,8), (5,2,8), (

(6,3,8).

For $A_1 A_2 A_5$: ((1,4,7)), 3/2/1, 6/5/4, 8/7, (3,2,1), (6,5,4),

(1,2,4,5,7,8)

For $2A_1 2A_3$: ((1,2,3)), 6/5/4/1, 7/2, 8/3, (7,2,3), (6,5,4,1,2,7),

(6,5,4,1,3,8)

For $E_6 A_2$:((1,6)), 5/4/3/2/1, 8/7/6, (3,2,1), (8,7,6)

For $A_3 D_5$: ((1,5)), 4/3/2/1, 8/7/6/5, (3,2,1), (1,2,5,6,7,8)

For $2A_4$:((1,3,5,7)), 2/1, 4/3, 6/5, 8/7, (2,1,7), (8,7,5),

(6,5,3), (4,3,1)

For $4A_2$:((1,2,3,8)), 5/4/1, 6/2, 7/3, (1,4,5,2,3,7), (1,4,5,2,6,8),

(6,2,3), (7,3,8)

For $2D_4$:((1,5)), 4/3/2/1, 8/7/6/5, (1,2,3,4,5,6), (1,2,5,6,7,8).

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