ON THE ISOTROPY IN THERMOELASTOVISCOPLASTICITY

by

Sanda CLEJA-ŢIGOIU

PREPRINT SERIES IN MATHEMATICS

No.63/1984

ON THE ISOTROPY IN THERMOELASTOVISCOPLASTICITY

by

Sanda CLEJA-ȚIGOIU*)

November 1984

*) University of Bucharest, Faculty of Mathematics, Str.Academiei 14, 70109 Bucharest, Romania.

ON THE ISOTROPY IN THERMOELASTOVISCOPLASTICITY

by Sanda CLEJA-ŢIGOIU

INTRODUCTION

In this paper we discuss the isotropy of the thermoelasto-viscoplastic body with instantaneous plasticity (or shorter thermoviscoplastic (t.e.v.p.) body) defined in the previous paper [1].

The point of view adopted there is the same one with that presented by M.Mihăilesc-Suliciu and I.Suliciu [2] and E.Sóos [3] in which the concepts of the plastic and elastic deformations are simultaneously introduced with the aid of the constitutive and evolution equations; these deformations are not thought of an kinematic concepts.

Here the gradient of deformation is multiplicatively decomposed into its components.

The local configuration [4,5] is used in [3] to defined the current local thermoplastic deformation F^p which as thermoplastic deformation F^e is not a gradient of some global deformation [9-11].

The term "instantaneous plasticity" is added to those of thermoviscoplastic as in [6,7,8] in order to specify the presence of the rate independent part in evolution equation of F^p and of the work hardening varialbe.

In the first section we briefly recall some of the basical assumptions of t.e.v.p. [1].

Two concepts of the material symmetry may be used for t.e.v.p. body, i.e. k-material symmetry related to k reference configuration and K_{Xt}-material symmetry corresponding to the current local relaxed configuration as both of them are involved in mathematical description of t.e.v.p. behaviour of the body.

Here we consider only the material isotropy.

We point out some questions: what is the relation which can exist between the two concepts of the isotropy, which of them is addequate to describe the behaviour of the appropiate t.e.v.p. body.

Since the admissible thermoelastic constitutive function by means of the evolution system becomes an operator depending on the history up to time t of the gradient of deformation, F_k, and of the temperature θ Noll's concept [12,13] of the isotropic body relative to the reference configuration k, as a simple material may seem naturally to be considered.

In the second section of the paper we consider the restrictions which can be imposed on the constitutive equations if the material has isotropy in its reference configuration. We prove there: if t.e.v.p. body is k-isotropic then the thermoelastic respons function is an isotropic one with respect to the left Cauchy-Green elastic tensor V^e and the proper values of the current local right stretch plastic tensor U^p are all equal. There are no favoured proper directions of the plastic deformation at X. In this last point of the proof we essentialy use the relaxation condition which is the usual adopted one in the description of the plastic deformation.

It follows that t.e.v.p. body k isotropic and plastically incompressible undergoes only thermoelastic (finite) deformation and on the other hand the evolution functions describing the rate of plastic deformation are skew symmetric valued tensor. But a thermoviscoplastic body maybe plastically incompressible deformed also by torsion.

In conclusion the concept of k isotropy is much more restrictive and it will be not addequate in the description of the t.e.v.p. behaviour of the body with specific material symmetry. Thus a t.e.v.p. body is essential dependent on the reference configuration k.

In the theory of dislocation it is assumed that the orientation of the crystallographic structure is maintened during slip deformation [14]. There exist one favoured configuration the local natural configuration at the initial moment t_o , this may be the fixed k.

We pass to the other concept of the isotropy this being independent of the previous one. We present K_{Xt}-isotropy in the fourth section. We conclude that the thermoelastic constitutive function is isotropic in V^e and F^p, and the evolution functions in π and F^p, where π is the Piola-Kirchhoff symmetric tensor relative to K_{Xt}.

If we consider that $\mathcal{B}$ satisfy the temporarly invariance condition [3] then all the functions will result isotropic in V^e and respectively in π , as there were assumed in the mentioned paper.

In this case we obtain the existence of one relaxed configuration $\tilde{K}_{Xt}$ deduced from the actual one by a pure elastic deformation and such that the rate of deformation is additively expressed by means of the rate of thermoelastic and thermoplastic strain.

This choise of the plastic deformation such that F^e - the elastic deformation - becomes a symmetric pure deformation is proposed by E.H.Lee [11]. Some cases when the additive representation of the rate of deformation following from the multiplicative decomposition of the deformation gradient are presented in [15].

This means we can develope the theory based on additive decomposition when we consider t.e.v.p. body K_{Xt}-isotropic verifying also the temporarly invariance condition. This was a basis for much works in elastic plastic analysis.

The second concept of isotropy is addequate for t.e.v.p. body. We can call this kind of the isotropy the plastical isotropy since it is related to K_{Xt} which is used to define the current local thermoplastic deformation tensor.

The equations describing the behavior of the isotropic elasto-plastical deformed body where considered in [6-8] . There the relaxed configurations are isoclinal.

We deal in the third section with the concept of the thermoviscoplastically equivalent configurations which will be used in the definition of K_{Xt}-isotropic t.e.v.p. body.

Here we use the following notations:

$\mathcal{B}$ - a body,

$\mathcal{E}$ - euclidian space with translation vector space $\mathcal{V}$,

Lin $= \{A: \mathcal{V} \to \mathcal{V} \mid$ linear mapping $\}$.

Orth $= \{Q \in$ Lin $\mid QQ^T=I\}$ the orthogonal group,

Invlin $\subset$ Lin - the sed of all invertible linear mappings,

Sym $= \{A \in$ Lin $\mid A=A^T\}$, where A^T is the transpose of A,

Unim $= \{A \in$ Lin $\mid |\det A|=1\}$,

Psym - the set of the all positive-defined symmetric mappings, Skew - the set of all skewsymmetric tensor,

$\{A\}^a$, $\{A\}^s$ - the skewsymmetric and respectiv symmetric parts of the tensor A,

K_X - a local configuration at the point X, i.e. the equivalence classe defined by the configuration K by the relation $K_X = \{\gamma \mid \gamma \text{ configuration of } \mathcal{B} : \nabla(\gamma \circ K^{-1})|_{K(X)} = I\}$,

$\mathcal{T}_X$ - the tangent space at X ,

R - the set of real numbers,

$\partial_{\pi}\mathcal{F}(\pi, \theta, \alpha) \in \text{Sym}$ is a tensor-partial derivative of $\mathcal{F}(\pi, \theta, \alpha)$ with respect to $\pi \in \text{Sym}$, i.e.,

$$\lim_{s \to 0} \frac{1}{s}(\mathcal{F}(\pi + sA, \theta, \alpha) - \mathcal{F}(\pi, \theta, \alpha)) = \partial_{\pi}\mathcal{F}(\pi, \theta, \alpha) \cdot A$$

1. THERMOELASTOVISCOPLASTIC BODY

We briefly recall some of the basical assumptions of the mathematical (axiomatic) presentation of the thermoviscoplastic bodies with instantaneous plasticity [1]; abbreviately denoted t.e.v.p.

Let $\mathcal{B}$ be a t.e.v.p. body. Then we have the following assumptions:

A.1. For a given material point X at time t for any (χ, θ) where χ is a motion of a neibourhood $\mathcal{N}_X \subset \mathcal{B}$ of X and θ the temperature field, there exist $K_{Xt} \in \text{Invlin}(\mathcal{T}_X, \mathcal{V})$ - the current local relaxed configuration (c.l.r.c.) and $\alpha_{K_{Xt}} \in R$ - the work -hardening variable (w.h.v.) and a function $f_{K_{Xt}}$ so that

A.2. The Cauchy stress tensor is given by

$$T(X,t) = f_{K_{Xt}}(F^e_{K_{Xt}}, \theta(X,t), \alpha_{K_{Xt}}) \tag{1.1}$$

where $F^{e}_{K_{Xt}}$ - the current local thermoelastic deformation at X is defined by

$$F^{e}_{K_{Xt}} = \nabla \chi(X,t) \circ K^{-1}_{Xt} \tag{1.2}$$

A.3. The actual rate of K_{xt} is given by the evolution equation

$$\dot{F}^{p}_{K_{Xt}} (F^{p}_{K_{Xt}})^{-1} = A_{K_{Xt}} (\tilde{\Pi}_{K_{Xt}}, \theta(X,t), \alpha_{K_{Xt}}) + \\ + \langle\lambda_{K_{Xt}}\rangle B_{K_{Xt}} (\tilde{\Pi}_{K_{Xt}}, \theta(X,t), \alpha_{K_{Xt}}) \tag{1.3}$$

together with initial data

$$K_{Xt_o} = Ko(X) \tag{1.4}$$

in which

$$F^{p}_{K_{Xt}} = K_{Xt} \circ (\nabla k(X))^{-1} \tag{1.5}$$

is the current local thermoplastic deformation at X, related to the local reference configuration $k \equiv \chi(.,t_o)$ and $\tilde{\Pi}_{K_{Xt}}$ is the symmetric Piola-Kirchhoff stress tensor corresponding to K_{Xt}:

$$\tilde{\Pi}_{K_{Xt}} = \det F^{e}_{K_{Xt}} (F^{e}_{K_{Xt}})^{-1} T(X,t) (F^{e}_{K_{Xt}})^{-T} \tag{1.6}$$

Here $\lambda_{K_{Xt}}$ is the plastic factor associated to the current yield surface $\mathcal{F}_{K_{Xt}} (\tilde{\Pi}_{K_{Xt}}, \theta(X,t), \alpha_{K_{Xt}}) = 0$ defined by

$$\lambda_{K_{Xt}} = \partial_{\tilde{\Pi}} \mathcal{F}_{K_{Xt}} (Z, \alpha_{K_{Xt}}) \cdot \dot{\tilde{\Pi}} + \partial_{\theta} \mathcal{F}_{K_{Xt}} (Z, \alpha_{K_{Xt}}) \dot{\theta} \tag{1.7}$$

in which $Z = (\tilde{\Pi}_{K_{Xt}}, \theta(X,t))$.

A.4. The evolution equation for $\alpha_{K_{Xt}}$ is given by

$$\dot{\alpha}_{K_{Xt}} = \ell_{K_{Xt}}(Z, \alpha_{K_{Xt}}) + \langle \lambda_{K_{Xt}} \rangle\, m_{K_{Xt}}(Z, \alpha_{K_{Xt}}) \tag{1.8}$$

with the initial data

$$\alpha_{K_{Xt_o}} = \alpha_o(x) \tag{1.9}$$

A.5. The configuration K_{Xt} and the function $f_{K_{Xt}}$ satisfy the relaxation condition:

the positive-defined symmetric tensor Uo is such that

$$f_{K_{Xt}}(Uo, \theta(X, to), \alpha_{K_{Xt}}) = 0 \tag{1.10}$$

if and only if

$$U_o = I \tag{1.11}$$

By taking into account some other assumptions [1] the following proposition and remarks hold:

P.1.1. The thermoelastic constitutive function has the "objectivity property", i.e.

$$f_{K_{Xt}}(QF^e, \theta, \alpha_{K_{Xt}}) = Qf_{K_{Xt}}(F^e, \theta, \alpha_{K_{Xt}})Q^T \tag{1.12}$$

for all $Q \in$ Orth.

