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ISSN 0250 3638

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PREPRINT SERIES IN MATHEMATICS
No. 66/1984

BUCUREȘTI

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November 1984

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0. Introduction

The theory of principal modules was developed by the author during the years 1980-1981 in the papers [8],[9] circulated as INCREST preprints. Some applications of it were presented ay the First Romanian-GDR Seminar on Banach space theory held in București, 1981 (see [10]), at the International Conference on operator algebras and ideals held in Leipzig, 1983 (see [13]) and at the 9th International Conference on operator theory held in Timișoara and Herculane, 1984 (see [15]).

This theory provides an unified framework for the proof of various results in the cycle of ideas which arouse with the Dodds-Fremlin theorem on compact operators ([4]). We mention its applications to M-tensor products ([10] and [13]), to an alternative proof of Schep's theorem on kernel operators, to approximable operators, to the characterization of the band generated by finite rank operators ([13]), to the relation between the order ideal and the closed algebraic ideal generated by an order bounded operator on a vector lattice ([14]) and on an ordered Banach space which is not a Banach lattice ([15]).

Recently, Buskes, Dodds, de Pagter and Schep proved in [2] some results which are in fact easy consequences of our theory. It is the purpose of this paper to clarify this point, showing how the results from [2] can be deduced from ours and even be improved and put in a more general framework.

1. Preliminaries

Let E be a Riesz space. We denote by $[x,y]$ the order interval $\{z \mid z \in E, x \leq z \leq y\}$. $E(x)$ will be the order ideal generated by $x \in E$. If $D \subset E$ we denote by $D^{\uparrow}$ the subset of those $x \in E$ for which there is a sequence $(x_n) \subset D$ such that $x_n \uparrow x$; similarly we define $D^{\downarrow}$. By $D^{\Uparrow}$ we denote the subset of those $x \in E$ for which there is a net $(x_\delta) \subset D$ such that $x_\delta \uparrow x$; similarly for $D^{\Downarrow}$.

If D is a subset of a topological space, $\overline{D}$ will be its closure.

We recall the monotone approximation theorem proved in [14]:

THEOREM 1.1. Let E be a Riesz space endowed with a locally solid topology τ and let $D \subset E$ be an order bounded sublattice.

a) Suppose that E is σ-order complete and τ is metrizable and σ-Fatou. Then $\overline{D} \subset D^{\downarrow\uparrow} \cap D^{\uparrow\downarrow}$.

b) Suppose that E is order complete and τ is separated and Fatou. Then $\overline{D} \subset D^{\downarrow\Uparrow\Downarrow} \cap D^{\uparrow\Downarrow\Uparrow}$.

Let E,F be Archimedian Riesz spaces. The tensor product $E\bar{\otimes}F$ in the sense of Fremlin ([5]) is an Archimedian Riesz space and there is a canonical Riesz bimorphism $\psi: E\times F \longrightarrow E\bar{\otimes}F$; we use the notation $x\otimes y$ for $\psi(x,y)$. The couple $(E\bar{\otimes}F,\psi)$ is universal in the following sense: for every Archimedian Riesz space G and every Riesz bimorphism $\varphi: E\times F \longrightarrow G$ there is a unique Riesz morphism $\bar{\varphi}: E\bar{\otimes}F \longrightarrow G$ such that $\varphi = \bar{\varphi}\psi$. The canonical morphism ψ induces a one-to-one map from the algebraic tensor product $E\otimes F$ into $E\bar{\otimes}F$; we shall identify $E\otimes F$ with its image in $E\bar{\otimes}F$. The vector sublattice generated by $E\otimes F$ equals $E\bar{\otimes}F$.

We recall that an f-algebra is an algebra which is also a Riesz space such that its multiplication is a Riesz bimorphism. In this paper, however, by an f-algebra we shall mean an Archimedian f-algebra A having an element e which is an algebraic unit and a strong order unit. We shall consider on A the norm associated with its order unit. If A_i are f-algebras, i=1,2 and e_i is the unit of A_i then $A_1\bar{\otimes}A_2$ is also an f-algebra; its unit is $e_1\otimes e_2$. Namely, by the universality property of $A_1\bar{\otimes}A_2$ there is a unique structure of f-algebra on $A_1\bar{\otimes}A_2$ such that $(a_1\otimes a_2)(b_1\otimes b_2) = a_1b_1\otimes a_2b_2$ for every $a_i,b_i\in A_i$, i=1,2. It is known ([5]) that $A_1\otimes A_2$ is dense in $A_1\bar{\otimes}A_2$.

