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Abstract. We establish the system of equations for the hygrothermomechanical behaviour of a composite material, using the homogenization method. The macroscopic coefficients are deduced and it is proved that the macroscopic system of equations is a coupled one, the temperature and moisture equations containing new terms. Finaly the convergence theorem for the homogenization process is proved.

1. INTRODUCTION

1.1. Generalities

In the general framework of the homogenization method [1,2] we consider the problem of linear elasticity of a composite material under the effects of combined moisture and thermal environments. The periodic structure of the composite material is associated with a small parameter ε. The asymptotic process, $\varepsilon \to 0$, implies that the number of periods is very large.

All hygrothermomechanical properties are different in the matrix and in the inclusions, and their magnitude is of order one.

1.2. General equations

We consider a parallelipipedic period Y of the space of the variables y_i (i=1,2,3) formed by two parts Y_1 and Y_2 (the matrix and the inclusion) separated by a smooth boundary Γ. We also denote by Y_i (i=1,2) the union of the Y_i parts of all periods. If Ω is the domain of the composite material in the space of the variables x_i, we introduce the small parameter ε and the domains $\Omega_{\varepsilon i}$ defined by

$$\Omega_{\varepsilon i}=\{x;\ x\in\Omega\ ,\ x\in\varepsilon Y_i\}\qquad (i=1,2)$$

In $\Omega_{\varepsilon i}$ we have the equations:

$$\frac{\partial \sigma^{\varepsilon}_{ij}}{\partial x_j} - \rho^{\varepsilon}\frac{\partial^2 u^{\varepsilon}_i}{\partial t^2} = -f_i \tag{1.1}$$

$$\frac{\partial}{\partial x_i}\left(k^{\varepsilon}_{ij}\frac{\partial \theta^{\varepsilon}}{\partial x_j}\right) - T_o\beta^{\varepsilon}_{ij}\frac{\partial e^{\varepsilon}_{ij}}{\partial t} - c^{\varepsilon}\frac{\partial \theta^{\varepsilon}}{\partial t} = -r \tag{1.2}$$

$$\frac{\partial}{\partial x_i}\left(d^{\varepsilon}_{ij}\frac{\partial H^{\varepsilon}}{\partial x_j}\right) - H_o\alpha^{\varepsilon}_{ij}\frac{\partial e^{\varepsilon}_{ij}}{\partial t} - b^{\varepsilon}\frac{\partial H^{\varepsilon}}{\partial t} = -h \tag{1.3}$$

and the constitutive equation

$$\sigma^{\varepsilon}_{ij}=c^{\varepsilon}_{ijkh}e^{\varepsilon}_{kh} - \beta^{\varepsilon}_{ij}\theta^{\varepsilon} - \alpha^{\varepsilon}_{ij}H^{\varepsilon} \tag{1.4}$$

with

$$e^{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial u^{\varepsilon}_i}{\partial x_j} + \frac{\partial u^{\varepsilon}_j}{\partial x_i}\right) \tag{1.5}$$

where $\sigma^{\varepsilon}_{ij}$ and e^{ε}_{ij} are the respective linear stress and strain tensors, θ^{ε} is the temperature, H^{ε} is the moisture concentration, u^{ε}_{i} are the components of the displacement vector, T_o and H_o are the respective absolute reference temperature and moisture content, f_i are the body force components, γ and h are the respective heat and moisture supply. The stiffness tensor c^{ε}_{ijkh}, the strain-temperature tensor β^{ε}_{ij} and the strain-moisture tensor $\alpha^{\varepsilon}_{ij}$ are symmetric tensors: $c^{\varepsilon}_{ijkh} = c^{\varepsilon}_{khij} = c^{\varepsilon}_{jikh}$, $\beta^{\varepsilon}_{ij} = \beta^{\varepsilon}_{ji}$, $\alpha^{\varepsilon}_{ij} = \alpha^{\varepsilon}_{ji}$.