2) The Piola-Kirchhoff symmetric stress tensor $\tilde{\pi}_{K_{Xt}}$ is given by

$$\tilde{\pi}_{K_{Xt}} = h_{K_{Xt}}(C^e_{K_{Xt}}, \theta(X,t), \alpha_{K_{Xt}}) \tag{1.13}$$

with

$$C^e_{K_{Xt}} = (F^e_{K_{Xt}})^T F^e_{K_{Xt}} \tag{1.14}$$

Remark 1.1. From (1.2) and (1.5) one gets the multiplicative decomposition of the deformation gradient F_k into its components

$$F_k = F^e_{K_{Xt}} F^p_{K_{Xt}} \tag{1.15}$$

Remark 1.2. Together with initial data one of the reference configurations of a neighborhood of a material points X must be given too.

Remark 1.3. In order to obtain some evolution system for the variables $F^p_{K_{xt}}$ and $\alpha_{K_{xt}}$ from (1.3) and (1.8) we rewrite all evolution and constitutive fonctions in the form:

$$A_{K_{Xt}}(Z,\alpha_{K_{Xt}}) = A_k(Z,\alpha_{K_{Xt}}, F^p_{K_{Xt}}) \tag{1.16}$$

So both reference configuration $k=\chi(.,to)$ and current relaxed local configuration K_{xt} are mentioned.

The evolution system for $(F^p_{K_{Xt}}, \alpha_{K_{Xt}})$ is obtained in the form:

$$\dot{F}^p_{K_{Xt}} (F^p_{K_{Xt}})^{-1} = A_k(Z,\alpha_{K_{Xt}}, F^p_{K_{Xt}}) + \langle \lambda_{K_{Xt}} \rangle B_k(Z,\alpha_{K_{Xt}}, F^p_{K_{Xt}}) \tag{1.17}$$

$$\dot{\alpha}_{K_{Xt}} = l_k(Z,\alpha_{K_{Xt}}, F^p_{K_{Xt}}) + \langle \lambda_{K_{Xt}} \rangle m_k(Z,\alpha_{K_{Xt}}, F^p_{K_{Xt}}) \tag{1.18}$$

in which

$$Z = (\tilde{\Pi}_{K_{Xt}}, \theta(X,t)) \tag{1.19}$$

with

$$\tilde{\Pi}_{K_{Xt}} = h_k(\{F_k(F^p_{K_{Xt}})^{-1})^T F_k(F^p_{K_{Xt}})^{-1}\}^s, \theta(X,t), \alpha_{K_{Xt}}, F^p_{K_{Xt}}) \tag{1.20}$$

as it follows from (1.13)-(1.15).

To (1.17)-(1.20) one adds the initial data.

Remark 1.4. By means of the complementary plastic factor [1] $\beta_{K_{Xt}}$ the plastic factor $\lambda_{K_{Xt}}$ may be written as

$$\lambda_{K_{Xt}} = a_{K_{Xt}} . L(X,t) + b_{K_{Xt}} \dot{\theta}(X,t) + c_{K_{Xt}} \qquad (1.21)$$

when $\lambda_{K_{Xt}} \geqslant 0$, therefore as depending on $(L(x,t), \dot{\theta}(X,t))$. Both $\beta_{K_{Xt}}$ and $\lambda_{K_{Xt}}$ have the same sign. All the functions in (1.21) are expressed by means of $h_{K_{Xt}}$ and $\mathcal{F}_{K_{Xt}}$ rewritten in the form (1.16) ■

In the above relations the dependency on g - the temperature gradient has not been mentioned in order to simplify this presentation although in the previous paper g was present ■

One may therefore draw to the following conclusion:

The admissible termoelastic constitutive function written under the form (1.16) by means of the evolution system becomes an operator depending on the history up to time t of the function

$$s \longrightarrow \xi(X,s) \equiv (F_k(X,s), \theta(X,s)). \qquad (1.22)$$

2. k-ISOTROPIC THERMOVISCOPLASTIC BODY

Noll's concept [12,13] of the material symmetry for a simple material may naturally be considered if we take the above conclusion into account.

Further on the thermomecanical response of the material will only be considered.

In order to discuss the concept of k-isotropic t.e.v.p. body we start from Noll's definition.

Let

$$T(X,t) = \mathcal{F}_k(F^t, \mathcal{X}) \qquad (2.1)$$

be the constitutive equation of a simple material at a particle X, where k is a local reference configuration with $\mathcal{X}=k(X)$.

D.2.1. Two local configurations k and $\bar{k}$ are materially isomorphic at X if

$$T(X,t)=\mathcal{F}_k(F^t,\mathbb{X})=\mathcal{F}_{\bar{k}}(F^t,\bar{\mathbb{X}}) \tag{2.2}$$

hold for all F^t in the domain of $\mathcal{F}_k$ and

$$\rho_k(\mathbb{X})=\rho_{\bar{k}}(\bar{\mathbb{X}}), \tag{2.3}$$

where $k(X)=\mathbb{X}$, $\bar{k}(X)=\bar{\mathbb{X}}$.

We suppose that the domain of $\mathcal{F}$ contains the history of all invertible linear mappings.

The local deformation from the configuration k to the local configuration $\bar{k}$ will be denoted by H:

$$H=\nabla(\bar{k}\circ k^{-1})(k(X))\equiv\bar{K}_o\circ K_o^{-1} \tag{2.4}$$

D.2.2. H∈Invlin is k-symmetry transformation at X if by the local deformation H to the reference configuration there corresponds a local configuration $\bar{k}$ that is materially isomorphis to k.

P.2.1. Let $g_k(\mathbb{X})$ be the set of all k-symmetry transformations. Then

1) $g_k(\mathbb{X})=\{H\in Unim|\mathcal{F}_k(F^tH,\mathbb{X})=\mathcal{F}_k(F^t,\mathbb{X})$, for all $F(\tau)\in Invlin\}$ (2.5)

2) The two symmetry groups $g_k(\mathbb{X})$ and $g_{\bar{k}}(\bar{\mathbb{X}})$ are related by

$$g_{\bar{k}}=Pg_kP^{-1} \tag{2.6}$$

with P-the local deformation from k to $\bar{k}$.

D.2.3. A t.e.v.p. body at X and at time t is elastically deformed if

$$U^p_{K_{Xt}}=I \tag{2.7}$$

where $U^p_{K_{Xt}}$ is the positive-defined symmetric tensor from the polar decomposition of $F^p_{K_{Xt}}$.

T.2.1. Let H be a k-symmetry transformation for $\mathcal{B}$ t.e.v.p. body with instantaneous plasticity at X. Then

$$f_{K_{Xt}}(F^e H_{K_{Xt}}, \theta, \alpha_{K_{Xt}}) = f_{K_{Xt}}(F^e, \theta, \alpha) \tag{2.8}$$

for all $(F^e, \theta, \alpha) \in \mathcal{D}_{K_{Xt}}$ - the domain of $f_{K_{Xt}}$, with

$$H_{K_{Xt}} = F^p_{K_{Xt}} H (F^p_{K_{Xt}})^{-1} \tag{2.9}$$

Proof. If $H \in g_k(X)$ there exists a local reference configuration $\bar{k}$ materially isomorphic to k. Therefore to a given history up to time t of the deformation gradient F and of the temperature θ the same values of the stress corresponding to the choice of k and $\bar{k}$ respectively as reference local configurations are equal. Thus two local motions χ and $\bar{\chi}$ may be defined as

$$F(\tau) = F_k(X, \tau) \equiv \nabla\chi(X, \tau) \circ (\nabla k(X))^{-1} \tag{2.10}$$

$$F(\tau) = \bar{F}_{\bar{k}}(X, \tau) \equiv \nabla\bar{\chi}(X, \tau) \circ (\nabla\bar{k}(X))^{-1} \tag{2.11}$$

The temperature field in two processes (χ, θ) and $(\bar{\chi}, \bar{\theta})$ is considered to be the same, i.e. $\theta(X,\tau) = \bar{\theta}(X,\tau)$, $\forall \tau \in R$.

By using A.2. two pairs $(K_{Xt}, \alpha_{K_{Xt}})$ and $(\bar{K}_{Xt}, \bar{\alpha}_{\bar{K}_{Xt}})$ may be determined such that

$$f_{\bar{K}_{Xt}}(\bar{F}^e_{\bar{K}_{Xt}}, \theta, \bar{\alpha}_{\bar{K}_{Xt}}) = f_{K_{Xt}}(F^e_{K_{Xt}}, \theta, \alpha_{K_{Xt}}) \tag{2.12}$$

The connection between the thermoelastic constitutive functions corresponding to a change of c.l.r.c. is given by formula [1]

$$f_{\bar{K}_{Xt}}(\bar{F}^e_{\bar{K}_{Xt}}, \theta, \bar{\alpha}_{\bar{K}_{Xt}}) = f_{K_{Xt}}(\bar{F}^e_{\bar{K}Xt} P(t), \theta, \alpha_{K_{Xt}}) \tag{2.13}$$

with $\theta = \theta(X,t)$ and

$$\overline{K}_{Xt} \circ K_{Xt}^{-1} = P(t) \in \text{Orth} \tag{2.14}$$

The relation between the corresponding current local thermoplastic deformations is given by

$$\overline{F}^{p}_{\overline{K}_{Xt}} = \overline{K}_{Xt} \circ (\nabla \overline{k}(X))^{-1} = P(t) F^{p}_{K_{Xt}} H^{-1} \tag{2.15}$$

where H is defined by (2.4).

Using the decomposition (1.15) of the gradient deformation into its parts

$$F^{e}_{K_{Xt}} F^{p}_{K_{Xt}} = \overline{F}^{e}_{\overline{K}_{Xt}} \overline{F}^{p}_{\overline{K}_{Xt}} \tag{2.16}$$

and connection (2.15) between the current local thermoplastic deformations when both motions are considered there results

$$\overline{F}^{e}_{\overline{K}_{Xt}} = F^{e}_{K_{Xt}} (F^{p}_{K_{Xt}} H (F^{p}_{K_{Xt}})^{-1}) P^{T}(t) \tag{2.17}$$

or, with (2.9)

$$\overline{F}^{e}_{\overline{K}_{Xt}} = F^{e}_{K_{Xt}} H_{K_{Xt}} P^{T}(t) \tag{2.18}$$

From (2.12), (2.13) and (2.18) one obtains formula (2.8).

The tensor $H_{K_{Xt}} \in \mathcal{U}\text{nim}$ is called a <u>K_{Xt}-symmetry transformation</u> if (2.8) holds for all $(F^{e}, \theta, \alpha) \in \mathcal{D}_{K_{Xt}}$.

The set of all K_{Xt}-symmetry transformations will be denoted by $g_{K_{Xt}}$.

<u>T.2.2.</u>

1) $F^{p}_{K_{Xt}} g_{k}(X) (F^{p}_{K_{Xt}})^{-1} \subset g_{K_{Xt}}$ (2.19)

2) $H_{K_{Xt}} \in g_{K_{Xt}}$ then

$$H^{-1}_{K_{Xt}} h_{K_{Xt}}(C^{e}_{K_{Xt}},\theta,\alpha_{K_{Xt}})H^{-T}_{K_{Xt}} = \tag{2.20}$$

$$=h_{K_{Xt}}(H^{T}_{K_{Xt}} C^{e}_{K_{Xt}} H_{K_{Xt}},\theta,\alpha_{K_{Xt}})\ .$$

with $C^{e}_{K_{Xt}}$ given by (1.14), for any $(C^{e}_{K_{Xt}},\theta,\alpha_{K_{Xt}})$ in the domain of $h_{K_{Xt}}$.

<u>Proof</u>. 1) If $H\in g_k(X)$ then $H_{K_{Xt}}$ defined by (2.9) belongs to $g_{K_{Xt}}$.

