INSTITUTUL
DE
MATEMATICA

INSTITUTUL NATIONAL
PENTRU CREATIE
STIINTIFICA SI TEHNICA

ISSN 0250 3638

ON THE VISCOELASTIC BEHAVIOUR OF A POROUS SATURATED MEDIUM

by

Horia ENE

PREPRINT SERIES IN MATHEMATICS

No.40/1985

BUCURESTI

ON THE VISCOELASTIC BEHAVIOUR OF A POROUS SATURATED MEDIUM

by

Horia ENE*)

June 1985

*) The National Institute for Scientific and Technical Creation, Department of Mathematics, Bd.Pacii 220, 79622 Bucharest, ROMANIA.

ON THE VISCOELASTIC BEHAVIOUR OF A POROUS SATURATED MEDIUM

by

Horia ENE

INCREST, Department of Mathematics

Bd.Pacii 220, 79622 Bucharest, Romania

Abstract. Using the homogenization method, we obtain the consitutive equation for a mixture formed by a viscoelastic skeleton and a viscous incompressible fluid. The macroscopic constitutive equation give us the effective stress tensor as the difference between the mean value of the stress tensor in the skeleton and the pore pressure multiplied by the porosity. The motion of the fluid is described by a Darcy's law with memory depending on the pressure gradient and the inertia force. It is deduced also the form of the conservation of mass and momentum.

1. INTRODUCTION

1.1. Generalities.

In the general framework of the homogenization method [1, 2] we consider the problem of the motion of a mixture formed by a viscoelastic skeleton and a viscous incompressible fluid. The geometric distribution of the solid and fluid parts is periodic, with small periods. The dimensions of the periods are then associated with the small parameter ε .

It is well known that a great variety of problems can arise if the orders of magnitude of the coefficients are very different or if the topological properties of the mixture are different [2]. In the case of the vibration of a mixture of an elastic body and a viscous barotropic fluid, it appears that the macrosscopic stress tensor is given also by a viscoelastic law with memory [2, 3], but depending only on the strain tensor.

In our case we consider that the solid part is connected, as well as the fluid one, and that the viscosity of the fluid is small (a slightly viscous fluid). In fact it is well known [2, 4] that Darcy's law hold only in the case of large viscosity and small velocity, or small viscosity and possible large velocity. More precisely, if the smaller magnitude is of order ε^2 than the larger is of order ε^0. As a consequence of the fact that the displacement vector in the solid part is of order ε^0, we take the velocity in the fluid part of the same order and the viscocity of the fluid of order ε^2.

<u>1.2. Mixture of a viscoelastic solid with a viscous fluid.</u>

In the solid part of the mixture, the equations are :

$$\rho_s \frac{\partial^2 u_i}{\partial t^2} - \frac{\partial \sigma^s_{ij}}{\partial x_j} = f_i \tag{1.1}$$

$$\sigma^s_{ij} = a^s_{ijkh}\, e_{kh}(\underline{u}) + b^s_{ijkh}\, e_{kh}\left(\frac{\partial \underline{u}}{\partial t}\right) \tag{1.2}$$

$$e_{kh}(\underline{u}) = \frac{1}{2}\left(\frac{\partial u_k}{\partial x_h} + \frac{\partial u_h}{\partial x_k}\right)$$

where $\underline{f}$ is the exterior body force, and the coefficients a^s_{ijkh}, b^s_{ijkh}, satisfy the usual properties of symmetry and positivity.

$$a^s_{ijkh} = a^s_{jikh} = a^s_{jihk} = a^s_{khij} \tag{1.3}$$

$$a^s_{ijkh}\, e_{ij}\, e_{kh} \geq \alpha\, e_{ij}\, e_{ij} \; ; \quad \alpha > 0 \tag{1.4}$$

and similar relations for b^s_{ijkh}.