The coefficients are k^{ε}_{ij} - the termal conductivity tensor, d^{ε}_{ij} - the hygroscopic conductivity tensor, c^{ε} the specific heat at constant deformation, b^{ε} the specific hygroscopic capacity and ρ^{ε} the mass density.

We look for Y-periodic coefficients in the varialbe $y = \frac{x}{\varepsilon}$: $c^{\varepsilon}_{ijkh}(x) \equiv c_{ijkh}(\frac{x}{\varepsilon})$, $\beta^{\varepsilon}_{ij}(x) \equiv \beta_{ij}(\frac{x}{\varepsilon})$, $\alpha^{\varepsilon}_{ij}(x) \equiv \alpha_{ij}(\frac{x}{\varepsilon})$, $k^{\varepsilon}_{ij}(x) \equiv k_{ij}(\frac{x}{\varepsilon})$, $d^{\varepsilon}_{ij}(x) \equiv d_{ij}(\frac{x}{\varepsilon})$, $\rho^{\varepsilon}(x) \equiv \rho(\frac{x}{\varepsilon})$, $c^{\varepsilon}(x) \equiv c(\frac{x}{\varepsilon})$ and $b^{\varepsilon}(x) \equiv b(\frac{x}{\varepsilon})$.

The boundary conditions on Γ are:

$$\begin{aligned} &[u^{\varepsilon}] = 0, && [\sigma^{\varepsilon}_{ij} n_j] = 0 \\ &[\theta^{\varepsilon}] = 0, && \left[k^{\varepsilon}_{ij} \frac{\partial \theta^{\varepsilon}}{\partial x_j} n_i\right] = 0 \\ &[H^{\varepsilon}] = 0, && \left[d^{\varepsilon}_{ij} \frac{\partial H^{\varepsilon}}{\partial x_j} n_i\right] = 0 \end{aligned} \tag{1.6}$$

1.3. Two-scale asymptotic process

In order to study the asymptotic process $\varepsilon \to 0$ we consider the classical expansions [1,2]:

$$\begin{aligned} \underset{\sim}{u}^{\varepsilon}(x,t) &= \underset{\sim}{u}^{o}(x,t) + \varepsilon \underset{\sim}{u}^{1}(x,y,t) + \dots \\ \theta^{\varepsilon}(x,t) &= \theta^{o}(x,y,t) + \varepsilon \theta^{1}(x,y,t) + \dots \\ H^{\varepsilon}(x,t) &= H^{o}(x,y,t) + \varepsilon H^{1}(x,y,t) + \dots \end{aligned} \tag{1.7}$$

where $y = \frac{x}{\varepsilon}$ and all functions are considered to be Y periodic with respect to the variable y. The two-scale asymptotic expansion is obtained by considering that the dependence in x is obtained directly and through the variable y. The derivatives must be considered as

$$\frac{d}{dx_i} \rightarrow \frac{\partial}{\partial x_i} + \frac{1}{\varepsilon} \frac{\partial}{\partial y_i}$$

and then

$$e_{ij}(\underset{-}{u}) = e_{ijx}(\underset{-}{u}) + \frac{1}{\varepsilon} e_{ijy}(\underset{-}{u})$$

From (1.5) we have

$$\begin{aligned} e_{ij}^{\varepsilon} \equiv e_{ij}(\underset{-}{u}^{\varepsilon}) &= e_{ij}^{o}(x,y,t) + \varepsilon e_{ij}^{1}(x,y,t) + \dots \\ e_{ij}^{o}(x,y,t) &= e_{ijx}(\underset{-}{u}^{o}) + e_{ijy}(\underset{-}{u}^{1}) \\ e_{ij}^{1}(x,y,t) &= e_{ijx}(\underset{-}{u}^{1}) + e_{ijy}(\underset{-}{u}^{2}) \end{aligned}$$

and from (1.4)

$$\sigma_{ij}^{\varepsilon} = \sigma_{ij}^{o}(x,y,t) + \varepsilon \sigma_{ij}^{1}(x,y,t) + \dots \tag{1.8}$$

$$\sigma_{ij}^{o}(x,y,t) = c_{ijkh}(y) e_{ij}^{o}(x,y,t) - \beta_{ij}(y) \theta^{o}(x,y,t) - \alpha_{ij}(y) H^{o}(x,y,t)$$

$$\sigma_{ij}^{1}(x,y,t) = c_{ijkh}(y) e_{ij}^{1}(x,y,t) - \beta_{ij}(y) \theta^{1}(x,y,t) - \alpha_{ij}(y) H^{1}(x,y,t)$$