2) With the notation

$$\tilde{F}^{e}_{K_{Xt}}=F^{e}_{K_{Xt}} H_{K_{Xt}} \tag{2.21}$$

introduced in (2.8) we obtain

$$T(X,t)=f_{K_{Xt}}(\tilde{F}^{e}_{K_{Xt}},\theta,\alpha_{K_{Xt}})=f_{K_{Xt}}(F^{e}_{K_{Xt}},\theta,\alpha_{K_{Xt}}) \tag{2.22}$$

The Piola-Kirchhoff symmetric stress tensors $\tilde{\Pi}_{K_{Xt}}$ and $\tilde{\tilde{\Pi}}_{K_{Xt}}$ associated to the same T(X,t) but corresponding to the tensors $F^{e}_{K_{Xt}}$ respectively are related by

$$\tilde{\tilde{\Pi}}_{K_{Xt}}=H^{-1}_{K_{Xt}} \tilde{\Pi}_{K_{Xt}} H^{-T}_{K_{Xt}} \tag{2.23}$$

since

$$\tilde{\tilde{\Pi}}_{K_{Xt}}=\det \tilde{F}^{e}_{K_{Xt}} (\tilde{F}^{e}_{K_{Xt}})^{-1} T(X,t) (\tilde{F}^{e}_{K_{Xt}})^{-T} \tag{2.24}$$

If we consider the thermoelastic constitutive equation (1.13) then from (2.22) and (2.23) the following condition on the function $h_{K_{Xt}}$ has to be satisfied:

$$\tilde{\tilde{\Pi}}_{K_{Xt}}=h_{K_{Xt}}(\tilde{C}^{e}_{K_{Xt}},\theta,\alpha_{K_{Xt}})=H^{-1}_{K_{Xt}} h_{K_{Xt}}(C^{e}_{K_{Xt}},\theta,\alpha_{K_{Xt}})H^{-T}_{K_{Xt}} \tag{2.25}$$

with

$$\tilde{C}^e_{K_{Xt}} = (\tilde{F}^e_{K_{Xt}})^T \tilde{F}^e_{K_{Xt}} = H^T_{K_{Xt}} C^e_{K_{Xt}} H_{K_{Xt}} \qquad (2.26)$$

So we state that $H_{K_{Xt}} \in g_{K_{Xt}}$ involves (2.20) ∎

We consider $\mathcal{B}$ an isotropic body at X and k an undistorted local configuration.

We state the following:

T.2.2. If $\mathcal{B}$ is a k-isotropic t.e.v.p. body at X then

1) The thermoelastic function $h_{K_{Xt}}$ is an isotropic mapping of $C^e \equiv C^e_{K_{Xt}} \in$ Psym, i.e.

$$h_{K_{Xt}}(HC^eH^T, \theta, \alpha_{K_{Xt}}) = Hh_{K_{Xt}}(C^e, \theta, \alpha_{K_{Xt}})H^T \qquad (2.27)$$

for all $H \in$ Orth.

2) The local current thermoviscoplastic deformation is conformal, i.e.

$$F^p_{K_{Xt}} = \mu_{K_{Xt}} R^p_{K_{Xt}} \qquad (2.28)$$

with $R^p_{K_{Xt}} \in$ Orth, $\mu_{K_{Xt}} \in R_{++}$.

3) The symmetry group with respect to K_{Xt} will contain the ortogonal group.

4) The symmetric parts of the evolution functions. $A_{K_{Xt}}$ and $B_{k_{Xt}}$ may be spherical, i.e. $\{A_{K_{Xt}}\}^s$ and $\{B_{K_{Xt}}\}^s$ satisfy this condition

$$\{A_{K_{Xt}}(Z, \alpha_{K_{Xt}})\}^s = a_{K_{Xt}}(Z, \alpha_{K_{Xt}})I$$

where $a_{K_{Xt}}(Z, \alpha_{K_{Xt}}) \in R$.

Proof. We begin with the second assertion. If $\mathcal{B}$ is a k-isotropic t.e.v.p. body at X then

$$H_{K_{Xt}}=F^p_{K_{Xt}}H(F^p_{K_{Xt}})^{-1}\in g_{K_{Xt}}$$

for all $H\in \mathrm{Orth}=g_k$ and (2.20) is satisfied. From (2.8) written for $F^e_{K_{Xt}}=I$ we obtain

$$f_{K_{Xt}}(H_{K_{Xt}},\theta_o,\alpha_{K_{Xt}})=0 \tag{2.29}$$

since the relaxation condition A.4. holds. The polar decomposition applied to the nonsingular tensor $H_{K_{Xt}}$ involves

$$H_{K_{Xt}}=R^H_{K_{Xt}}U^H_{K_{Xt}} \tag{2.30}$$

with $R^H_{K_{Xt}}\in$ Orth and $U^H_{K_{Xt}}\in$ Psym. The "objectivity property"(1.12) of the function $f_{K_{Xt}}$ gives

$$R^H_{K_{Xt}}f_{K_{Xt}}(U^H_{K_{Xt}},\theta_o,\alpha_{K_{Xt}})(R^H_{K_{Xt}})^T=0 \tag{2.31}$$

But $f_{K_{Xt}}(U^H_{K_{Xt}},\theta_o,\alpha_{K_{Xt}})=0$ if and only if $U^H_{K_{Xt}}=I$ since A.4 holds. Therefore

$$I=(U^H_{K_{Xt}})^2\equiv H^T_{K_{Xt}}H_{K_{Xt}}=(F^p_{K_{Xt}})^{-T}H^TC^p_{K_{Xt}}H(F^p_{K_{Xt}})^{-1} \tag{2.32}$$

for any $H\in$ Orth, or

$$C^p_{K_{Xt}}H=HC^p_{K_{Xt}} \tag{2.33}$$

where

$$C^p_{K_{Xt}}=(F^p_{K_{Xt}})^TF^p_{K_{Xt}} \tag{2.34}$$

Since $C^p_{K_{Xt}}\in$ Sym (2.33) holds for all $H\in$ Orth if and only if

$$C^p_{K_{Xt}} = \mu^2_{K_{Xt}} I \qquad (2.35)$$

So

$$U^p_{K_{Xt}} = \mu_{K_{Xt}} I \qquad (2.36)$$

with $\mu_{K_{Xt}} \in R_{++}$ and the polar decomposition of $F^p_{K_{Xt}}$ involves

$$F^p_{K_{Xt}} = \mu_{K_{Xt}} R^p_{K_{Xt}} \qquad (2.37)$$

as the right stretch tensor $U^p_{K_{Xt}}$ is spherical.

With (2.37) introduced in the evolution equation for $F^p_{K_{Xt}}$ we obtain 4).

3) We have proved that $H_{K_{Xt}} = R^H_{K_{Xt}} \in$ Orth because of $U^H_{K_{Xt}} = I$.

So we have obtained $g_{K_{Xt}} \supset$ Orth.

1) As $H_{K_{Xt}}$ may be any orthogonal tensor the relation (2.20) implies that $h_{K_{Xt}}$ is an isotropic mapping of its first argument.

Remark 1.5. If $\mathcal{B}$ is k-isotropic at X then the thermoelastic constitutive function is an isotropic one for any K_{Xt}-c.l.r.c. and the proper values of the current local right stretch termoplastic tensor $U^p_{K_{Xt}}$ are all equal. There are no favoured proper directions of the plastic deformation at X.

P.2.2. Let $\mathcal{B}$ be a k-isotropic body at X with k an undistorted reference configuration.

1) If all the thermoplastic deformations are incompressible then the body undergoes only thermoelastic deformation.

2) The evolution functions $A_{K_{Xt}}$ and $B_{K_{Xt}}$ are skewsymmetric tensor valued.

Proof. We work in the hypotesis of T.1.2 and we suppose that k is an undistorted, reference configuration. Therefore (2.28) holds. Since all the plastic deformations are incompressible i.e.tr $L^p_{K_{Xt}}=0$ then by (2.28) there results

$$L^p_{K_{Xt}} \equiv \dot{F}^p_{K_{Xt}} (F^p_{K_{Xt}})^{-1} = \frac{\dot{\mu}_{K_{Xt}}}{\mu_{K_{Xt}}} I + \dot{R}^p_{K_{Xt}} (R^p_{K_{Xt}})^T \qquad (2.28)$$

so

$$\dot{\mu}_{K_{Xt}}=0$$

As $F^p_{K_{Xt_o}}=I$ (or $F^p_{K_{Xt_o}}=R_o \in$ Orth) then $\mu_{K_{Xt}}=1$. The gradient of deformation is therefore expressed by

$$F_k=F^e_{K_{Xt}} R^p_{K_{Xt}} = U^e_{K_{Xt}} \tilde{R} \qquad (2.29)$$

with

$$\tilde{R}=R^e_{K_{Xt}} R^p_{K_{Xt}} \qquad (2.40)$$

So t.e.v.p. body at any time t undergoes only thermoelastic deformation since (2.7) holds.

From (2.38) and (1.17) we obtain that

$$A_{K_{Xt}}(Z,\alpha_{K_{Xt}}) \; , \; B_{K_{Xt}}(Z,\alpha_{K_{Xt}}) \in \text{Skew} \; ■$$

In conclusion the concept of k-isotropy is much more restrictive and will be not adequate in the description of the t.e.v.p. behaviour of the body■

So t.e.v.p. body depend on its reference configuration k. Further on this will be fixed.

3. CURRENT LOCAL RELAXED CONFIGURATIONS THERMOVISCO-PLASTICALLY EQUIVALENT

The definition of K_{Xt}-isotropy which will be adopted in [4] uses the configurations QK_{Xt} thermoelastoplastically equivalent to K_{Xt}. This means that QK_{Xt} as well as K_{Xt} may be used in order to characterize the thermomecanical response of the t.e.v.p. body at X.

Therefore it will be necessary to analyse the consequences that follow from the definition of thermoviscoplastically (t.e.v.p.-ally) equivalent configurations.

D.3.1. Two sets of configurations K_{Xt} and $\bar{K}_{Xt}$ ($t\in R$) are t.e.v.p.-ally equivalent at X if they correspond to the same process (χ, θ) and

$$\alpha_{\bar{K}_{Xt}} = \alpha_{K_{Xt}} \tag{3.1}$$

$$T(X,t)=f_{\bar{K}_{Xt}}(F^{e}_{\bar{K}_{Xt}}, \theta, \alpha_{\bar{K}_{Xt}})=f_{K_{Xt}}(F^{e}_{K_{Xt}}, \theta, \alpha_{K_{Xt}}) \tag{3.2}$$

with $\theta(X,t)\equiv\theta$ and

$$F^{e}_{K_{Xt}} = \nabla\chi(X,t)\circ K_{Xt}^{-1} \quad , \quad F^{e}_{\bar{K}_{Xt}} = \nabla\chi(X,t)\circ \bar{K}_{Xt}^{-1} \tag{3.3}$$

where

$$\dot{\bar{K}}_{Xt}(\bar{K}_{Xt})^{-1}=A_{\bar{K}_{Xt}}(\tilde{\pi}_{\bar{K}_{Xt}}, \theta, \alpha_{\bar{K}_{Xt}})+ \langle\lambda_{\bar{K}_{Xt}}\rangle\, B_{\bar{K}_{Xt}}(\tilde{\pi}_{\bar{K}_{Xt}}, \theta, \alpha_{\bar{K}_{Xt}}) \tag{3.4}$$

$$\dot{\alpha}_{\bar{K}_{Xt}} = l_{\bar{K}_{Xt}}(\tilde{\pi}_{\bar{K}_{Xt}}, \theta, \alpha_{\bar{K}_{Xt}})+ \langle\lambda_{\bar{K}_{Xt}}\rangle\, m_{\bar{K}_{Xt}}(\tilde{\pi}_{\bar{K}_{Xt}}, \theta, \alpha_{\bar{K}_{Xt}}) . \tag{3.5}$$

and at the initial moment

$$\bar{K}_{Xt_o}=\bar{K}_o(X), \quad \bar{\alpha}_{\bar{K}_{Xt_o}} = \alpha_o \tag{3.6}$$

The pair $(K_{Xt}, \alpha_{K_{Xt}})$ is also the solution of (1.3) and (1.8) with the initial data

$$K_{Xt_o}=K_o(X)\ ,\quad \alpha_{K_{Xt_o}}=\alpha_o \tag{3.7}$$

One necessary has

$$\bar{K}_o(K_o)^{-1}=P_o\in \text{Orth} \tag{3.8}$$

as a consequence of the below proposition.