In the fluid part, the equations are:

$$\rho_f \frac{\partial^2 u_i}{\partial t^2} - \frac{\partial \sigma^f_{ij}}{\partial x_j} = f_i \tag{1.5}$$

$$(1.6) \quad \sigma^f_{ij} = -p\,\delta_{ij} + \varepsilon^2\,\mu\,\delta_{ik}\,\delta_{jk}\,e_{kh}(\underline{v})$$

$$(1.7) \quad \operatorname{div} \underline{v} = 0, \quad \underline{v} = \frac{\partial u}{\partial t}$$

Moreover, at the interface between the solid and the fluid, we must have the continuity of displacement and stress:

$$(1.8) \quad [\underline{u}] = 0, \qquad [\sigma_{ij} n_j] = 0$$

We must adjoin initial and boundary conditions:

$$(1.9) \quad \underline{u} = 0 \quad \text{on } \partial\Omega$$

$$(1.10) \quad \underline{u} = \frac{\partial u}{\partial t} = 0 \quad \text{for } t = 0$$

where Ω is the domain occupied by the mixture, and is formed by Ω_s and Ω_f.

The variational formulation of the problem (1.1) (1.5) (1.8) (1.9) (1.10) is: find $\underline{u}$, function of t with values in $H^1_0(\Omega)$ such that:

$$(1.11) \quad \int_\Omega \rho \frac{\partial^2 u_i}{\partial t^2} w_i\,dx + a(\underline{u}, \underline{w}) + b\left(\frac{\partial \underline{u}}{\partial t}, \underline{w}\right) - \int_{\Omega_f} p \operatorname{div} \underline{w}\,dx = \int_\Omega \underline{f}\,\underline{w}\,dx \qquad \forall\, \underline{w} \in H^1_0(\Omega)$$

$$(1.12) \quad \operatorname{div} \underline{v} = 0 \text{ in } \Omega_f$$

$$(1.13) \quad a(\underline{u}, \underline{w}) = \int_{\Omega_s} a^s_{ijkh}\, e_{kh}(\underline{u})\, e_{ij}(\underline{w})\,dx$$

$$(1.14) \quad b(\underline{v}, \underline{w}) = \int_\Omega b_{ijkh}\, e_{kh}(\underline{v})\, e_{ij}(\underline{w})\,dx$$

$$(1.15) \quad b_{ijkh} = \begin{cases} b^s_{ijkh} & \text{in } \Omega_s \\ \varepsilon^2\,\mu\,\delta_{ik}\,\delta_{jh} & \text{in } \Omega_f \end{cases}$$

1.3. Two-scale asymptotic process

We consider a parallelipipedic period Y of the space of variables y_i (i=1,2,3) formed by a fluid part Y_f and a solid one Y_s, separated by a smooth boundary Γ. We look for Y-periodic coefficients in the variable $y = \frac{x}{\varepsilon}$: $\rho^\varepsilon(x) \equiv \rho(\frac{x}{\varepsilon})$, $a^\varepsilon_{ijkh}(X) \equiv a_{ijkh}(\frac{x}{\varepsilon})$ and $b^\varepsilon_{ijkh}(X) \equiv b_{ijkh}(\frac{x}{\varepsilon})$.

In order to study the asymptotic process $\varepsilon \to 0$, we assume that the appropiate asymptotic expansion in the solid part is analogons to that of viscoelastic mixture, and in the fluid part analogous to that of flow through porous media. But the first term of the displacement vector expansion in the solid do not depend on y in the solid region. Contrary it does in the fluid region. It is than natural to introduce the relative displacement of the fluid with respect to the solid : $\underline{u}^r(x, y, t) = \underline{u}^0(x, y, t) - \underline{u}^0(x, t)$, and consequently we search for a two-scale asymptotic expansion suitable in $\Omega_{\varepsilon f}$ as well as in $\Omega_{\varepsilon s}$:

(1.16) $\underline{u}^\varepsilon(x, t) = \underline{u}^0(x,t) + \underline{u}^r(x,y,t) + \varepsilon\, \underline{u}^1(x,y,t) + \ldots$

where $y = \frac{x}{\varepsilon}$ and all functions are Y-periodic in y. The vector $\underline{u}^r$ takes values in $H^1(Y)$, is zero on Y_s and on Γ.