2. MACROSCOPIC EQUATIONS

2.1. Macroscopic balance of momentum

In order to obtain the macroscopic equation we use (1.7) and (1.8) in (1.1) and (1.6) and we identified the succesive powers of ε. At orders ε^{-1} and ε^{o} we have

$$\frac{\partial \sigma^{o}_{ij}}{\partial y_j} = 0 \tag{2.1}$$

$$\frac{\partial \sigma^{o}_{ij}}{\partial x_j} + \frac{\partial \sigma^{1}_{ij}}{\partial y_j} - \rho(y)\frac{\partial^2 u^{o}_{i}}{\partial t^2} = -f_i \tag{2.2}$$

The mean operator

$$\tilde{\cdot} = \frac{1}{|Y|}\int_Y \cdot\, dy \tag{2.3}$$

applied to (2.2) give us the macroscopic equation of balance of momentum:

$$\frac{\partial \tilde{\sigma}^{o}_{ij}}{\partial x_j} - \tilde{\rho}\,\frac{\partial^2 u^{o}_{i}}{\partial t^2} = -f_i \tag{2.4}$$

Remark 2.1. The equation (2.4) is the classical homogeneized equation for the linear elasticity [2].

2.2. Macroscopic conductivity tensors

First it is necessary to observe that the equations (1.2) and (1.3) are of the same type. As it usually happens in homogenization problem [1,2], θ^{o} and H^{o} does not depend on y. Using the same computation as in the case of linear thermoelasticity of composite materials [3] we obtain

$$\left[k_{ij}(y)\left(\frac{\partial\theta^{o}}{\partial x_j}+\frac{\partial\theta^{1}}{\partial y_j}\right)\right]^{\sim}=k^{o}_{ij}\frac{\partial\theta^{o}}{\partial x_j} \qquad (2.5)$$

$$k^{o}_{ij}=\tilde{k}_{ij}+\left[k_{ie}(y)\frac{\partial w^{j}}{\partial y_e}\right]^{\sim} \qquad (2.6)$$

$$\theta^{1}(x,y,t)=w^{j}(y)\frac{\partial\theta^{o}}{\partial x_j}+c(x,t) \qquad (2.7)$$

$$\left[d_{ij}(y)\left(\frac{\partial H^{o}}{\partial x_j}+\frac{\partial H^{1}}{\partial y_j}\right)\right]^{\sim}=d^{o}_{ij}\frac{\partial H^{o}}{\partial x_j} \qquad (2.8)$$

$$d^{o}_{ij}=\tilde{d}_{ij}+\left[d_{ie}(y)\frac{\partial h^{j}}{\partial y_e}\right]^{\sim} \qquad (2.9)$$

$$H^{1}(x,y,t)=h^{j}(y)\frac{\partial H^{o}}{\partial x_j}+c(x,t) \qquad (2.10)$$

where w^j and h^j are the solutions of $H^1_{per}(Y)$, with $\tilde{w}^j=0$ and $\tilde{h}^j=0$, of the equations:

$$\int_Y k_{ie}(y)\frac{\partial w^{j}}{\partial y_e}\frac{\partial\varphi}{\partial y_i}dy=\int_Y\frac{\partial k_{ie}}{\partial y_e}\varphi dy \qquad (\forall)\varphi\in H^{1}_{per}(Y) \qquad (2.11)$$

$$\int_Y d_{ie}(y)\frac{\partial h^{j}}{\partial y_e}\frac{\partial\varphi}{\partial y_i}dy=\int_Y\frac{\partial d_{ie}}{\partial y_e}\varphi dy \qquad (\forall)\varphi\in H^{1}_{per}(Y) \qquad (2.12)$$

Remark 2.2. The macroscopic thermal conductivity tensor k^o_{ij} and the macroscopic hygroscopic conductivity tensor d^o_{ij} are different from the simply mean values of the microscopic tensors, and they was obtained in classical way [2, 3, 4].