P.3.1. If K_{Xt} and $\bar{K}_{Xt}$ are t.e.v.p-ally equivalent configurations at X then

1) $(\exists)$ $P(t)\in$ Orth such that

$$\bar{K}_{Xt}K_{Xt}^{-1}=P(t) \tag{3.9}$$

2) $$F^e_{\bar{K}_{Xt}}=F^e_{K_{Xt}}\ P^T(t) \tag{3.10}$$

3) $$\pi_{\bar{K}_{Xt}}=P(t)\ \pi_{K_{Xt}}\ P^T(t) \tag{3.11}$$

4) $$F^p_{\bar{K}_{Xt}}=P(t)F^p_{K_{Xt}} \tag{3.12}$$

Proof. The first property is a consequence of the relaxation conditions and of the "objectivity property" [see 1].

2) From (3.3) we have the following equalities

$$F^e_{\bar{K}_{Xt}} = \nabla\chi(X,t)\circ K_{Xt}^{-1}\cdot(K_{Xt}\cdot\bar{K}_{Xt}^{-1})=F^e_{K_{Xt}}(K_{Xt}\circ\bar{K}_{Xt}^{-1}) \tag{3.13}$$

So (3.10) follows from (3.13) with (3.9).

3) As a consequence of the definition (1.6) of the Piola-Kirchhoff stress tensor $\pi_{\bar{K}_{Xt}}$ relative to $\bar{K}_{Xt}$ and the formulae

(3.2) and (3.10) the following relation holds

$$\mathcal{T}_{\overline{K}_{Xt}} = dt\, F^e_{K_{Xt}} P(t) (F^e_{K_{Xt}})^{-1} T(X,t) (F^e_{K_{Xt}})^{-T} P^T(t) = P(t)\, \mathcal{T}_{K_{Xt}} P^T(t) \qquad (3.14)$$

By using the definition (1.5) of the current local thermoplastic deformation and taking into account the relation (3.9) the tensors $F^p_{\overline{K}_{Xt}}$ and $F^p_{K_{Xt}}$ will be related to each other by

$$F^p_{\overline{K}_{Xt}} = \overline{K}_{Xt} \circ (\nabla k(X))^{-1} = P(t) K_{Xt} (\nabla k(X))^{-1} = P(t) F^p_{K_{Xt}} \qquad (3.15)$$

<u>T.3.1.</u> If K_{Xt} and $\overline{K}_{Xt}$ are t.e.v.p.-ally equivalent then

1) $$f_{K_{Xt}}(F^e P(t), \theta, \alpha_{K_{Xt}}) = f_{K_{Xt}}(F^e, \theta, \alpha_{K_{Xt}}) \qquad (3.16)$$

2) The pair $Z \equiv (\mathcal{T}_{K_{Xt}}, \theta(X,t))$ lies on the current yield surface, i.e. $\mathcal{F}_{K_{Xt}}(Z, \alpha_{K_{Xt}}) = 0$, if and only if $\overline{Z} \equiv (\mathcal{T}_{\overline{K}_{Xt}}, \theta)$ lies on the current yield surface $S_{\overline{K}_{Xt}}(\alpha_{\overline{K}_{Xt}})$ i.e. $\mathcal{F}_{\overline{K}_{Xt}}(\overline{Z}, \alpha_{\overline{K}_{Xt}}) = 0$.

3) If $\{\mathcal{B}_{K_{Xt}}(Z, \alpha_{K_{Xt}})\}^S \neq 0$ for all points $Z \in S_{K_{Xt}}(\alpha_{K_{Xt}})$ then

$$\operatorname{sign} \lambda_{K_{Xt}} = \operatorname{sign} \lambda_{\overline{K}_{Xt}} \qquad (3.17)$$

<u>Proof</u>. We assume that the process (χ, θ) is such that at the moment t, $Z \in S_{K_{Xt}}(\alpha_{K_{Xt}})$ and the process is a loading one and on the other hand $\overline{Z} = (\mathcal{T}_{\overline{K}_{Xt}}, \theta) \notin S_{\overline{K}_{Xt}}(\alpha_{\overline{K}_{Xt}})$. From the assumptions [1] concerning a t.e.v.p. body there result $\mathcal{F}_{K_{Xt}}(Z, \alpha_{K_{Xt}}) = 0$, $\mathcal{F}_{\overline{K}Xt}(\overline{Z}, \alpha_{\overline{K}_{Xt}}) < 0$ and the following evolution functions are zero

$$\mathcal{B}_{\overline{K}_{Xt}}(\overline{Z}, \alpha_{\overline{K}_{Xt}}) = 0, \quad m_{\overline{K}_{Xt}}(\overline{Z}, \alpha_{\overline{K}_{Xt}}) = 0 \qquad (3.18)$$

So we have

$$\dot{K}_{Xt}K_{Xt}^{-1}=A_{K_{Xt}}(Z,\alpha_{K_{Xt}})+\lambda_{K_{Xt}}B_{K_{Xt}}(Z,\alpha_{K_{Xt}}) \qquad (3.19)$$

$$\dot{\bar{K}}_{Xt}\bar{K}_{Xt}^{-1}=A_{\bar{K}_{Xt}}(\bar{Z},\alpha_{\bar{K}_{Xt}}) \qquad (3.20)$$

and

$$\dot{\alpha}_{K_{Xt}}=l_{K_{Xt}}(Z,\alpha_{K_{Xt}})+\lambda_{K_{Xt}}m_{K_{Xt}}(Z,\alpha_{K_{Xt}}),\ \dot{\alpha}_{\bar{K}_{Xt}}=l_{\bar{K}_{Xt}}(\bar{Z},\alpha_{\bar{K}_{Xt}}) \qquad (3.21)$$

By means of (3.19) written for the plastic complementary factor (1.21) the evolution system becomes

$$\dot{F}^{p}_{K_{Xt}}(F^{p}_{K_{Xt}})^{-1}=\tilde{A}_{k}(M)+(\tilde{a}_{k}\cdot L+\tilde{b}_{k}\dot{\theta}+\tilde{c}_{k})B_{k}(M) \qquad (3.22)$$

Here $\tilde{A}_k$, $\tilde{B}_k$, $\tilde{a}_k$, $\tilde{b}_k$ and respectively $\tilde{c}_k$ are composite functions, i.e. for instance

$$\tilde{A}_{k}(M)\equiv A_{k}(h_{k}(C^{e}_{K_{Xt}},\theta,\alpha_{K_{Xt}},F^{p}_{K_{Xt}}),\theta,\alpha_{K_{Xt}},F^{p}_{K_{Xt}}) \qquad (3.23)$$

Let us consider now a linear continuation of (χ,θ) of direction (L_1,θ_1) at time t on the interval $(t,t+\varepsilon)$ with $\varepsilon>0$ that corresponds to a loading process; i.e. $(\tilde{\chi},\tilde{\theta})$.

Let $(\tilde{K}_{X\tau},\alpha_{\tilde{K}_{X\tau}})$ with $\tau\in(t,t+\varepsilon_1)$ and $\varepsilon_1<\varepsilon$ be a solution of the system (3.19) and $(3.21)_1$ for $(\tilde{\chi},\tilde{\theta})$ which satisfies, at the moment $\tau=t$, the following conditions

$$\tilde{K}_{Xt}=K_{Xt}\ ,\quad \alpha_{\tilde{K}_{Xt}}=\alpha_{K_{Xt}}\ .$$

This solution loclly exist since all constitutive and evolution functions are continuous with respect to $(F^{p}_{K_{Xt}},\alpha_{K_{Xt}})$ [1].

Therefore $\dot{\tilde{K}}_{Xt+0}$ may be arbitrary as (L_1,θ_1) is an arbitrary given direction of the linear continuation of (χ,θ) at time t.

Similarly we consider one locally solution $(\tilde{\bar{K}}_{X\tau}, \tilde{\alpha}_{\tilde{\bar{K}}_{X\tau}})$ of the system (3.20), $(3.21)_2$ for $(\tilde{\chi}, \tilde{\theta})$, with the conditions $\tilde{\bar{K}}_{Xt}=\bar{K}_{Xt}$, $\tilde{\alpha}_{\tilde{\bar{K}}_{Xt}}=\alpha_{\bar{K}_{Xt}}$ at time $\tau=t$.

The right hand derivative at time t $\dot{\tilde{\bar{K}}}_{Xt+0}$ does therefore not depend on (L_1, θ_1): $\dot{\tilde{\bar{K}}}_{Xt+0}=\dot{\bar{K}}_{Xt}$.

On the other hand there exists $P(s)\in$ Orth such that

$$\tilde{\bar{K}}_{Xs}(\tilde{K}_{Xs})^{-1}=P(s) \qquad (\forall)\ s\in(t,t+\varepsilon_2) \tag{3.24}$$

If we diferentiate (3.24) with respect to s and we change s into t we get

$$\dot{\bar{K}}_{Xt}(\bar{K}_{Xt})^{-1}=\dot{P}(t+0)P^T(t)+P(t)\dot{\tilde{K}}_{Xt+0}K_{Xt}^{-1}P^T(t) \tag{3.25}$$

Here

$$\dot{P}(t+0)P^T(t)\equiv W(t)\in \text{SKew} \tag{3.26}$$

By means of the equations (3.19) and (3.20) the relation (3.25) becomes

$$\begin{aligned} &A_{\bar{K}_{Xt}}(\bar{Z},\alpha_{\bar{K}_{Xt}})-P(t)A_{K_{Xt}}(Z,\alpha_{K_{Xt}})P^T(t)= \\ &=W(t)+P(t)\tilde{\lambda}_{K_{Xt}}B_{K_{Xt}}(Z,\alpha_{K_{Xt}})P^T(t) \end{aligned} \tag{3.27}$$

By considering the symmetric and the skew-symmetric parts in (3.27) we obtain

$$\{A_{\bar{K}_{Xt}}(\bar{Z},\alpha_{\bar{K}_{Xt}})-P(t)A_{K_{Xt}}(Z,\alpha_{K_{Xt}})P^T(t)\}^S=0 \tag{3.28}$$

and

$$\tilde{\lambda}_{K_{Xt}}P(t)\{B_{K_{Xt}}(Z,\alpha_{K_{Xt}})\}^S P^T(t)=0 \tag{3.29}$$

As $\tilde{\lambda}_{K_{Xt}}$ may be arbitrary we get $\{B_{K_{Xt}}(Z,\alpha_{K_{Xt}})\}^S=0$ which contradicts the hypotesis. Therefore $\mathcal{F}_{\bar{K}_{Xt}}(\bar{Z},\alpha_{K_{\bar{X}t}})=0$ as

$\mathcal{F}_{K_{Xt}}(Z, \alpha_{K_{Xt}})=0.$

In order to prove assertion 3) we suppose that $Z \equiv (\pi_{K_{Xt}}, \theta(X,t)) \in S_{K_{Xt}}(\alpha_{K_{Xt}})$; $\lambda_{K_{Xt}} > 0$ and $\lambda_{\bar{K}_{Xt}} \leq 0$. Then the formulae (3.19), (3.20), (3.21) hold. By a similar argument to that used in 2) we obtain (3.29). This leads to $\lambda_{K_{Xt}}=0$ which contradicts the assumption $\lambda_{K_{Xt}}=0$. So if $\lambda_{K_{Xt}} > 0$ then $\lambda_{\bar{K}_{Xt}} > 0$. And we also have $\lambda_{\bar{K}'_{X_t}} \leq 0$ if and only if $\lambda_{K_{Xt}} \leq 0$.