For the pressure we have also:

(1.17) $p^\varepsilon(x,t) = p^0(x,t) + \varepsilon p^1(x,y,t) + \cdots$

Now, with standard notation in homogenization theory, for fixed ε, the problem (1.11) (1.12) may be considered for $\underline{u}^\varepsilon$ and p^ε.

2. MACROSCOPIC EQUATIONS

2.1. Balance of mass

If we replace (1.16) into (1.12) we have

(2.1) $\operatorname{div}_y \underline{v}^r = 0$

(2.2) $\operatorname{div}_x (\underline{v}^0 + \underline{v}^r) + \operatorname{div}_y \underline{v}^1 = 0$ in $\Omega_{\varepsilon f}$

Note that (2.2) only hold in the fluid part. But using the fact that

$$\int_Y \operatorname{div}_y \underline{v}^1 dy = \int_{\partial Y} \underline{v}^1 \underline{n}\, ds = 0$$

we take the mean value of (2.2) over Y and we have:

$$(2.3)\quad n\,\mathrm{div}_x\,\underline{v}^0 + \mathrm{div}_x\,\underline{\tilde{v}}^r = \frac{1}{|Y|}\int_{Y_s} \mathrm{div}_y\,\underline{v}^1\,dy;\quad n = \frac{|Y_f|}{|Y|}$$

which is the balance of mass.

After that, taking test functions depending on ε in the form

$$(2.4)\quad \underline{w}(x) = \underline{w}^0(x) + \underline{w}^r(x,y) + \varepsilon\,\underline{w}^1(x,y);\quad \mathrm{div}_y\,\underline{w}^r = 0$$

at order ε^0, from the equation analogons to (1.11), using (1.16) and (1.17) we obtain:

$$(2.5)\quad \int_\Omega \rho^\varepsilon \frac{\partial^2(u_i^0+u_i^r)}{\partial t^2}(w_i^0+w_i^r)\,dx + \int_{\Omega_{\varepsilon s}} a^s_{ijkh}\left(\frac{\partial u_k^0}{\partial x_h}+\frac{\partial u_k^1}{\partial y_h}\right)\left(\frac{\partial w_i^0}{\partial x_j}+\frac{\partial w_i^1}{\partial y_j}\right)dx -$$

$$- \int_{\Omega_{\varepsilon f}} p^0(\mathrm{div}_x\,\underline{w}^0 + \mathrm{div}_x\,\underline{w}^r + \mathrm{div}_y\,\underline{w}^1)\,dx +$$

$$+ \int_{\Omega_{\varepsilon s}} b^s_{ijkh}\frac{\partial}{\partial t}\left(\frac{\partial u_k^0}{\partial x_h}+\frac{\partial u_k^1}{\partial y_h}\right)\left(\frac{\partial w_i^0}{\partial x_j}+\frac{\partial w_i^1}{\partial y_j}\right)dy +$$

$$+ \int_{\Omega_{\varepsilon f}} \mu\frac{\partial}{\partial t}\frac{\partial u_i^r}{\partial y_j}\frac{\partial w_i^r}{\partial y_j}\,dx = \int_\Omega f_i(w_i^0+w_i^r)\,dx$$

2.2. Relative velocity

The relative motion of the fluid may be obtained if we take in (2.5) $\underline{w}^0 = \underline{w}^1 = 0$, $\underline{w}^r = \theta(x)\,\underline{\varpi}\left(\frac{x}{\varepsilon}\right)$, $\theta \in \mathcal{D}(\Omega)$, $\mathrm{div}_y\,\underline{\varpi} = 0$, $\underline{\varpi}$ Y-periodic and zero on Y_s. To this end, it is also useful to modify the coresponding pressure term in (2.5) into $\int_\Omega (\mathrm{grad}_x\,p^0 + \mathrm{gard}_y\,p^1)\underline{w}^r\,dx$. Then (2.5) gives:

$$\int_{\Omega_{\varepsilon f}} \rho_f \frac{\partial^2(u_i^0+u_i^r)}{\partial t^2}\varpi_i\,\theta\,dx + \int_{\Omega_{\varepsilon f}} (\mathrm{grad}_x\,p^0 + \mathrm{grad}_y\,p^1)\,\underline{\varpi}\,\theta\,dx +$$

$$+ \int_{\Omega_{\varepsilon f}} \mu\frac{\partial}{\partial t}\frac{\partial u_i^r}{\partial y_j}\frac{\partial \varpi_i}{\partial y_j}\,\theta\,dx = \int_\Omega f_i\,\varpi_i\,\theta\,dx$$

and for $\varepsilon \to 0$ we have the local problem for the relative velocity:

$$(2.6)\quad \int_{Y_f} \rho_f\left(\frac{\partial v_i^0}{\partial t} + \frac{\partial v_i}{\partial t}\right) \varpi_i\, dy + \int_{Y_f} \frac{\partial p^0}{\partial x_i} \varpi_i\, dy +$$

$$+ \mu \int_{Y_f} \frac{\partial v_i^r}{\partial y_j} \frac{\partial \varpi_i}{\partial y_j}\, dy = \int_{Y_f} f_i\, \varpi_i\, dy$$

If we define the space $V_y = \{\underline{u};\ \underline{u} \in H^1(Y_f),\ \underline{u}|_\Gamma = 0,$ $\mathrm{div}_y \underline{u} = 0$, Y-periodic $\}$ and H_y, the completion of V_y for the norm associated with the scalar product

$$(\underline{u}, \underline{w})_{H_y} = \int_{Y_f} u_i\, w_i\, dy$$

we obtain the evolution problem : find $\underline{v}^r$, function of t with values in V_y, such that :

$$(2.7)\quad \begin{cases} \rho\left(\dfrac{\partial \underline{v}^r}{\partial t}, \underline{\varpi}\right)_{H_y} + \mu(\underline{v}^r, \underline{\varpi})_{V_y} = \\ = \left(f_i - \dfrac{\partial p^0}{\partial x_i} - \rho_f \dfrac{\partial v_i^0}{\partial t}\right) \displaystyle\int_{Y_f} \varpi_i\, dy \qquad \forall\, \underline{\varpi} \in V_y \\ \underline{v}^r(0) = 0 \end{cases}$$

If we introduce the vectors $\underline{\phi}^i\ (i = 1,2,3)$, elements of H_y defined by

$$(2.8)\quad \int_{Y_f} \varpi_i\, dy = (\underline{\phi}^i, \underline{\varpi})_{H_y} \qquad (\forall)\ \underline{\varpi} \in H_y$$

and the selfadjoint operator A_1 of H_y associated by the representation theorem with the form $(\underline{v}, \underline{w})_{V_y}$, (2.7) becomes:

$$(2.9)\quad \begin{cases} \rho \dfrac{\partial \underline{v}^r}{\partial t} + \mu A_1 \underline{v}^r = \left(f_i - \dfrac{\partial p^0}{\partial x_i} - \rho \dfrac{\partial v_i^0}{\partial t}\right) \underline{\phi}^i \\ \underline{v}^r(0) = 0 \end{cases}$$

The solution of (2.9), by standard semigroup theory, is:

$$(2.10)\quad \underline{v}^r(t) = \rho^{-1} \int_0^t e^{-\rho^{-1} \mu A_1 (t-s)}\, \underline{\phi}^i \left(f_i - \frac{\partial p^0}{\partial x_i} - \rho \frac{\partial v_i^0}{\partial t}\right)_{(s)} ds$$

Taking the mean value of (2.10) we have the macroscopic relative velocity :

(2.11) $\tilde{v}^r_k(t) = \int_0^t g_{ki}(t-s)\left(f_i - \frac{\partial p^0}{\partial x_i} - \rho \frac{\partial v^0_i}{\partial t}\right)(s)\,ds$

(2.12) $g_{ki}(\xi) \equiv \rho^{-1}\left(e^{-\rho^{-1}\mu A_1 \xi}\, \underline{\phi}^i, \underline{\phi}^k\right)_{H_Y}$

<u>Remark 2.1.</u> (2.10) give $\underline{v}^r(t)$ as a functional of exterior body forces, gradient presure and inertia term. The mean value (2.11) contain a well-defined function of ξ, $g_{ki}(\xi)$ which decrease exponentially as $\xi \to \infty$, and $g_{ki} = g_{ik}$. The proof is similar as in the case of acoustics in porous media [2] .