2.3. Macroscopic constitutive equation

As in the case of thermoelasticity of composite materials [3] we must return to the equation (2.1) named the local equation. Using (1.8), the equation (2.1) takes the form

$$-\frac{\partial}{\partial y_j}\left[c_{ijkh}(y)e_{khy}(\underline{u}^1)\right] = e_{khx}(\underline{u}^o)\frac{\partial c_{ijkh}}{\partial y_j} - \theta^o\frac{\partial \beta_{ij}}{\partial y_j} - H^o\frac{\partial \alpha_{ij}}{\partial y_j} \quad (2.13)$$

or in the variational formulation

$$\int_Y c_{ijkh}(y)e_{khy}(\underline{u}^1)e_{ijy}(\underline{v})dy = e_{khx}(\underline{u}^o)\int_Y \frac{\partial c_{ijkh}}{\partial y_j}v_i dy -$$

$$-\theta^o\int_Y \frac{\partial \beta_{ij}}{\partial y_j}v_i dy - H^o\int_Y \frac{\partial \alpha_{ij}}{\partial y_j}v_i dy \quad (\forall)\ \underline{v}\in H^1_{per}(Y) \quad (2.14)$$

If we define $\underline{W}^{kh}\in H^1_{per}(Y)$, $\underline{\Theta}\in H^1_{per}(Y)$ and $\underline{\chi}\in H^1_{per}(Y)$, with $\tilde{\underline{W}}^{kh}=0$, $\tilde{\underline{\Theta}}=0$, $\tilde{\underline{\chi}}=0$, solutions of the equations:

$$\int_Y c_{ijmn}(y)e_{mny}(\underline{W}^{kh})e_{ijy}(\underline{v})dy = \int_Y \frac{\partial c_{ijkh}}{\partial y_j}v_i dy \quad (\forall)\ \underline{v}\in H^1_{per}(Y) \quad (2.15)$$

$$\int_Y c_{ijmn}(y)e_{mny}(\underline{\Theta})e_{ijy}(\underline{v})dy = \int_Y \frac{\partial \beta_{ij}}{\partial y_j}v_i dy \quad (\forall)\ \underline{v}\in H^1_{per}(Y) \quad (2.16)$$

$$\int_Y c_{ijmn}(y)e_{mny}(\underline{\chi})e_{ijy}(\underline{v})dy = \int_Y \frac{\partial \alpha_{ij}}{\partial y_j}v_i dy \quad (\forall)\ \underline{v}\in H^1_{per}(Y) \quad (2.17)$$

the solution of (2.14) is

$$\underline{u}^1(x,y,t) = e_{khx}(\underline{u}^o)\underline{W}^{kh} - \theta^o\underline{\Theta} - H^o\underline{\chi} \quad (2.18)$$

abstraction of a function depending on x and t.

Using (2.18) we have

$$e_{mny}(\underline{u}^1) = e_{khx}(\underline{u}^o)e_{mny}(\underline{W}^{kh}) - \theta^o e_{mny}(\underline{\Theta}) - H^o e_{mny}(\underline{\chi})$$

$$\sigma^o_{ij} = c_{ijmn}\left[\delta_{mk}\delta_{nh} + e_{mny}(\underline{W}^{kh})\right]e_{khx}(\underline{u}^o) -$$

$$-\left[\beta_{ij} + c_{ijmn}e_{mny}(\underline{\Theta})\right]\theta^o - \left[\alpha_{ij} + c_{ijmn}e_{mny}(\underline{\chi})\right]H^o$$

and applying the mean operator (2.3) to the last equation we obtain the macroscopic constitutive equation