Further on we show that for a given process (χ, θ) there is not possible to have $\lambda_{\bar{K}_{Xt}}=0$ and $\lambda_{K_{Xt}} < 0$ simultaneously. By using P.4.4 from [1] there results that for any $\varepsilon>0$ there exist a $\tau_\varepsilon \in (t, t+\varepsilon]$ such that

$$\mathcal{F}_{K_{X\tau_\varepsilon}}(\pi_{K_{X\tau_\varepsilon}}, \theta(X, \tau_\varepsilon), \alpha_{K_{X\tau_\varepsilon}})<0 \text{ and } \mathcal{F}_{\bar{K}_{X\tau_\varepsilon}}(\pi_{\bar{K}_{X\tau_\varepsilon}}, \theta(X, \tau_\varepsilon), \alpha_{\bar{K}_{X\tau_\varepsilon}})=0$$

We have already proved that leads to a contradiction.

<u>P.3.2.</u> If $z \in S_{K_{Xt}}(\alpha_{K_{Xt}})$ then

$$\lambda_{\bar{K}_{Xt}} = \lambda_{K_{Xt}} + 2\{\pi\, \partial_{\pi} \mathcal{F}_{K_{Xt}}(Z, \quad _{K_{Xt}})\}^a . \omega(t)=0 \tag{3.30}$$

with

$$\omega(t)=\dot{P}^T(t)P(t) \tag{3.31}$$

<u>Proof</u>. By the definition of the plastic factor $\lambda_{K_{Xt}}$ it follows that

$$\lambda_{\bar{K}_{Xt}} = \partial_{\bar{\pi}} \mathcal{F}(\bar{Z}, \alpha_{\bar{K}_{Xt}}) \circ \dot{\pi}_{\bar{K}_{Xt}} + \partial_\theta \mathcal{F}(\bar{Z}, \alpha_{\bar{K}_{Xt}})\, \dot{\theta} \tag{3.32}$$

with the notation

$$\partial_{\bar{\pi}} \mathcal{F}(\bar{Z}, \alpha_{\bar{K}_{Xt}}) \equiv \partial_{\bar{\pi}_{\bar{K}_{Xt}}} \mathcal{F}_{\bar{K}_{Xt}}(\bar{\pi}_{\bar{K}_{Xt}}, \theta, \alpha_{\bar{K}_{Xt}}) ,$$

$$\partial_{\theta} \mathcal{F}(\bar{Z}, \alpha_{\bar{K}_{Xt}}) \equiv \partial_{\theta} \mathcal{F}_{\bar{K}_{Xt}}(\bar{\pi}_{\bar{K}_{Xt}}, \theta, \alpha_{\bar{K}_{Xt}}) .$$

By differentiating (2.11) with respect to τ, $\tau > t$, we obtain

$$\dot{\pi}_{\bar{K}_{Xt+0}} = P(t) \left[\dot{\pi}_{K_{Xt}} + \pi_{K_{Xt}} \omega(t) - \omega(t) \pi_{K_{Xt}} \right] P^T(t) \tag{3.33}$$

On the other hand from T.2.1 we have

$$\partial_{\theta} \mathcal{F}_{\bar{K}_{Xt}}(\pi_{\bar{K}_{Xt}}, \theta, \alpha_{\bar{K}_{Xt}}) = \partial_{\theta} \mathcal{F}_{K_{Xt}}(\pi_{K_{Xt}}, \theta, \alpha_{K_{Xt}}) \tag{3.34}$$

and

$$P^T(t) \partial_{\bar{\pi}} \mathcal{F}(\bar{Z}, \alpha_{\bar{K}_{Xt}}) P(t) = \partial_{\pi} \mathcal{F}(Z, \alpha_{K_{Xt}}) \tag{3.35}$$

If we come back to (3.32) with (3.33) - (3.35) one obtains the following relation between $\lambda_{\bar{K}_{Xt}}$ and $\lambda_{K_{Xt}}$:

$$\lambda_{\bar{K}_{Xt}} = \lambda_{K_{Xt}} + 2 \{ \pi \partial_{\pi} \mathcal{F}(Z, \alpha_{K_{Xt}}) \}^a \cdot \omega(t) .$$

<u>Remark 3.1</u>. If $\pi \partial_{\pi} \mathcal{F}(Z, \alpha_{K_{Xt}}) \in \mathrm{Sym}$ then $\lambda_{\bar{K}_{Xt}} = \lambda_{K_{Xt}}$ for any $Z \in S_{K_{Xt}}(\alpha_{K_{Xt}})$. For instance if $\mathcal{F}_{K_{Xt}}(Z, \alpha_{K_{Xt}}) \equiv$ $\mathcal{F}_{K_{Xt}}(j(\pi_{K_{Xt}}), \theta(X,t), \alpha_{K_{Xt}})$ with $j(\pi_{K_{Xt}})$ the invariants of $\pi_{K_{Xt}}$ then the above property holds.

<u>Remark 3.2</u>. If $\lambda_{K_{Xt}} = 0$ then $\{ \pi \partial_{\pi} \mathcal{F} \}^a \cdot \omega(t) = 0$ since $\lambda_{\bar{K}_{Xt}}$ also vanishes.

<u>T.3.2</u>. Let K_{Xt} and $\bar{K}_{Xt}$ be two t.e.v.p.-ally equivalent configurations at X and at time t. Then

1) The symmetric parts of the functions $A_{K_{Xt}}$ and $B_{K_{Xt}}$ satisfy the following relations

$$\{A_{\overline{K}_{Xt}}(\overline{Z},\alpha)\}^{s}=P(t)\{A_{K_{Xt}}(Z,\alpha)\}^{s}P^{T}(t) \qquad (3.36)$$

when we pass from one c.l.r.c. to an equivalent one, $\overline{K}_{Xt}$.

2) The rate of the transformation $P(T)=\overline{K}_{Xt}K_{Xt}^{-1}$

$$-\dot{P}^{T}(t+0)P(t)=\{P(t)A_{K_{Xt}}(Z,\alpha)P^{T}(t)-A_{\overline{K}_{Xt}}(\overline{Z},\alpha)\}^{a}+$$
$$+\langle\lambda_{K_{Xt}}\rangle\{P(t)B_{K_{Xt}}(Z,\alpha)P^{T}(t)-B_{\overline{K}_{Xt}}(\overline{Z},\alpha)\}^{a} \qquad (3.37)$$

with the functions $A_{K_{Xt}}$ and $B_{K_{Xt}}$ satisfying

$$\{\widetilde{\pi}\,\partial_{\widetilde{\pi}}\mathcal{F}(Z,\alpha)\}^{a}\cdot\{A_{K_{Xt}}(z,\alpha)-P^{T}(t)A_{\overline{K}_{Xt}}(\overline{Z},\alpha)P(t)\}^{a}=0 \qquad (3.38)$$

on the yield surface.

3) For any $z\in S_{K_{Xt}}(\alpha_{K_{Xt}})$ the plastic factor corresponding to K_{Xt} and $\overline{K}_{Xt}$ are equal, i.e.

$$\lambda_{\overline{K}_{Xt}}=\lambda_{K_{Xt}} \qquad (3.39)$$

4) The evolution functions corresponding to the two considered t.v.e.p.-ally equivalent configurations are also equal

$$l_{K_{Xt}}(Z,\alpha)=l_{\overline{K}_{Xt}}(\overline{Z},\alpha),\ m_{K_{Xt}}(Z,\alpha)=M_{\overline{K}_{Xt}}(Z,\alpha) \qquad (3.40)$$

here

$$Z=(\widetilde{\pi}_{K_{Xt}},\theta(X,t))\ \text{and}\ \overline{Z}=(P(t)\widetilde{\pi}_{K_{Xt}}P^{T}(t),\theta(X,t)).$$

Proof. Let $(\tilde{\lambda}, \tilde{\theta})$ be a linear continuation of (λ, θ) at time t of direction (L_1, θ_1). We denote by $(K_{Xs}, \bar{K}_{Xs})$ for some $t < s < t+\varepsilon$ the t.v.p. equivalent configurations which satisfy at the moment t $\tilde{K}_{Xt} = K_{Xt}$, $\tilde{\bar{K}}_{Xt} = \bar{K}_{Xt}$. Therefore from (2.4), (1.19) and (2.9) the following equality results

$$A_{\bar{K}_{Xt}}(\bar{Z}, \bar{\alpha}) + \langle \tilde{\lambda}_{\bar{K}_{Xt}} \rangle B_{\bar{K}_{Xt}}(\bar{Z}, \bar{\alpha}) = \dot{\tilde{P}}(t) P^T(t) + P(t) \Big[A_{K_{Xt}}(Z, \alpha) + \langle \tilde{\lambda}_{K_{Xt}} \rangle B_{K_{Xt}}(Z, \alpha) \Big] P^T(t) \tag{3.41}$$

But $\bar{\alpha} = \tilde{\alpha}_{\tilde{\bar{K}}_{Xt}}$ is equal to $\alpha = \tilde{\alpha}_{\tilde{K}_{Xt}}$ by definition (3.1) of t.e.v.p-ally equivalent configurations.