<u>2.3. Stress tensor</u>

In order to study the local state in the solid, we take in (2.5) $\underline{w}^0 = \underline{w}^r = 0$, $\underline{w}^1 = \theta(x)\,\underline{\omega}(x,y)$, $\theta \in \mathcal{D}(\Omega)$, $\underline{\omega}$ Y-periodic. In the same way, the asymptotic process $\varepsilon \to 0$, give us:

$$(2.13) \quad \int_{Y_s} \left(a^s_{ijkh} + b^s_{ijkh}\frac{\partial}{\partial t}\right)\frac{\partial u^1_k}{\partial y_h}\frac{\partial \omega_i}{\partial y_j}\,dy + \int_{Y_s} a^s_{ijkh}\frac{\partial u^0_k}{\partial x_k}\frac{\partial \omega_i}{\partial y_j}\,dy + \int_{Y_s} b^s_{ijkh}\frac{\partial}{\partial t}\frac{\partial u^0_k}{\partial x_h}\frac{\partial \omega_i}{\partial y_j}\,dy + p^0\int_{Y_s}\delta_{ij}\frac{\partial \omega_i}{\partial y_j}\,dy = 0$$

Note that $p^0(x,t)$ is defined in Ω (does not depend on y). In fact we continue p^ε in the solid part, with the periodicit condition, and we use that $\int_Y \mathrm{div}_y\,\underline{\omega}\,dy = 0$. If we introduce the space $\tilde{V}_y$ of functions from $H^1(Y_s)$ with zero mean value and the scalar product

$$(2.14) \quad (\underline{u}, \underline{v})_{\tilde{V}_y} = \int_{Y_s} b^s_{ijkh}\frac{\partial u_k}{\partial y_h}\frac{\partial v_i}{\partial y_j}\,dy$$

and $A_2 \in \mathcal{L}(\tilde{V}_y, \tilde{V}_y)$, $\underline{m}^{kh} \in \tilde{V}_y$, $\underline{n}^{kh} \in \tilde{V}_y$, $\underline{\psi} \in \tilde{V}_y$ by:

$$(2.15)\quad (A_2\, \underline{u}^1, \underline{\omega}) = \int_{Y_s} a^s_{ijkh} \frac{\partial u^1_k}{\partial y_h} \frac{\partial \omega_i}{\partial y_j}\, dy$$

$$(2.16)\quad (\underline{m}^{kh}, \underline{\omega})_{\tilde{V}_y} = \int_{Y_s} a^s_{ijkh} \frac{\partial \omega_i}{\partial y_j}\, dy$$

$$(2.17)\quad (\underline{n}^{kh}, \underline{\omega})_{\tilde{V}_y} = \int_{Y_s} b^s_{ijkh} \frac{\partial \omega_i}{\partial y_j}\, dy$$

$$(2.18)\quad (\underline{\psi}, \underline{\omega})_{\tilde{V}_y} = \int_{Y_s} \delta_{ij} \frac{\partial \omega_i}{\partial y_j}\, dy$$

the relation (2.13) is equivalent to :

$$(2.19)\quad \left(\frac{\partial \underline{u}^1}{\partial t} + A_2 \underline{u}^1 + \underline{m}^{kh} \frac{\partial u^0_k}{\partial x_h} + \underline{n}^{kh} \frac{\partial}{\partial t} \frac{\partial u^0_k}{\partial x_h} + p^0 \underline{\psi}\ , \underline{\omega}\right)_{\tilde{V}_y} =$$

$$= 0 \quad \forall\ \underline{\omega} \in \tilde{V}_y$$

Thus the first factor in (2.19) must be zero.