The center Z(E) of a Riesz space is an f-algebra.

1_E will be the identity map on a set E.

Let E,F be Riesz spaces such that F is order complete. $L_r(E,F)$ will be the Riesz space of all order bounded linear maps from E to F. If E and F are endowed with locally solid topologies, $L_r'(E,F)$ will be the order ideal of $L_r(E,F)$ consisting of continuous maps. We shall write $L_r(F)$ for $L_r(F,F)$; similarly for $L_r'(F)$. If M is a subset of $L_r(E,R)$, $|\sigma|(E,M)$ will be the topology on E generated by the seminorms $x\longmapsto f(|x|)$ with $f\in M\cap L_r(E,\mathbb{R})_+$. By $E_n^{\sim}$ we shall denote the Riesz space of all order continuous linear forms on E.

The dual of a Banach space will be denoted by E'; T' will be the transpose of a linear map T.

For a compact space X, C(X) will be the space of all continuous functions on X.

2. Principal modules

We recall here some basic definitions and results from principal modules theory.

Let A be an f-algebra with unit e. By an A-module we shall mean a Riesz space E which is an algebraic module over A such that $A_+E_+\subset E_+$. For E Archimedian this definition implies that the map $(a,x)\longmapsto ax$ (from $A\times E$ into E) is a Riesz bimorphism. If $x\in E$ and $M\subset A$, Mx will be the set $\{ax \mid a\in M\}$.

Suppose that E is an Archimedian A-module and also a B-module. Then by the universality property of $A\bar{\otimes}B$ we can give a unique structure of $A\bar{\otimes}B$-module on E such that $(a\otimes b)x = a(bx)$ for every $a\in A$, $b\in B$, $x\in E$.

A principal A-module is an A-module endowed with a locally solid topology such that for every $x \in E$, Ax is dense in $E(x)$. This is equivalent to each of the following requirements:

a) $[0,x] = \overline{[0,e]x}$ for every $x \in E_+$.

b) $[-|x|,|x|] = \overline{[-e,e]x}$ for every $x \in E$.

From these conditions it follows easily that if F is a principal dense A-submodule of an A-module E, then E is also principal.

The notion of a principal module was introduced in [8] (see also [11]).

Every Banach lattice E with a quasi interior point is a principal Z(E)-module.

The following monotone approximation theorem was proved in [14]:

THEOREM 2.1. Let E be a principal A-module and let $x \in E_+$.

a) Suppose that E is σ-order complete and its topology is σ-Fatou and metrizable. Then

$$[0,x] = ([0,e]x)^{\uparrow\downarrow} = ([0,e]x)^{\downarrow\uparrow}.$$

b) Suppose that E is order complete and its topology is Fatou and separated. Then

$$[0,x] = ([0,e]x)^{\uparrow\downarrow\uparrow} = ([0,e]x)^{\downarrow\uparrow\downarrow}.$$

This result is deduced from theorem 1.1 observing that $[0,e]x$ is a bounded sublattice. As $[-e,e]x$ is also a bounded sublattice for every $x \in E_+$ we have the following complement to theorem 2.1:

THEOREM 2.2. Let E be a principal A-module and let $x \in E_+$.

a) Suppose we are in case a) of theorem 2.1. Then

$$[-x,x] = ([-e,e]x)^{\uparrow\downarrow} = ([-e,e]x)^{\downarrow\uparrow}.$$

b) Suppose we are in case b) of theorem 2.1. Then

$$[-x,x] = ([-e,e]x)^{\uparrow\downarrow\uparrow} = ([-e,e]x)^{\downarrow\uparrow\downarrow}.$$

Let E be an A-module and let F be an order complete B-module. We define structures of A-module and of B-module on $L_r(E,F)$ by

$$(Ua)(x) = U(ax), \quad (bU)(x) = bU(x).$$

In this way, $L_r(E,F)$ becomes an $A\bar{\otimes}B$-module.

If E,F are endowed with locally solid topologies, the solid strong topology on $L_r'(E,F)$ is the topology having as a basis for 0 the sets

$$\{U \mid U \in L_r'(E,F),\ |U|(x) \in V\}$$

for every $x \in E_+$ and every neighborhood V of 0 in F.