$$\tilde{\sigma}^{o}_{ij}=c^{o}_{ijkh}\tilde{e}^{o}_{kh} - \beta^{o}_{ij}\,\theta^{o} - \alpha^{o}_{ij}H^{o} \tag{2.19}$$

with:

$$c^{o}_{ijkh}=\tilde{c}_{ijkh} + \left[c_{ijmn}e_{mny}(\underline{W}^{kh})\right]^{\sim} \tag{2.20}$$

$$\beta^{o}_{ij}=\tilde{\beta}_{ij}+\left[c_{ijmn}e_{mny}(\underline{\Theta})\right]^{\sim} = \tilde{\beta}_{ij}+\left[\beta_{mn}e_{mny}(\underline{W}^{ij})\right]^{\sim} \tag{2.21}$$

$$\alpha^{o}_{ij}=\tilde{\alpha}_{ij}+\left[c_{ijmn}e_{mny}(\underline{\chi})\right]^{\sim} = \tilde{\alpha}_{ij}+\left[\alpha_{mn}e_{mny}(\underline{W}^{ij})\right]^{\sim} \tag{2.22}$$

Remark 2.3. The macroscopic constitutive equation (2.19) is also linear. (2.20) is the macroscopic stiffness tensor obtained first in the classical linear elasticity [2]. The macroscopic strain-temperature tensor (2.21) and the macroscopic strain-moisture tensor (2.22) depends also of the microscopic stiffness. The equalities (2.21) and (2.22) results from (2.15), (2.16) and (2.17) by taking as test functions succesively $\underline{W}^{kh}$, $\underline{\Theta}$ and $\underline{\chi}$ and using the symmetry of c_{ijmn}, β_{ij} and α_{ij}.

2.4. Macroscopic equations for temperature and moisture

From (1.2) and (1.3), using (1.7) at order ε^{o} we have the equations:

$$\frac{\partial}{\partial x_i}\left[k_{ij}(y)\left(\frac{\partial\theta^{o}}{\partial x_j}+\frac{\partial\theta^{1}}{\partial y_j}\right)\right] + \frac{\partial}{\partial y_i}\left[k_{ij}(y)\left(\frac{\partial\theta^{1}}{\partial x_j}+\frac{\partial\theta^{2}}{\partial y_j}\right)\right] -$$

$$- T_o\beta_{ij}(y)\frac{\partial e^{o}_{ij}}{\partial t} - c(y)\frac{\partial\theta^{o}}{\partial t} = -r(x) \tag{2.23}$$

$$\frac{\partial}{\partial x_i}\left[d_{ij}(y)\left(\frac{\partial H^o}{\partial x_j}+\frac{\partial H^1}{\partial y_j}\right)\right]+\frac{\partial}{\partial y_i}\left[d_{ij}(y)\left(\frac{\partial H^1}{\partial x_j}+\frac{\partial H^2}{\partial y_j}\right)\right] -$$
$$- H_o \alpha_{ij}(y)\frac{\partial e^o_{ij}}{\partial t} - b(y)\frac{\partial H^o}{\partial t} = -h(x) \tag{2.24}$$

If we take the mean value of (2.23) and (2.24) we obtain, as in [3], the macroscopic equations:

$$\frac{\partial}{\partial x_i}\left(k^o_{ij}\frac{\partial \theta^o}{\partial x_j}\right) - T_o \beta^o_{ij}\frac{\partial e^o_{ij}}{\partial t} - (\tilde{c}-T_o\gamma)\frac{\partial \theta^o}{\partial t}+T_o\delta\frac{\partial H^o}{\partial t}=-r \tag{2.25}$$

$$\frac{\partial}{\partial x_i}\left(d^o_{ij}\frac{\partial H^o}{\partial x_j}\right)-H_o\alpha^o_{ij}\frac{\partial e^o_{ij}}{\partial t} - (\tilde{b}-H_o\lambda)\frac{\partial H^o}{\partial t}+H_o\delta\frac{\partial \theta^o}{\partial t}=-h \tag{2.26}$$