$$(2.20)\quad \begin{cases} \dfrac{\partial \underline{u}^1}{\partial t} + A_2 \underline{u}^1 = -\underline{m}^{kh} \dfrac{\partial u^0_k}{\partial x_h} - \underline{n}^{kh} \dfrac{\partial}{\partial t} \dfrac{\partial u^0_k}{\partial x_h} - p^0 \underline{\psi} \\[2ex] \underline{u}^1(0) = 0 \end{cases}$$

The solution of (2.20) is:

$$(2.21)\quad \underline{u}^1 = -\underline{n}^{kh} \frac{\partial u^0_k}{\partial x_h} + \int_0^t e^{-A_2(t-s)} \left(\underline{r}^{kh} \frac{\partial u^0_k}{\partial x_h} - p^0 \underline{\psi}\right)(s)\, ds$$

$$(2.22)\quad \underline{r}^{kh} = A_2\, \underline{n}^{kh} - \underline{m}^{kh}$$

$$(2.23)\quad \frac{\partial \underline{u}^1}{\partial t} = -\underline{n}^{kh} \frac{\partial}{\partial t} \frac{\partial u^0_k}{\partial x_h} + \underline{r}^{kh} \frac{\partial u^0_k}{\partial x_h} - p^0 \underline{\psi} -$$

$$- \int_0^t A_2 e^{-A_2(t-s)} \left(\underline{r}^{kh} \frac{\partial u^0_k}{\partial x_h} - p^0 \underline{\psi}\right)(s)\, ds$$

Remark 2.2. The right hand side of the equation (2.3) is well defined as function of $\underline{u}^0$ and p^0.

The macroscopic stress tensor is defined as the mean value of

$$(2.24)\quad \sigma^0_{ij} = (a^s_{ijkh} + b^s_{ijkh}\ \frac{\partial}{\partial t})\ (\frac{\partial u^0_k}{\partial x_h} + \frac{\partial u^1_k}{\partial y_h})$$

If we introduce the coefficients and the functions:

$$(2.25)\quad \alpha^0_{ijkh} = \Big[a_{ijkh} - a_{ijep} \frac{\partial}{\partial y_p} (\underline{n}^{kh})_e + $$

$$+ b_{ijep} \frac{\partial}{\partial y_p} (\underline{r}^{kh})_e \Big]^{\sim}$$

$$(2.26)\quad \alpha^1_{ijkh} = \Big[b_{ijkh} - b_{ijep} \frac{\partial}{\partial y_p} (\underline{n}^{kh})_e \Big]^{\sim}$$

$$(2.27)\quad \alpha^2_{ij} = \Big[b_{ijkh} \frac{\partial \psi_k}{\partial y_h} \Big]^{\sim}$$

$$(2.28)\quad g_{ijkh}(\xi) = \Big[a_{ijep} \frac{\partial}{\partial y_p} (e^{-A_2 \xi} \underline{r}^{kh})_e -$$

$$- b_{ijep} \frac{\partial}{\partial y_p} (A_2\ e^{-A_2 \xi}\ \underline{r}^{kh})_e \Big]^{\sim}$$

$$(2.29)\quad g^*_{ij}(\xi) = \Big[b_{ijep} \frac{\partial}{\partial y_p} (A_2\ e^{-A_2 \xi}\ \psi)_e -$$

$$- a_{ikep} \frac{\partial}{\partial y_p} (e^{-A_2 \xi}\ \psi)_e \Big]^{\sim}$$

we have

$$(2.30)\quad \tilde{\sigma}^0_{ij} = \alpha^0_{ijkh}\ e_{kh}(\underline{u}^0) + \alpha^1_{ijkh}\ e_{kh}(\frac{\partial \underline{u}^0}{\partial t}) -$$

$$- \alpha^2_{ij} p^0 + \int_0^t g_{ijkh}(t-s)\ e_{kh}(\underline{u}^0)(s)ds +$$

$$+ \int_0^t g_{ij}(t-s) p^0(s) ds$$

<u>Remark 2.3.</u> The constitutive equation (2.30) contains an elastic term α^0, a viscoelastic term with instantaneous memory α^1, a pressure term α^2, and two terms with long memory g, g^*, functions of strain and pressure. Because A_2 is a positive defined operator, $g(\xi)$ decays exponentially

for $\xi \to \infty$ (also for $g^*(\xi)$). The strain stress law (2.30) is very different than (1.2).