THEOREM 2.3. Let E be a principal A-module, F an order complete principal B-module such that its topology is order continuous. Then $L_r'(E,F)$ is a principal $A\bar{\otimes}B$-module with respect to the solid strong topology.

This result was proved in [9]; it was also mentioned (without proof) and used in [10] and [13].

3. The results of Buskes, Dodds, de Pagter and Schep

First, some notations from [2]. If E,F are Riesz spaces with F order complete, C^+ will be the set of all linear endomorphisms μ of $L_r(E,F)$ with the following properties:

a) $0 \le \mu \le 1_{L_r(E,F)}$.

b) μ lies in the linear hull of the maps $T \longmapsto PT\pi$ where $\pi \in Z(E)$ and P is a projection on a band in F.

Let E,F be Banach lattices and let F_1 be the order ideal generated by F in F''. Q^+ will be the set of all linear endomorphisms μ of $L_r(E,F_1)$ with the following properties:

a) $0 \le \mu \le 1_{L_r(E,F_1)}$.

b) μ lies in the closure of the linear hull of the maps $T \longmapsto \sigma'' T\pi$ where $\pi \in Z(E)$ and $\sigma \in Z(F)$, the closure being taken with respect to the norm associated to the order unit $1_{L_r(E,F_1)}$ of $Z(L_r(E,F_1))$.

If $T \in L_r(E,F)$, $C^+(T)$ will be the set $\{\mu(T) \mid \mu \in C^+\}$; similarly for $Q^+(T)$. Now we can list the mentioned results.

THEOREM 3.1. Let E be a Banach lattice with a quasi interior point and let F be an order complete Riesz space. Suppose that $F_n^{\sim}$ separates the points of F. Then

a) $[0, 1_{L_r(E,F)}] = C^{+\uparrow\downarrow\uparrow\downarrow}$.

b) $[0,T] = C^+(T)^{\uparrow\downarrow\uparrow\downarrow}$ for every $T \in L_r(E,F)_+$.

THEOREM 3.2. Let E,F be Banach lattices with quasi interior points. Then

a) $[0, 1_{L_r(E,F_1)}] = Q^{+\uparrow\downarrow\uparrow\downarrow}$.

b) $[0,T] = Q^+(T)^{\uparrow\downarrow\uparrow\downarrow}$ for every $T \in L_r(E,F_1)_+$.

THEOREM 3.3. Let E,F be Banach lattices with quasi interior points. Suppose that $S,T \in L_r(E,F_1)$ with $0 \le |S| \le |T|$ and let ρ be an order continuous Riesz seminorm on the order ideal generated by $|T|$ in $L_r(E,F_1)$. If $\varepsilon > 0$ is given there exist elements $\pi_1,\dots,\pi_n \in Z(E)$, $\sigma_1,\dots,\sigma_n \in Z(F)$ such that

$$\Big|\sum_{i=1}^{n} \sigma''_i U \pi_i\Big| \le U \text{ for every } U \in L_r(E,F_1)_+ \text{ and } \rho\Big(S - \sum_{i=1}^{n} \sigma''_i T \pi_i\Big) < \varepsilon .$$

Theorem 3.3 is an extension of a result of Kalton and Saab ([6]).

4. A principality result

PROPOSITION 4.1. Let E be an Archimedian Riesz space on which there are two structures of A-module: $(a,x)\mapsto ax$ and $(a,x)\mapsto xa$. Let $a\in A$ be given. Then the set $E_a=\{x\mid x\in E,\ ax=xa\}$ is a band in E.

PROOF. We consider first the particular case $A=C(X)$, $E=C(Y)$. By corollary 2.2 in [8], there are continuous maps $h_1,h_2:Y\to X$ such that $(ax)(t)=a(h_1(t))x(t)$, $(xa)(t)=a(h_2(t))x(t)$ for every $a\in A$, $x\in E$ and $t\in Y$; from this it is obvious that E_a is a band.

Now, for the general case it suffices to prove that E_a is an order ideal, because the order continuity of the maps $x\mapsto ax$, $x\mapsto xa$ will imply that it is a band. To this end, it suffices to prove that $E_a\cap E(x)$ is an order ideal for every $x\in E_+$. So let $x\in E_+$ be given and let $\overline{E(x)}$ be the completion of $E(x)$ with respect to the norm associated with its order unit x, $\overline{A}$ be the norm completion of A; the two structures of A-module of $E(x)$ can be extended to structures of $\overline{A}$-module on $\overline{E(x)}$. As $\overline{A}$ is isomorphic to a $C(X)$ and $\overline{E(x)}$ to a $C(Y)$, the result follows from the particular case considered above.