where

$$\gamma = [\beta_{ij}e_{ijy}(\underset{-}{\Theta})]^{\sim} \tag{2.27}$$

$$\delta = [\beta_{ij}e_{ijy}(\underset{-}{\chi})]^{\sim} = [\alpha_{ij}e_{ijy}(\underset{-}{\Theta})]^{\sim} \tag{2.28}$$

$$\lambda = [\alpha_{ij}e_{ijy}(\underset{-}{\chi})]^{\sim} \tag{2.29}$$

Remark 2.4. The macroscopic equation for the temperature (2.25) contain the time derivative of the moisture, and the macroscopic equation for the moisture concentration (2.26) contain the time derivative of the temperature. Then we have a complet coupled system of equations (2.4), (2.19), (2.25), (2.26). This system was obtained for the first time by Chung and Bradshaw [5] using the classical theory of mechanics of continua and the thermodynamic restrictions imposed by the entropy inequality. In that case the macroscopic coefficients must be determined by the experiences. In our case, the macroscopic coefficients (2.6), (2.9) (2.20), (2.21), (2.22), (2.27), (2.28) and (2.29) may be computed directly starting from the microscopic values in the matrix and in the inclusion.

3. THE CONVERGENCE THEOREM

The equations (1.1), (1.2), (1.3) admit a unique solution $(\underline{u}^{\varepsilon}, \theta^{\varepsilon}, H^{\varepsilon})$. If the initial conditions are zero, as in the case of thermoelasticity [6,7] we have: there exists $(\underline{u}^{*}, \theta^{*}, H^{*})$ such that $\underline{u} \to \underline{u}^{*}$ weakly in $L^2(0,T:H^1(\Omega))$, $\theta^{\varepsilon} \to \theta^{*}$ weakly $*$ in $L^2(0,T-\alpha;\ H^1(\Omega))$ $(\forall)\ \alpha > 0$ and $H^{\varepsilon} \to H^{*}$ weakly$*$ in $L^2(0,T-\alpha;H^1(\Omega))$. Let $h(\varphi) \equiv \int_0^T h(t)\varphi(t)dt$ for all $\varphi \in C_0^{\infty}([0,T])$.

<u>Theorem</u>. Let $(\underline{u}^{\varepsilon}, \theta^{\varepsilon}, H^{\varepsilon})$ be the solution of (1.1), (1.2), (1.3) with homogeneous initial conditions and $(\underline{f},r,h) \in L^2(Q_T) \times L^2(Q_T) \times L^2(Q_T)$, $Q_T = [0,T] \times \Omega$. Let $(\underline{u}^{o}, \theta^{o}, H^{o})$ be the solution of (2.4), (2.25), (2.26). Then

$$\left.\begin{array}{l} \underline{u}^{\varepsilon} \to \underline{u}^{o} \\ \theta^{\varepsilon} \to \theta^{o} \\ H^{\varepsilon} \to H^{o} \end{array}\right\} \text{ weakly in } L^2(0,T-\alpha;H^1(\Omega)), \quad (\forall)\ \alpha > 0$$

<u>Proof</u>

We shall prove that $\underline{u}^{*} = \underline{u}^{o}$, $\theta^{*} = \theta^{o}$ and $H^{*} = H^{o}$. Using the classical method of [2], we define the vector $\underline{w}_{\varepsilon}$, with the components $w_{\varepsilon i} = \delta_{ik}x_h + \varepsilon W_i^{kh}(\frac{x}{\varepsilon})$, where $\underline{W}^{kh}$ is the solution of (2.15). For $\underline{w}_{\varepsilon}$ we obtain the equation

$$\int_{\Omega} c^{\varepsilon}_{ijmn} e_{mn}(\underline{w}_{\varepsilon}) e_{ij}(\underline{v})\, dx = 0 \quad (\forall)\ \underline{v} \in L^2(0,T-\alpha;H^1(\Omega)) \qquad (3.1)$$