2.4. Balance of momentum.

Now it is easy to obtain the balance of momentum. For this we take in (2.5) $\underline{w} = \underline{w}^1 = 0$. Then, for $\varepsilon \to 0$ we have:

$$(2.32) \quad \int_\Omega \tilde{\rho} \frac{\partial^2 (u_i^0 + u_i^r)}{\partial t^2} w_i^0 dx + \int_\Omega \tilde{\sigma}^0_{ij} \frac{\partial w_i^0}{\partial x_j} dx -$$

$$-n \int_\Omega p^0 \operatorname{div} w^0 dx = \int_\Omega f_i w_i^0 dx \qquad \forall \; \underline{w}^0 \in H_0^1(\Omega)$$

$$\int_\Omega \tilde{\sigma}^0_{ij} \frac{\partial w_i^0}{\partial x_j} dx - \int_\Omega n\, p^0 \operatorname{div} \underline{w}^0 dx = -$$

$$- \int_\Omega \frac{\partial}{\partial x_j} (\tilde{\sigma}^0_{ij} - n\, p^0 \delta_{ij}) w_i^0 dx = - \int_\Omega \frac{\partial \sigma^T_{ij}}{\partial x_j} w_i^0 dx$$

$$(2.32) \quad \sigma^T_{ij} = \tilde{\sigma}^0_{ij} - n\, p^0 \delta_{ij}$$

$$(2.33) \quad \tilde{\rho} \frac{\partial^2 u_i^0}{\partial t^2} + \rho_f \frac{\partial v_i^r}{\partial t} - \frac{\partial \sigma^T_{ij}}{\partial x_j} = f_i$$

Remark 2.4. σ^T_{ij} is the effective (or total) stress tensor [5,6]. In the same time (2.32) prove that in the effective stress tensor appears the pore pressure multiplied by the porosity.

3. CONCLUSION.

The macroscopic (or homogenized) motion of the mixture may be described by the displacement vector in the solid $\underline{u}^0(x,t)$, the pore pressure in the fluid $p^0(x,t)$ and the mean value of the relative velocity $\tilde{\underline{v}}^r$. These quantites satisfy the equations (2.3) conservation of mass, (2.11), Darcy's law and (2.33), conservation of momentum, the effective stress tensor being defined by (2.32) and (2.30).

The coefficients α^0_{ijkh} and α^1_{ijkh} and the function g_{ijkh}

are the same as in the case of homogenization in viscoelasticity [2]. In the particular case of an elastic skeleton our results reduce to those obtained in [2], but the conservatio of mass is different. In fact it was proved that a Darcy's law of the from (2.11) is not only a consequence of the ~~compressibi~~ compressibility of the fluid. Also in the case of incompressible fluid it appears as valid.

REFERENCES

1 A. Bensoussan, J.L.Lions and G.Papanicolaou, Asymptotic Analysis for Periodic Structure , North-Holland, Amsterdam (1978).

2 E. Sanchez - Palencia, Topics in Non-Homogeneous Media and Vibration Theory, Lecture Notes in Physics 127, Springer-Verlag Berlin (1980)

3 Th. Levy, Intern.J. Enging. Sci., 17, p.1005-1014 (1979)

4 H.I.Ene et E.Sanchez-Palencia, Jour.Mecan. 14, p.73-108 (1975)

5 M.A. Biot, Indiana Univ. Math.J. 21(7), p.597(1972)

6 M.A.Biot, J.Geoph.Res., 78(23), p.4924 (1973).