If E,F are A-modules, $\mathrm{Hom}_A(E,F)$ will be the set of all A-linear maps from E to F; we shall write $\mathrm{End}_A(E)$ for $\mathrm{Hom}_A(E,E)$.

PROPOSITION 4.2. If F is order complete them $\mathrm{Hom}_A(E,F)\cap L_r(E,F)$ is a band in $L_r(E,F)$.

PROOF. Follows from proposition 4.1 observing that $\mathrm{Hom}_A(E,F)=\bigcap_{a\in A} L_r(E,F)_a$.

A structure of A-module can be given on $\mathrm{Hom}_A(E,F)$ by $(aU)(x)=aU(x)$. If E,F are endowed with locally solid topologies and F is order complete then $\mathrm{Hom}_A(E,F)\cap L_r'(E,F)$ is a submodule of $\mathrm{Hom}_A(E,F)$.

THEOREM 4.1. Let E,F be principal A-modules such that F is order complete and its topology is order continuous. Then $\mathcal{H}=\mathrm{Hom}_A(E,F)\cap L_r'(E,F)$ is a band of $L_r'(E,F)$ and a principal A-module with respect to the solid strong topology. We have the relations

$$[0,U]=([0,e]U)^{\downarrow\downarrow\uparrow}=([0,e]U)^{\downarrow\uparrow\downarrow},$$

$$[-U,U]=([-e,e]U)^{\downarrow\downarrow\uparrow}=([-e,e]U)^{\downarrow\uparrow\downarrow}$$

for every $U\in\mathcal{H}_+$.

PROOF. By proposition 4.2, $\mathcal{H}$ is a band in $L_r'(E,F)$. By theorem 2.3, $L_r'(E,F)$ is a principal $A\bar{\otimes}A$-module.

By the universality property of $A\bar{\otimes}A$ there is a Riesz morphism $\varkappa:A\bar{\otimes}A\to A$ such that $\varkappa(a_1\otimes a_2)=a_1a_2$ for every $a_1,a_2\in A$. Using the fact that $A\bar{\otimes}A$ is the vector sublattice generated by $A\otimes A$ we can see that $cU=\varkappa(c)U$ for every $c\in A\bar{\otimes}A$ and every $U\in\mathcal{H}$ (in the left side of the equality U is regarded as a member of the $A\bar{\otimes}A$-module $L_r'(E,F)$ and in the right side, as a member of the A-module $\mathcal{H}$). Let $U\in\mathcal{H}_+$ be given and let e be the unit of A. We have

$$\overline{[0,U]} = \overline{[0,e\otimes e]U} = \overline{\mathcal{K}([0,e\otimes e])U} \subset \overline{[0,e]U} \subset \overline{[0,U]}$$

hence $\overline{[0,U]} = \overline{[0,e]U}$ which shows that $\mathcal{H}$ is a principal A-module.

The last assertion is a consequence of theorems 2.1 and 2.2.

COROLLARY 4.1. Let E be a principal A-module and let F be a principal B-module. Let e_1 be the unit of A and let e_2 be the unit of B. Suppose that F is order complete and its topology is separated and order continuous. Consider the $A\bar{\otimes}B$-module $\mathcal{H} = \mathrm{End}_{A\bar{\otimes}B}(L_r^!(E,F)) \cap L_r^!(L_r^!(E,F))$ (the topology on $L_r^!(E,F)$ and on $L_r^!(L_r^!(E,F))$ being the solid strong topology). Then $\mathcal{H}$ is a band in $L_r^!(L_r^!(E,F))$ a principal $A\bar{\otimes}B$-module and we have the relations

$$[0,\mu] = ([0,e_1\otimes e_2]\mu)^{\uparrow\downarrow\uparrow} = ([0,e_1\otimes e_2]\mu)^{\downarrow\uparrow\downarrow},$$

$$[-\mu,\mu] = ([-e_1\otimes e_2, e_1\otimes e_2]\mu)^{\uparrow\downarrow\uparrow} = ([-e_1\otimes e_2, e_1\otimes e_2]\mu)^{\downarrow\uparrow\downarrow}$$

for every $\mu \in \mathcal{H}_+$.

PROOF. Follows from theorem 4.1 applied to the principal $A\bar{\otimes}B$-module $L_r^!(E,F)$.