By writing the symmetric and the skewsymmetric parts of (3.41) we obtain

$$\left\{ P(t) A_{K_{Xt}}(Z, \alpha) P^T(t) - A_{\bar{K}_{Xt}}(\bar{Z}, \bar{\alpha}) \right\}^s = 0 \tag{3.42}$$

and

$$\langle \tilde{\lambda}_{K_{Xt}} \rangle \left\{ P(t) B_{K_{Xt}}(Z, \alpha) P^T(t) \right\}^s = \langle \tilde{\lambda}_{\bar{K}_{Xt}} \rangle \left\{ B_{\bar{K}_{Xt}}(\bar{Z}, \bar{\alpha}) \right\}^s \tag{3.43}$$

since the factors $\tilde{\lambda}_{K_{Xt}}$ and $\tilde{\lambda}_{\bar{K}_{Xt}}$ depend linearly on (L_1, θ_1). The skewsymmetric part of (3.41) is nothing else than (3.37). From (3.37) and (3.30) we obtain

$$\tilde{\lambda}_{\bar{K}_{Xt}} - \lambda_{K_{Xt}} = 2 \left\{ \mathcal{T} \partial_{\mathcal{T}} \mathcal{F} \right\}^a . \left\{ A_{K_{Xt}} - P^T A_{\bar{K}_{Xt}} P \right\}^a \tag{3.44}$$

$$2 \langle \tilde{\lambda}_{K_{Xt}} \rangle \left\{ \mathcal{T} \partial_{\mathcal{T}} \mathcal{F} \right\}^a . \left\{ B_{K_{Xt}} \right\}^a - 2 \langle \tilde{\lambda}_{\bar{K}_{Xt}} \rangle \left\{ \mathcal{T} \partial_{\mathcal{T}} \mathcal{F} \right\}^a . \left\{ P^T B_{\bar{K}_{Xt}} P \right\}^a$$

We recall that $\tilde{\lambda}_{\bar{K}_{Xt}}$ has the same sign as $\tilde{\lambda}_{K_{Xt}}$ (see 3.17)

The particular case $\tilde{\lambda}_{\bar{K}_{Xt}} = \tilde{\lambda}_{K_{Xt}} = 0$ leads to (3.38). Since (3.44) all the functions without $\tilde{\lambda}_{K_{Xt}}$ and $\tilde{\lambda}_{\tilde{K}_{Xt}}$ are independent of (L_1, θ_1) the relation (3.44) with (3.38) imply

$$\tilde{\lambda}_{\bar{K}_{Xt}} + 2 < \tilde{\lambda}_{\bar{K}_{Xt}} > \{\pi \partial_{\pi} \mathcal{F}\}^a . \{P^T B_{\bar{K}_{Xt}} P\}^a = \tag{3.45}$$

$$= \tilde{\lambda}_{K_{Xt}} + 2 < \tilde{\lambda}_{K_{Xt}} > \{\pi \partial_{\pi} \mathcal{F}\}^a . \{B_{K_{Xt}}\}^a$$

If $\tilde{\lambda}_{K_{Xt}}$ and $\tilde{\lambda}_{\bar{K}_{Xt}}$ are both negative then from there results $\tilde{\lambda}_{\bar{K}_{Xt}} = \tilde{\lambda}_{K_{Xt}}$. If both $\tilde{\lambda}_{K_{Xt}}$ and $\tilde{\lambda}_{\bar{K}_{Xt}}$ are positive then

$$\tilde{\lambda}_{\bar{K}_{Xt}} (1+2 \{\pi \partial_{\pi} \mathcal{F}\}^a . P^T \{B_{\bar{K}_{Xt}}\}^a P) = \tilde{\lambda}_{K_{Xt}} (1+2 \{\pi \partial_{\pi} \mathcal{F}\}^a . \{B_{K_{Xt}}\}^a) \tag{3.46}$$

By using relation (1.21) for the positive value of plastic, (3.46) may be written in the form

$$(a_{\bar{K}_{Xt}} . L_1 + b_{\bar{K}_{Xt}} \theta_1 + c_{\bar{K}_{Xt}}) (1+2 \{\pi \partial_{\pi} \mathcal{F}\}^a . P^T \{B_{\bar{K}_{Xt}}\}^a P) = \tag{3.47}$$

$$= (a_{K_{Xt}} . L_1 + b_{K_{Xt}} \theta_1 + c_{K_{Xt}}) (1+2 \{\pi \partial_{\pi} \mathcal{F}\}^a . \{B_{K_{Xt}}\}^a)$$

there (L_1, θ_1) is arbitrary but defining a loading process. If $1+2 \{\pi \partial_{\pi} \mathcal{F}\}^a . \{B_{K_{Xt}}\}^a$ is not zero then $\alpha \tilde{\lambda}_{\bar{K}_{Xt}} = \tilde{\lambda}_{K_{Xt}}$, when $\tilde{\lambda}_{K_{Xt}} > 0$, with

$$\alpha = (1+2 \{\pi \partial_{\pi} \mathcal{F}\}^a . P^T \{B_{\bar{K}_{Xt}}\}^a P) / 1+2 \{\pi \partial_{\pi} \mathcal{F}\}^a . \{B_{K_{Xt}}\}^a$$

independent of (L_1, θ_1). We have $\tilde{\lambda}_{\bar{K}_{Xt}} = \tilde{\lambda}_{K_{Xt}}$ if both $\tilde{\lambda}_{\bar{K}_{Xt}}$ and $\lambda_{K_{Xt}}$ are negative. Therefore $\alpha = 1$. Thus we obtain from (3.48) that the function $B_{K_{Xt}}$ satisfy (3.38) too and the equality of the plastic factors $\tilde{\lambda}_{K_{Xt}}$ and $\tilde{\lambda}_{\bar{K}_{Xt}}$.

4) Since the corresponding w.h.v. are equal by (3.1) the evolution functions that enter into equations (1.8) and (3.5) have the same values, i.e. (3.40) holds.

Remark 3.3. The t.e.v.p. equivalence of K_{Xt} and $\bar{K}_{Xt}$ impose the restrictions (3.36) on the symmetric parts of the functions $A_{K_{Xt}}$ and $B_{K_{Xt}}$. Their skewsymmetric parts get into (3.37) and are subjected to the restrictions (3.38).

Remark 3.4. If $\mathcal{J}\partial_{\mathcal{J}}\mathcal{F} \in$ Sym then (3.38) holds and the actual rate of the orthogonal transformation $P(t)=\bar{K}_{Xt}\ K_{Xt}^{-1}$ is given by (3.37) without any conditions imposed on the skewsymmetric parts of the functions $A_{K_{Xt}}$ and $B_{K_{Xt}}$.

4. THERMOVISCOPLASTIC K_{Xt}-ISOTROPIC BODY WITH INSTANTANEOUS PLASTICITY

D.4.1. The t.e.v.p. body $\mathcal{B}$ is K_{Xt}-isotropic at X and at time t if QK_{Xt} and K_{Xt} are t.e.v.p.-ally equivalent for all $Q \in$ Orth.

T.4.1. If $\mathcal{B}$ is a t.e.v.p. isotropic body at X and at time t then

1) The evolution functions $A_{K_{Xt}}$ and $B_{K_{Xt}}$ are such that their corresponding functions A_k and B_k respectively

$$A_k(\mathcal{J}, \theta, \alpha, F^p) = A_{K_{Xt}}(\mathcal{J}, \theta, \alpha) \tag{4.1}$$

are isotropic in their tensorial arguments, i.e.

$$A_k(Q\mathcal{J}Q^T,\theta,\alpha,QF^p)=QA_k(\mathcal{J},\theta,\alpha,F^p)A^T \tag{4.2}$$

for all $Q\in$ Orth.

2) The thermoelastic constitutive function $h_{K_{Xt}}$ is such that the function h_k is an isotropic mapping in its arguments, i.e.

$$h_k(QC^eQ^T,\theta,\alpha,QF^p)=Qh_k(C^e,\theta,\alpha,F^p)Q^T \tag{4.3}$$

for all $Q\in$ Orth with

$$C^e=(F^e)^T(F^e)^T . \tag{4.4}$$

3) The functions $\mathcal{F}_{K_{Xt}}$, $m_{K_{Xt}}$ and $l_{K_{Xt}}$ are such that their associated functions ℓ_k, m_k and l_k are isotropic mappings, i.e.

$$\mathcal{F}_k(Q\mathcal{J}Q^T,\theta,\alpha,QF^p)=\mathcal{F}_k(\mathcal{J},\theta,\alpha,F^p) \tag{4.5}$$

<u>Proof</u>. This theorem follows from the definitions D.4.1., D.3.1., by using the assertions of the theorems T.3.1. and T.3.2. One get

$$\alpha_{QK_{Xt}}=\alpha_{K_{Xt}} \tag{4.6}$$

The relation (3.36) written for the two configurations $\bar{K}_{Xt}=QK_{Xt}$ and K_{Xt} give us

$$Q\{A_{K_{Xt}}(\mathcal{J},\theta,\alpha)\}^sQ^T=\{A_{\bar{K}_{Xt}}(Q\mathcal{J}Q^T,\theta,\alpha)\}^s \tag{4.7}$$

and one similar relation for $B_{K_{Xt}}$ since the theorem T.3.2. holds. By using (3.37) in which the orthogonal transformation $\bar{K}_{Xt}\bar{K}_{Xt}^{-1}=Q$ must be constant we have

$$\{QA_{K_{Xt}}(\mathcal{T},\theta,\alpha)Q^T-A_{\overline{K}_{Xt}}(Q\mathcal{T}Q^T,\theta,\alpha)\}^a+ \tag{4.8}$$
$$+\langle\lambda_{K_{Xt}}\rangle\{QB_{K_{Xt}}(\mathcal{T},\theta,\alpha)A^T-B_{\overline{K}_{Xt}}(Q\mathcal{T}Q^T,\theta,\alpha)\}^a=0$$

Since K_{Xt} depend on the history up to time t of (χ,θ) and it is independent of $(L(x,t+0),\ \dot{\theta}(X,t+0))$ from (4.8) we obtain the connection of the skewsymmetric parts of $A_{K_{Xt}}$ (and $B_{K_{Xt}}$ respectively) with $A_{\overline{K}_{Xt}}$ (and $B_{\overline{K}_{Xt}}$ respectively). Thus the evolution functions $A_{K_{Xt}}$ and $B_{K_{Xt}}$ are changing by this rule

$$QA_{K_{Xt}}(\mathcal{T},\theta,\alpha)Q^T=A_{\overline{K}_{Xt}}(Q\mathcal{T}Q^T,\theta,\alpha) \tag{4.9}$$

when the c.l.r.c. K_{Xt} is replaced by $\overline{K}_{Xt}=QK_{Xt}$. On the other hand if $Z=(\mathcal{T},\theta)\in S_{K_{Xt}}(\alpha_{K_{Xt}})$ then

$$\mathcal{F}_{K_{Xt}}(\mathcal{T},\theta,\alpha)=0 \Leftrightarrow \mathcal{F}_{\overline{K}_{Xt}}(Q\mathcal{T}Q^T,\theta,\alpha)=0 \tag{4.10}$$

since T.3.1 holds.

Already the reference configuration has fixed and by using the representation (1.16) and the formulae (3.1), (3.11) we have

$$A_{K_{Xt}}(\mathcal{T}_{K_{Xt}},\theta,\alpha_{K_{Xt}})=A_k(\mathcal{T}_{K_{Xt}},\theta,\alpha_{K_{Xt}},F^p_{K_{Xt}}) \tag{4.11}$$

and

$$A_{\overline{K}_{Xt}}(\mathcal{T}_{\overline{K}_{Xt}},\theta,\alpha_{\overline{K}_{Xt}})=A_k(Q\mathcal{T}_{K_{Xt}}Q^T,\theta,\alpha_{K_{Xt}},QF^p_{K_{Xt}}) \tag{4.12}$$

From (4.9), (4.10) and (4.11) it follows the isotropy of the functions A_k and B_k with respect to their arguments $(\mathcal{T},F^p)$, i.e.

$$QA_k(\mathcal{T},\theta,\alpha,F^p)Q^T=A_k(Q\mathcal{T}Q^T,\theta,\alpha,QF^p)$$

for any pair (θ,α) and for all orthogonal mappings Q.

3) By using (4.10) with (1.16) one gets the isotropy of the scalar function $\mathcal{F}_k$ with respect to $(\mathcal{T}, F^p)$, i.e. (4.5). Starting from the equality (3.40) the isotropy of the functions l_k and m_k also result.