The equation (1.1) is equivalent with the equation

$$\int_\Omega c^\varepsilon_{ijkh} e_{kh}(\underline{u}^\varepsilon(\varphi)) e_{ij}(\underline{v})dx + \int_\Omega \rho^\varepsilon u^\varepsilon_i(\varphi'')v_i dx - \int_\Omega \beta^\varepsilon_{ij}\,\theta^\varepsilon(\varphi)e_{ij}(\underline{v})dx -$$

$$- \int_\Omega \alpha^\varepsilon_{ij} H^\varepsilon(\varphi) e_{ij}(\underline{v})dx = \int_\Omega f_i(\varphi)v_i dx \qquad (3.2)$$

We take, as in [2], $v_i = u_i(\varphi)\psi$ in (3.1) and $v_i = w_{\varepsilon i}\psi$ in (3.2) with $\psi \in C^\infty_o(\Omega)$. By substraction, taking into account the symmetry of the coefficients and passing to the limit with $\varepsilon \to o$ we obtain

$$\sigma^*_{ij}(\varphi) = c^o_{ijkh}\frac{\partial u^*_k(\varphi)}{\partial x_h} - \left[\beta_{kh} e_{khy}(\underline{W}^{ij})\right]^\sim \theta^*(\varphi) -$$

$$- \left[\alpha_{kh} e_{khy}(\underline{W}^{ij})\right]^\sim H^*(\varphi) \qquad (3.3)$$

Here $\sigma^*_{ij}(\varphi)$ is the weak limit of $c^\varepsilon_{ijkh}\dfrac{\partial u^\varepsilon_k(\varphi)}{\partial x_h}$ in $L^2(\Omega)$ and we used the fact that $w_{\varepsilon i} \to \delta_{ik} x_h$ strongly in $L^2(\Omega)$ and the well known result: if $f(x,\frac{x}{\varepsilon})$ is a y periodic function then $f(x,\frac{x}{\varepsilon}) \to \tilde{f}(x)$ weakly in $L^2(\Omega)$. Taking into account (3.3) and passing to the limit with $\varepsilon \to 0$ in (3.2) we obtain that $(\underline{u}^*, \theta^*, H^*)$ verify the equation

$$\frac{\partial}{\partial x_j}\left(c^o_{ijkh}\frac{\partial u^*_k}{\partial x_h}\right) - \left\{\tilde{\beta}_{ij} + \left[\beta_{kh} e_{khy}(\underline{W}^{ij})\right]^\sim\right\}\frac{\partial \theta^*}{\partial x_j} -$$

$$- \left\{\tilde{\alpha}_{ij} + \left[\alpha_{kh} e_{khy}(\underline{W}^{ij})\right]^\sim\right\}\frac{\partial H^*}{\partial x_j} - \tilde{\rho}\, u^{*''}_i = - f_i \qquad (3.4)$$

We denote by p^*_i and q^*_i, respectively, the weak limit in $L^2(\Omega)$ of $k^\varepsilon_{ij}\dfrac{\partial \theta^\varepsilon}{\partial x_j}$ and $d^\varepsilon_{ij}\dfrac{\partial H^\varepsilon}{\partial x_j}$. We introduce the functions

$z_\varepsilon = x_i + \varepsilon w^i(\frac{x}{\varepsilon})$ and $t_\varepsilon = x_i + \varepsilon h^i(\frac{x}{\varepsilon})$, with w^i and h^i solutions of (2.11), (2.12). Using the equations (2.11) and (2.12) with test functions of the form $z_\varepsilon \psi$, respectively $t_\varepsilon \psi$, substracting it from (1.2) and (1.3) and passing to the limit with $\varepsilon \to 0$ we obtain:

$$p_i^* = k_{ij}^o \frac{\partial \theta^*}{\partial x_j} \tag{3.5}$$

$$q_i^* = d_{ij}^o \frac{\partial H^*}{\partial x_j} \tag{3.6}$$

$$\beta_{ij}^\varepsilon e_{ij}(\underline{u}^\varepsilon(\varphi')) \to \mu(\varphi') \text{ weakly in } L^2(\Omega) \tag{3.7}$$

$$\alpha_{ij}^\varepsilon e_{ij}(\underline{u}^\varepsilon(\varphi')) \to \nu(\varphi') \text{ weakly in } L^2(\Omega) \tag{3.8}$$