Now part a) of theorem 3.1 is a special case of this corollary. Indeed, let E and F be as in theorem 3.1, let $A = Z(E)$ and let B be the f-algebra generated by the projections on bands in F; in this case, $A\bar{\otimes}B = A\otimes B$. If we consider the norm topology on E and the order continuous separated topology $|\sigma|(F,F_n^{\sim})$ on F then E is a principal A-module, F a principal B-module and $L_r^!(E,F) = L_r(E,F)$. Apply corollary 4.1 to $\mu = 1_{L_r(E,F)}$; then C^+ is precisely $[0,1_E\otimes 1_F]1_{L_r(E,F)}$, hence $[0,1_{L_r(E,F)}] = C^{+\uparrow\downarrow\uparrow}$. As concerns part b) of theorem 3.1, this is already a special case of theorem 2.3 combined with theorem 2.1; indeed, they imply that $[0,T] = ([0,e_1\otimes e_2]T)^{\uparrow\downarrow\uparrow}$ for every $T\in L_r(E,F)_+$ and it remains to observe that $[0,e_1\otimes e_2]T$ is precisely $C^+(T)$.

Comparing theorem 3.1 with the above corollary, we remark the following improvements: first, the condition that F_n separates the points of F is replaced by the existence of a locally solid separated order continuous (possibly not locally convex) topology on F. Second, we may take any $\mu\in\mathcal{H}$ instead of $1_{L_r(E,F)}$. Third, the monotone closure $\uparrow\downarrow\uparrow\downarrow$ is replaced by $\uparrow\downarrow\uparrow$.

We present now a monotone approximation result in principal modules which complements theorems 2.1 and 2.2. Some notations first: if D is a subset in the Riesz space E we let D^o be the subset of those $x\in E$ for which there are sequences $(x_n)\subset D$ and $(u_n)\subset E_+$ such that $|x-x_n|\le u_n$ and $u_n\downarrow 0$. We let D^ω be the subset of those $x\in E$ for which there are nets $(x_\delta)\subset D$ and $(u_\delta)\subset E_+$ such that $|x-x_\delta|\le u_\delta$ and $u_\delta\downarrow 0$. If E,F are Riesz spaces, $D\subset E$ and $U:E\to F$ is an order continuous linear map then $U(D^o)\subset U(D)^o$ and $U(D^\omega)\subset U(D)^\omega$.

THEOREM 4.2. Let E be an order complete principal A-module such that its topology is order continuous. Then $[-|x|,|x|] = ([-e,e]x)^{o\omega\omega}$ for every $x\in E$.

PROOF. Clearly $([-e,e]x)^{o\omega\omega}\subset[-|x|,|x|]$. For the converse inclusion, take any $y\in[-|x|,|x|]$. There is $U\in[-1_E,1_E]$ such that $y=U(x)$. By theorem 4.1, $U\in([-e,e]1_E)^{\uparrow\downarrow\uparrow}$; as the map $V\mapsto V(x)$ from $L_r(E)$ into E is order continuous, it follows that $y\in([-e,e]x)^{o\omega\omega}$.

5. The proofs of theorems 3.2 and 3.3

For a Banach lattice F, $\hat{F}$ will be the closure of the order ideal F_1 generated by F in F'' with respect to the topology $|\sigma|(F'',F')$. It is known ([3]) that $\hat{F}$ equals the closure of F with respect to $|\sigma|(F'',F')$.

PROPOSITION 5.1. Let F be a Banach lattice with a quasi interior point. Then F_1 and $\hat{F}$ are principal Z(F)-modules for the topologies $|\sigma|(F_1,F')$ and $|\sigma|(\hat{F},F')$.

PROOF. It suffices to prove that F_1 is a principal Z(F)-module. As F has a quasi interior point, it is a principal Z(F)-module for the norm topology. By theorem 2.2, F' is a principal Z(F)-module for $|\sigma|(F',F)$. Again by theorem 2.2 $F_1=L_r'(F',\mathbb{R})$ is a principal Z(F)-module for $|\sigma|(F_1,F')$.