2) Finally we discuss the property of the thermoelastic function $h_{K_{Xt}}$ when the body is K_{Xt}-isotropic. There are $\bar{K}_{Xt}=QK_{Xt}$ and K_{Xt} t.e.v.p.-ally equivalent configurations corresponding to the same process (χ, θ). Therefore the appropiate thermoelastic constitutive equations are

$$\mathcal{T}_{\bar{K}_{Xt}} = h_{\bar{K}_{Xt}}(C^e_{\bar{K}_{Xt}}, \theta(X,t), \alpha_{\bar{K}_{Xt}}) \tag{4.13}$$

and

$$\tilde{\mathcal{T}}_{K_{Xt}} = h_{K_{Xt}}(C^e_{K_{Xt}}, \theta(X,t), \alpha_{K_{Xt}}) \tag{4.14}$$

with the Cauchy-Green right thermoelastic corresponding deformation tensors

$$C^e_{\bar{K}_{Xt}} = (F^e_{\bar{K}_{Xt}})^T F^e_{\bar{K}_{Xt}} \quad \text{and} \quad C^e_{K_{Xt}} = (F^e_{K_{Xt}})^T F^e_{K_{Xt}} \tag{4.15}$$

From (3.11) with (4.13)-(4.15), (3.10) we obtain

$$h_{\bar{K}_{Xt}}(QC^e_{K_{Xt}}Q^T, \theta, \alpha_{\bar{K}_{Xt}}) = Qh_{K_{Xt}}(C^e_{K_{Xt}}, \theta, \alpha_{K_{Xt}})Q^T \tag{4.16}$$

If we rewrite (4.16) in the form (1.16) it results

$$h_k(QC^eQ^T, \theta, \alpha_{\bar{K}_{Xt}}, F^p_{\bar{K}_{Xt}}) = Qh_k(C^e, \theta, \alpha_{K_{Xt}}, F^p_{K_{Xt}})Q^T \tag{4.17}$$

with (3.12) the last is nothing else than (4.3).

<u>P.4.1.</u> Let $\mathcal{B}$ be a thermoviscoplastic K_{Xt}-isotropic body. Then

1) There is a c.l.c.r., $\tilde{K}_{Xt}$ such that

$$F^e_{K_{Xt}} = V^e_{K_{Xt}} \in \text{Psym} \tag{4.16}$$

$$F^p_{\tilde{K}_{Xt}} = \tilde{R} U^p_{K_{Xt}} \tag{4.17}$$

with

$$\tilde{R} = R^e_{K_{Xt}} \, R^p_{K_{Xt}} \tag{4.18}$$

and

$$\pi_{\tilde{K}_{Xt}} = R^e_{K_{Xt}} \, \pi_{K_{Xt}} \, (R^e_{K_{Xt}})^T \tag{4.19}$$

2) The functions A_k and B_k respectively satisfy this relation

$$A_k(Q\pi^{R^p}Q^T, \theta, \alpha, QU^p) = QA_k(\pi^{R^p}, \theta, \alpha, U^p)Q^T \tag{4.20}$$

for all $Q \in \text{Orth}$, where π^{R^p} denotes the tensor

$$\pi^{R^p} \equiv (R^p_{K_{Xt}})^T \pi_{K_{Xt}} (R^p_{K_{Xt}}) = \tilde{R}^T \pi_{\tilde{K}_{Xt}} \tilde{R} \tag{4.21}$$

3) Thermoelastic constitutive equation relativ to $\tilde{K}_{Xt}$ becomes

$$\pi_{\tilde{K}_{Xt}} = h_k(B^e, \theta, \alpha, RU^p) \tag{4.22}$$

with

$$B^e = (V^e_{K_{Xt}})^2 \tag{4.23}$$

Proof. We can pass to the configuration K_{Xt} from K_{Xt} by the local deformation $R^e_{K_{Xt}} \in \text{Orth}$

$$\tilde{K}_{Xt} = R^e_{K_{Xt}} K_{Xt} \tag{4.24}$$

as follows from the definition D.4.1. Thus from (3.11) and (4.24) we obtain (4.19). Therefore the current thermoplastic deformation corresponding to $\tilde{K}_{Xt}$ will be given by

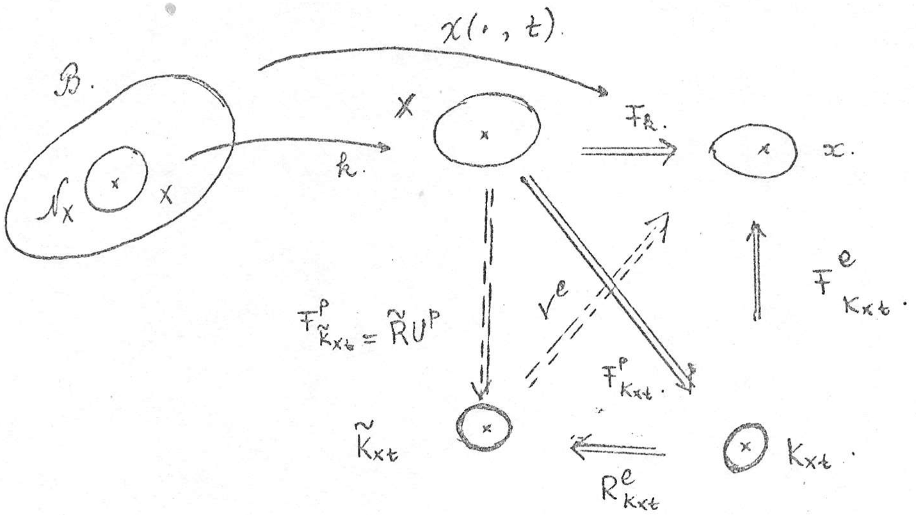
$$F^p_{\tilde{K}_{Xt}} = \tilde{K}_{Xt} (\nabla k(X))^{-1} = R^e_{K_{Xt}} F^p_{K_{Xt}} = \tilde{R} U^p_{K_{Xt}} \tag{4.25}$$

with $\tilde{R}$ introduced by (4.18), and $R^p_{K_{Xt}} \in$ Orth, $U^p_{K_{Xt}} \in$ Psym the tensors from the polar decomposition of $F^p_{K_{Xt}}$.

The current thermoelastic deformation corresponding to K_{Xt} will be equal to

$$F^e_{\tilde{K}_{Xt}} = \nabla \chi(X,t) \circ \tilde{K}^{-1}_{Xt} = F^e_{K_{Xt}} (R^e_{K_{Xt}})^T = V^e_{K_{Xt}} \tag{4.26}$$

where $V^e_{K_{Xt}}$ is the left thermoelastic stretch tensor. In the following picture there are ploted the above mentioned configurations and their corresponding deformation tensors:



2) If we put in (4.2) $Q(R^p_{K_{Xt}})^T$ for Q we obtain

$$A_k(Q \mathcal{J}^{R^p} Q^T, \theta, \alpha, QU^p) = Q(R^p)^T A_k(\mathcal{J}, \theta, \alpha, F^p) R^p Q^T \tag{4.27}$$

since

$$Q(R^p_{K_{Xt}})^T F^p_{K_{Xt}} = QU^p_{K_{Xt}} \tag{4.28}$$

and

$$Q(R^p_{K_{Xt}})^T \tilde{\mathcal{J}}_{K_{Xt}} R^p_{K_{Xt}} Q^T = Q \tilde{\mathcal{J}}^{R^p}_{K_{Xt}} Q^T \tag{4.29}$$

By using the isotropy of the function A_k in the right hand of (4.27) we obtain (4.20).

3) If we use the thermoviscoplastic equivalent configuration $\tilde{K}_{Xt}$ then

$$\pi_{\tilde{K}_{Xt}} = h_{\tilde{K}_{Xt}}(C^e_{\tilde{K}_{Xt}}, \theta, \alpha_{\tilde{K}_{Xt}}) = h_k(C^e_{\tilde{K}_{Xt}}, \theta, \alpha_{\tilde{K}_{Xt}}, F^p_{\tilde{K}_{Xt}}) \qquad (4.30)$$

From (4.16) one gets

$$C^e_{\tilde{K}_{Xt}} = (F^e_{\tilde{K}_{Xt}})^T F^e_{\tilde{K}_{Xt}} = (V^e_{K_{Xt}})^2 = B^e \qquad (4.31)$$

By using (4.31), (4.17) the relation (4.30) becomes (4.22) with $\alpha_{\tilde{K}_{Xt}} = \alpha_{K_{Xt}}$.

<u>Remark 4.1.</u> Here $\tilde{K}_{Xt}$ is the local current relaxed configuration deduced by <u>a pure elastic deformation</u> from the actual one. In such way the case considered in [11,15] has been obtained.

<u>P.4.2.</u> Let $\mathcal{B}$ be t.e.v.p. K_{Xt}-isotropic body with all constitutive and evolution functions satisfying the <u>temporarly invariance condition</u>, i.e.

$$f_{K_{Xt}}(M) = f_{K_{Xt'}}(M) \qquad (4.23)$$

for any $t, t' \in R$ and for any M in the domain of the function $f_{K_{Xt}}$.

Then

1) The functions $A_{K_{Xt}}$ and $B_{K_{Xt}}$ are isotropic in Piola-Kirchhoff stress tensor, i.e.

$$A_{K_{Xt}}(Q\pi Q^T, \theta, \alpha) = QA_{K_{Xt}}(\pi, \theta, \alpha)Q^T \qquad (4.24)$$

for all $Q \in$ Orth.

2) The thermoelastic function $f_{K_{Xt}}$ is also an isotropic function with respect to the right Cauchy-Green thermoelastic tensor, i.e.

$$f_{K_{Xt}}(QU^eQ^T,\theta,\alpha)=Qf_{K_{Xt}}(U^e,\theta,\alpha)Q^T \quad (4.25)$$

3) The yieeld function $\mathcal{F}_{K_{Xt}}$ and the evolution function $m_{K_{Xt}}$, $l_{K_{Xt}}$ are isotropic scalar functions, i.e.

$$\mathcal{F}_{K_{Xt}}(Q\pi Q^T,\theta,\alpha)=\mathcal{F}_{K_{Xt}}(\pi,\theta,\alpha) \quad (4.26)$$

4) All the constitutive and evolution functions are independent of $F^p_{K_{Xt}}$.

The proof draw immediately from T.4.1 if we observe the independency of F^p of the constitutive and evolution functions. It follows

$$f_k(M,K_{Xt}\circ(\nabla k(X))^{-1})=f_k(M,K_{Xt'}\circ(\nabla k(X))^{-1})$$

or

$$f_k(M,F^p_{K_{Xt}})=f_k(M,F^p_{K_{Xt'}}) \quad (4.27)$$

from the temporally invariance condition in which has been used (1.16). If we take t_o for t' in (4.27) the independency of F^p results.

T.4.2. Let $\mathcal{B}$ be t.e.v.p. K_{Xt}-isotropic body which satisfy the temporally invariance condition.

Then

1) There exists a c.l.r.c. $\tilde{K}_{Xr}$ deduced from the actual one by a pure elastic deformation and such that,

2) The deformation rate is additively expressed

$$D=D^e+D^p \quad (4.28)$$

by means of the rate of thermoelastic strain

$$D^e = \{\dot{F}^e_{\tilde{K}_{Xt}} (F^e_{\tilde{K}_{Xt}})^{-1}\}^s = \{\dot{V}^e (V^e)^{-1}\}^s \qquad (4.29)$$

of the rate of thermoplastic strain

$$D^p = \{\dot{F}^p_{K_{Xt}} (F^p_{K_{Xt}})^{-1}\}^s \qquad (4.30)$$

3) The thermoelastic constitutive equation is an isotropic function in its tensorial argument B^e, i.e.

$$\tilde{\pi}_{\tilde{K}_{Xt}} = h_{\tilde{K}_{Xt}} (B^e, \theta, \alpha) \qquad (4.31)$$

and

$$h_{\tilde{K}_{Xt}} (QB^eQ^T, \theta, \alpha) = Qh_{K_{Xt}} (B^e, \theta, \alpha) Q^T \qquad (4.32)$$

for all $Q \in$ Orth.