Now we shall determine $\mu(\varphi)$ and $\nu(\varphi)$. For that we take (2.16) and (2.17) under the global form

$$\int_\Omega c_{ijmn}^\varepsilon e_{mn}(\underline{\Theta}^\varepsilon) e_{ij}(\underline{v})dx = -\int_\Omega \beta_{ij}^\varepsilon e_{ij}(\underline{v})dx \tag{3.9}$$

$$\int_\Omega c_{ijmn}^\varepsilon e_{mn}(\underline{\chi}^\varepsilon) e_{ij}(\underline{v})dx = -\int_\Omega \alpha_{ij}^\varepsilon e_{ij}(\underline{v})dx \tag{3.10}$$

where $\underline{\Theta}^\varepsilon = \varepsilon \underline{\Theta}(\frac{x}{\varepsilon})$ and $\underline{\chi}^\varepsilon = \varepsilon \underline{\chi}(\frac{x}{\varepsilon})$. Now we take $\underline{v} = \underline{\Theta}^\varepsilon \psi$ in (3.2) and $\underline{v} = \underline{u}^\varepsilon \psi$ in (3.9) with $\psi \in C_o^\infty(\Omega)$, substract it and pass to the limit with $\varepsilon \to 0$ (note that $\underline{\Theta}^\varepsilon \to 0$ strongly in $L^2(\Omega)$). After that we take $\underline{v} = \underline{\chi}^\varepsilon \psi$ in (3.2) and $\underline{v} = \underline{u}^\varepsilon \psi$ in (3.10) with $\psi \in C_o^\infty(\Omega)$ and proceed as before. Then we obtain, using (2.21), (2.22), (2.27), (2.28) and (2.29):

$$\mu(\varphi) = \beta^{o}_{ij} e_{ij}(\underline{u}^{*}) - \gamma\,\theta^{*} - \delta\, H^{*} \tag{3.11}$$

$$\gamma(\varphi) = \alpha^{o}_{ij} e_{ij}(\underline{u}^{*}) - \lambda\, H^{*} - \delta\,\theta^{*} \tag{3.12}$$

The equations (3.5), (3.6), (3.11), (3.12), (1.2), (1.3), as $\varepsilon \to 0$, implies:

$$\frac{\partial}{\partial x_j}\left(k^{o}_{ij}\frac{\partial \theta^{*}}{\partial x_j}\right) - T_o\,\beta^{o}_{ij} e_{ij}(\underline{u}^{*\prime}) + T_o\gamma\,\theta^{*\prime} + T_o\delta\, H^{*\prime} - \tilde{c}\,\theta^{*\prime} = -r \tag{3.13}$$

$$\frac{\partial}{\partial x_j}\left(d^{o}_{ij}\frac{\partial H^{*}}{\partial x_j}\right) - H_o\alpha^{o}_{ij} e_{ij}(\underline{u}^{*\prime}) + H_o\lambda H^{*\prime} + H_o\delta\,\theta^{*\prime} - \tilde{b}H^{*\prime} = -h \tag{3.14}$$

The equations (3.4), (3.13), (3.14) having a unique solution and the coefficients being the same of (1.1), (1.2), (1.3) we obtain $(\underline{u}^{*}, \theta^{*}, H^{*}) = (\underline{u}^{o}, \theta^{o}, H^{o})$.

4. CONCLUSION

A composite material subjected to hygrothermomechanic loadings has been investigated. The obtained macroscopic equations and the macroscopic constitutive equation, are of the same type as those obtained by using the classical theory of mechanics of continua [5]. The macroscopic coefficients may be calculated directly by the explicite formula obtained here. In [5] a numerical example was studied, consisting in a composite with graphite fibers in an epoxy matrix. The conclusion of this example was very explicite "the effect of temperature and moisture is significant in the deformation and stress field". That justifie our rigurous mathematical deduction of these equations.

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