THEOREM 5.1. Let E,F be Banach lattices with quasi interior points. The following are true:

a) Let $\mathcal{H}=\mathrm{End}_{Z(E)\bar{\otimes}Z(F)}(L_r(E,\hat{F}))\cap L_r'(L_r(E,\hat{F}))$. Then $\mathcal{H}$ is a principal $Z(E)\bar{\otimes}Z(F)$-module for the solid strong topology (the topology on F being $|\sigma|(\hat{F},F')$ and the topology on $L_r(E,\hat{F})$ and on $L_r'(L_r(E,\hat{F}))$ being the solid strong topology) and we have

$$[0,\mu]=([0,1_E\otimes 1_F]\mu)^{\uparrow\downarrow\uparrow}=([0,1_E\otimes 1_F]\mu)^{\downarrow\uparrow\downarrow}$$

for every $\mu\in\mathcal{H}_+$.

b) $$[0,T]=([0,1_E\otimes 1_F]T)^{\uparrow\downarrow\uparrow}=([0,1_E\otimes 1_F]T)^{\downarrow\uparrow\downarrow}$$

for every $T\in L_r(E,\hat{F})_+$.

c) $$[-|T|,|T|]=([-1_E\otimes 1_F,1_E\otimes 1_F]T)^{o\omega\omega}$$

for every $T\in L_r(E,\hat{F})$.

The same results hold replacing $\hat{F}$ by F_1.

PROOF. Part a) follows from corollary 4.1, taking into account proposition 5.1; part b) from theorems 2.1 and 2.2; part c) from theorem 4.2.

Theorem 3.2 is now a special case of theorem 5.1; indeed take $\mu=1_{L_r(E,F_1)}$ and observe that $[0,1_E\otimes 1_F]1_{L_r(E,F_1)}\subset Q^+$ and $[0,1_E\otimes 1_F]T\subset Q^+(T)$ by the density of $Z(E)\otimes Z(F)$ in $Z(E)\bar{\otimes}Z(F)$.

As concerns theorem 3.3, observe that by part c) of theorem 5.1 there is $c\in[-1_E\otimes 1_F,1_E\otimes 1_F]$ such that $\rho(S-cT)<\varepsilon$. As $Z(E)\otimes Z(F)$ is dense in $Z(E)\bar{\otimes}Z(F)$, there is $d\in Z(E)\otimes Z(F)\cap[-1_E\otimes 1_F,1_E\otimes 1_F]$ such that $\rho(S-dT)<\varepsilon$; this concludes the proof.

We remark that in our version of theorem 3.3, F_1 is replaced by F.

REFERENCES

1. C.D.Aliprantis, O.Burkinshaw — The components of a positive operator, Math.Z. 184, 245-257 (1983).

2. G.J.H.M.Buskes, P.G.Dodds, B.de Pagter, A.R.Schep — Up-down theorems in the center of $L_b(E,F)$, preprint.

3. P.G.Dodds — o-weakly compact mappings of Riesz spaces, Trans. of the Am. Math. Soc. 214, 389-402 (1975).

4. P.G.Dodds, D.H.Fremlin — Compact operators in Banach lattices, Israel J. of Math. 34, 287-320 (1979).

5. D.H.Fremlin — Tensor products of Archimedian vector lattices, American J. Math. 94, 777-798 (1972).

6. N.J.Kaltan, P.Saab — Ideal properties of regular operators between Banach lattices, to appear in Ill. J. Math.

7. B.de Pagter — The components of a positive operator, Indag. Math. 45, 229-241 (1983).

8. D.Vuza — Strongly lattice-ordered modules over function algebras, Preprint Series Math. INCREST, București, 62 (1980).

9. D.Vuza — Extension theorems for strongly lattice-ordered modules and applications to linear operators, Preprint Series Math. INCREST, București, 100 (1981).

10. D.Vuza — The perfect M-tensor product of perfect Banach lattices, Proceedings of the First Romanian-GDR Seminar, Lecture Notes in Math. 991, Springer, 272-295 (1983).

11. D.Vuza — Strongly lattice-ordered modules over function algebras I, Analele Univ. Craiova, XI, 52-63 (1983).

12. D.Vuza — Module peste inele ordonate comutative, thesis, București, 1983.

13. D.Vuza — Principal modules of linear maps and their applications, Proceedings of the Second International Conference on Operator Algebras, Ideals and their Applications in Theoretical Physics, Teubner-Texte zur Math. 67, 212-219 (1984).

14. D.Vuza — Ideals and bands in principal modules, Preprint Series Math. INCREST, București, 42 (1984).

15. D.Vuza — Ideal properties of order bounded operators on ordered Banach spaces which are not Banach lattices

to appear in the Proceedings of the 9th International Conference of Operator Theory, Timişoara and Herculane, 1984.