4) The constitutive and evolution functions are independent of $F^p_{K_{Xt}}$.

Proof. In the hypotesis of this theorem T.4.1 and P.4.2 hold. So the existence of $\tilde{K}_{Xt}$ such that

$$\nabla\chi(X,t) \circ \tilde{K}_{Xt} \equiv F^e_{\tilde{K}_{Xt}} = V^e_{K_{Xt}} \equiv V^e \in \text{Psym}$$

has been proved by P.4.1. Therefore $\tilde{K}_{Xt} = (V^e)^{-1} \circ \nabla\chi(X,t)$; this means that the configuration $\tilde{K}_{Xt}$ is deduced from the actual one by a pure elastic deformation.

The isotropy of the functions $h_{K_{Xt}}$, $A_{K_{Xt}}$ and $B_{K_{Xt}}$ with respect to their tensorial argument has been proved by P.4.2. The thermoelastic constitutive equation (4.22) relative to $\tilde{K}_{Xt}$ becomes (4.31) with $h_{\tilde{K}_{Xt}}$ isotropic in B^e.

These functions will be symmetric valued with their appropiate representation. For instance the representation of $A_{K_{Xt}}$ is given by

$$A_{K_{Xt}}(\tilde{\pi}_{K_{Xt}},\theta,\alpha)= \varphi_0(j(\tilde{\pi}_{K_{Xt}}),\theta,\alpha)I+\varphi_1(j(\tilde{\pi}_{K_{Xt}}),\theta,\alpha)\tilde{\pi}_{K_{Xt}}+ \tag{4.33}$$

$$+\varphi_2(j(\tilde{\pi}_{K_{Xt}}),\theta,\alpha)\tilde{\pi}^2_{K_{Xt}}$$

where $j(\tilde{\pi})$ denotes the invariants of $\tilde{\pi}$.

Therefore $\tilde{\pi}_{\tilde{K}_{Xt}}$ has the same principal axes parallel to those of B^e or V^e.

From (1.15) the expression of the velocity gradient

$$L(x,t)=\dot{F}^e_{K_{Xt}}(F^e_{K_{Xt}})^{-1}+F^e_{K_{Xt}}\dot{F}^p_{K_{Xt}}(F^p_{K_{Xt}})^{-1}(F^e_{K_{Xt}})^{-1} \tag{4.34}$$

results. We consider in the second term of L the evolution equation (1.17) for $F^p_{K_{Xt}}$

$$F^e_{K_{Xt}}\dot{F}^p_{K_{Xt}}(F^p_{K_{Xt}})^{-1}(F^e_{K_{Xt}})^{-1}=V^eR^e_{K_{Xt}}\Big[A_{K_{Xt}}(\tilde{\pi}_{K_{Xt}},\theta,\alpha)+ \tag{4.35}$$

$$+<\lambda_{K_{Xt}}>B_{K_{Xt}}(\tilde{\pi}_{K_{Xt}},\theta,\alpha)\Big](R^e_{K_{Xt}})^T(V^e_{K_{Xt}})^{-1}$$

By using the isotropy of $A_{K_{Xt}}$ and $B_{K_{Xt}}$, the temporarly invariance condition and formulae (4.19) from (4.35) one gets

$$F^e_{K_{Xt}}\dot{F}^p_{K_{Xt}}(F^p_{K_{Xt}})^{-1}(F^e_{K_{Xt}})^{-1}=$$

$$=V^e\{A_{\tilde{K}_{Xt}}(\tilde{\pi}_{\tilde{K}_{Xt}},\theta,\alpha)+<\lambda_{K_{Xt}}>B_{\tilde{K}_{Xt}}(\tilde{\pi}_{\tilde{K}_{Xt}},\theta,\alpha)\}(V^e)^{-1} \tag{4.36}$$

Since $A_{\tilde{K}_{Xt}}$ and $B_{\tilde{K}_{Xt}}$ are isotropic we obtain that they have the same principal axes as $\tilde{\pi}_{\tilde{K}_{Xt}}$, as also has V^e. Thus the multiplication of V^e with $A_{\tilde{K}_{Xt}}$ and $B_{\tilde{K}_{Xt}}$ respectively is commutative; so V^e and $(V^e)^{-1}$ in (4.36) cancel. Thus (4.36) becomes

$$F^e_{K_{Xt}} \dot{F}^p_{K_{Xt}} (F^p_{K_{Xt}})^{-1} (F^e_{K_{Xt}})^{-1} = \tag{4.37}$$
$$= A_{\tilde{K}_{Xt}} (\tilde{\Pi}_{\tilde{K}_{Xt}}, \theta, \alpha) + \langle \lambda_{K_{Xt}} \rangle B_{\tilde{K}_{Xt}} (\tilde{\Pi}_{\tilde{K}_{Xt}}, \theta, \alpha)$$

Concerning the rate of $F^p_{K_{Xt}}$ we state that

$$\dot{F}^p_{\tilde{K}_{Xt}} (F^p_{\tilde{K}_{Xt}})^{-1} = \dot{R}^e_{K_{Xt}} (R^e_{K_{Xt}})^T + R^e_{K_{Xt}} \dot{F}^p_{K_{Xt}} (F^p_{K_{Xt}})^{-1} (R^e_{K_{Xt}})^T \equiv \tag{4.38}$$
$$\omega_e + A_{K_{Xt}} (\tilde{\Pi}_{\tilde{K}_{Xt}}, \theta, \alpha) + \langle \lambda_{K_{Xt}} \rangle B_{K_{Xt}} (\tilde{\Pi}_{\tilde{K}_{Xt}}, \theta, \alpha)$$

or

$$\dot{F}^p_{\tilde{K}_{Xt}} (F^p_{\tilde{K}_{Xt}})^{-1} = \omega_e + A_{\tilde{K}_{Xt}} (\tilde{\Pi}_{\tilde{K}_{Xt}}, \theta, \alpha) + \langle \lambda_{K_{Xt}} \rangle B_{\tilde{K}_{Xt}} (\tilde{\Pi}_{\tilde{K}_{Xt}}, \theta, \alpha) \tag{4.39}$$

since $F^p_{\tilde{K}_{Xt}}$ is related to $F^p_{K_{Xt}}$ by (4.25) and the function $A_{K_{Xt}}$ satisfy the temporally invariance condition.

The first term of (4.34) may be calculated by means of $F^e_{\tilde{K}_{Xt}} = V^e_{K_{Xt}}$ as

$$\dot{F}^e_{K_{Xt}} (F^e_{K_{Xt}})^{-1} = \dot{R}^e_{K_{Xt}} (R^e_{K_{Xt}})^T + \dot{V}^e_{K_{Xt}} (V^e_{K_{Xt}})^{-1} \tag{4.40}$$
$$= \omega_e + \dot{F}^e_{\tilde{K}_{Xt}} (F^e_{\tilde{K}_{Xt}})^{-1}$$

The relation (4.34) with (4.37), (4.39) and (4.40) becomes:

$$L(x,t) = \dot{F}^e_{\tilde{K}_{Xt}} (F^e_{\tilde{K}_{Xt}})^{-1} + \dot{F}^p_{\tilde{K}_{Xt}} (F^p_{\tilde{K}_{Xt}})^{-1} \equiv L^e(x,t) + L^p(x,t) \tag{4.41}$$

By taking the symmetric parts of (4.41) the rate of deformation is additively expressed in (4.28) by means of the rate of thermoelastic strain (4.29) and thermoplastic strain (4.30) ■

In conclusion the concept of K_{Xt}-isotropy leads to the isotropic constitutive and evolution functions in the pair (V^e, F^p) or $(\tilde{\pi}, F^p)$ respectively. If we add the temporally invariance condition they become isotropic with respect to their first argument. When we suppose that the constitutive and evolution functions depend on F^p only trought its positive-symmetric part we obtain the same.

In this last cases there exist a c.l.r.c. K_{Xt} deduced from the actual one by a pure elastic deformation and such that the rate of deformation is additively expressed by means of the rate of thermoelastic and thermoplastic strain.

Acknowledgements. The author wishes to thank to I.Suliciu, M.Suliciu, E.Sóos and V.Ţigoiu for the helpful discussions which have led to the present work.

REFERENCES

[1] S.Cleja-Ţigoiu, Thermoviscoplastic matherials with instantaneous plasticity, Preprint series in mathematics nr.6, 1983, INCREST, Bucureşti.

[2]. M.Mihăilescu-Suliciu, I.Suliciu, Hypoelasticity and rate--independent plasticity theory, Preprint series in mathematics no.88/1981, INCREST, Bucureşti.

[3]. E.Sóos, Thermoelastoviscoplasticitatea metalelor (Reconstrucţie axiomatică), St. Cerc. Mec. Apl., tom 42, nr.5, 445-463, 1983.

[4]. W.Noll, Materially Uniform Simple Bodies with Inhomogeneities, Arch. Rat. Mech. Anal., 27, 1-32, 1967 or pp. 211-242 in W.Noll: The foundations of mechanics and thermodinamics, selected papers (S.P.), Springer-Verlag, Berlin, Heidelberg, New York, 1974.

[5]. - A new mathematical theory of simple materials, Arch. Rat. Mech. Anal. 48, 1-50, 1972, or pp. 243-392 in Ref 4 .

[6]. J.Mandel, Sur les relations de comportement d'un milieu élastique viscoplastic, C.R. Acad. Sc. Paris, t.272, p. 1596-1598, 1971.

[7]. - , Plasticité classique et viscoplasticité, Udine 1971, Springer-Verlag, Wienn, New-York, 1972.

[8]. - , Equations constitutives et directeurs dans les milieux plastiques et viscoplastique, Int. J. Solids Structures, vol.9, 725-740, 1973.

[9]. E.H.Lee, D.T.Liu, Finite-Strain Elastic-Plastic theory, pp. 117-131 in Proc. IUTAM Symp. Vienna 1966, Irreversible aspects of continuum mechanics and transfer of physical characteristics in moving fluids, Ed. H.Parkus,

L.I.Sedov, Springer-Verlag, Wienn, New-York, 1968.

[10]. - , Finite-strain Elastic-Plastic theory with application to plane-wave analysis,, J.Appl. Phys. 38, 19-27, 1967.

[11]. E.H.Lee, Elastic plastic deformation at finite strain, ASME Journal of Appl. Mech. 36, 1-6, 1969.

[12]. W.Noll, A mathematical theory of the mechanical behavior of continuous media, Arch. Rat. Mech. Anal. 48, 1-50, 1972, or pp. 243-292 in Ref. 3 .

[13]. C.Truesdell, W.Noll, The non linear field theories of mechanics, Handbuch der physik III/3, Springer-Verlag, Berlin-Heidelberg, New York, 1965.

[14]. C.Teodosiu, A dynamic theory of dislocations and its applications to the theory of the elastic-plastic continuum, pp. 837-876 in Fundamental aspects of dislocation theory, Ed. J.A. Simmons, R. de Wit, R.Bullough, Nat. Bar. Stand. (U.S.), Spec. Publ. 317, II, 1970.

[15]. E.H.Lee, R.M.McMeeking, Concerning elastic and plastic components of deformation, Int. J. Solids Structures, vol. 16, 715-721, 1980.